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Risk taking under heterogenous revenue sharing

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ABSTRACT

We examine the impact of informal risk sharing on risk taking incentives when transfers are organized through a social network. A *bilateral partial sharing rule* satisfies that neighbors share equally a part of their revenue. In such a society, correlated technologies generate interdependent risk levels. We obtain three findings. First, there is a unique and interior Nash-equilibrium risk profile, and it is in general differentiated and related to the Bonacich measure of the risk sharing network. Second, more revenue sharing enhances risk taking on average, although some agents may lower their risk level. Last, we find that under investment might often be observed.

1. Introduction

The adoption of new technologies in rural economies raises serious policy concern, since low rates of adoption potentially contribute to poverty traps. Actually, innovation adoption is risky, and many factors discourage risky investments. On the one hand, some imperfections in the access of credit market are due to asymmetric information (adverse selection, ex ante moral hazard) and imperfect enforcement (ex post moral hazard).[1] On the other hand, there also exist limitations in the demand for innovation, due to risk-aversion. Farmers fear the consequences of bad events, and the effect of perceived risk on behaviors in developing economies has been widely documented.[2]

Risk aversion creates demand for insurance. Whereas markets for insurance exist in developed countries, developing villages often have no formal institution to make insurance mechanisms operational. In such a context, households often set up informal insurance mechanisms.[3] In general, facing income fluctuations, risk averse agents should put all their income in a common pool and share the pool equally. However, the empirical literature has stressed that villagers do not proceed to full equal sharing of incomes.[4] Rather, social networks are channels for informal risk-sharing, and relevant networks are often composed of relatives and friends. These networks are generally not completely connected, and that agents occupy asymmetric positions on the network.[5] This nonanonymity makes transfers heterogenous: the shares of revenues transferred to neighbors may differ across households; two households facing the same adverse shock may not receive the same amount of transfers from a common neighbor. Many reasons explain why full equal sharing at the village level is not implemented. Households often use historical social networks as channels for informal insurance. Furthermore, moral hazard or self-enforcement issues can explain that two neighbors do not share equally their whole income, choosing rather to reduce informal insurance in order to restore individual

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[1] For surveys on the issue, see Conning and Udry (2005), Ghosh et al. (2000).

[2] Sandmo (1971) suggests that risk aversion leads to under-investment and underproduction. Dreze and Modigliani (1972) show that, absent insurance markets, risk-averse farmers should concentrate their portfolio in safe assets, even if the return is lower. Related empirical literature contains Dercon and Christiaensen (2011), Fafchamps et al. (1998), Kazianga and Udry (2004), or Hill (2005).

[3] On the competition between informal and formal insurance, see Giné et al. (2010) or Rosenzweig and Wolpin (1993).

[4] See Rosenzweig (1988), Townsend (1994), Udry (1994).

[5] In Fafchamps and Lund (2003), each household of some village of rural Philippine listed, as number of individuals on which it could rely in case of need or to whom the respondent gives help when called upon to do so, 4.6 individuals on average, with a minimum of 1 and a maximum of 8. In Dercon and De Weerdt (2006), households of Tanzanian village mentioned between 2 and 22 intra-village network partners in their interviews, with a mean equal to 6.5.

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incentives to produce effort or to enforce the contract. Other reasons explain that informal insurance is heterogenous across households, like heterogeneity in trust, in correlated incomes, etc.

The risk sharing network allows agents to obtain insurance against income fluctuations. This creates a diversification effect, which encourages them to take risk. Now, when agents' incomes are positively correlated, the risk sharing network makes agents exposed to the risk of their neighbors. Furthermore, when one agent increases her risk, this also increases the risk of the transferred income to her neighbors. As a consequence, the covariance between transferred revenue and neighbors' revenues increases. Then, neighbors would react by reducing their risk. That is, in presence of positive correlations, risk levels are strategic substitutes. The strategic interaction aspect of risk taking behaviors has not been addressed in the literature. This paper analyzes the impact of the structure of the risk sharing network on risk taking incentives of homogenous-type agents. It shows that differentiated risk levels may arise among homogenous-type agents, as the result of the geometry of the risk sharing network.

To proceed formally, we consider a society of risk averse agents (like farmer or entrepreneurs) with mean–variance utility function. Each agent has one divisible unit to invest in a project through a portfolio of two technologies. One is risk-free, the other is more profitable but risky. The returns of the risky technology are positively correlated across projects. Each agent chooses the share to invest in the risky technology, which we interpret as individual level of risk. After income realizations, agents proceed to transfers. For instance, the risky technology can be interpreted as an innovation like a fertilizer, or a new crop variety, and the agent can be a farmer who experiment the innovation on a part of her land. Given the abstractness of the model, the risky technology may also represent an investment in entrepreneurial activities, in human capital as the level of school of children, etc. Modeling risk sharing on networks is hardly compatible with optimal transfers.[6] We opt for a simple approach, in which transfers are state-contingent and possibly heterogeneous, taking into account the social network, but do not depend on the realizations of third parties. More precisely, we consider a simple set of bilateral risk sharing rules adapted to transfers on social networks, that we call *bilateral partial sharing rules*. Start with a social network describing a set of bilateral relationships, which can be used to share risk. Considering any pair of neighbors, the rule expresses that both partners put the same fixed share of revenue in a bilateral common pool and that they share the pool equally. A bilateral partial sharing rule can be represented as a network of exchanges, in which the value of the connection *ij* is the share of agent *j*'s revenue that she gives to agent *i*. We call own share the share of revenue that each individual keeps for herself. Importantly, a bilateral partial sharing rule satisfies that every own share exceeds one half.

Through transfers, agents are exposed to the risk of their neighbors' revenues. A crucial ingredient of our model is the existence of correlations between risky investments across projects. Positive correlations generate strategic substitutability, that is, the return of a marginal increase of individual risk level is a decreasing function of the risk level of neighbors.

To study the impact of revenue sharing on risk taking, we start by the case of homogenous own shares. We show that equilibrium risk levels are homogenous, and decreasing in the value of own shares. That is, more revenue sharing enhances risk taking. In particular, agents take more risk than under autarky regime. The economic intuition is as follows. Two factors shape incentives to take risk. First, when agents exchange more, they are less exposed to own risk, thus they increase risk. Second, when neighbors increase risk, this pushes toward a reduction of own risk because of strategic substitutability. Actually, when own shares are identical, the first effect always dominates.

Then, we pursue the analysis with the case of heterogenous own shares, which entails asymmetric interaction. We show the existence of a unique and interior equilibrium. In particular, risk levels are differentiated and related to the structure of transfers in the society. Individual risk is an affine function of a Bonacich measure defined over a slight transformation of the network of transfers.[7] From a theoretical perspective, we give news conditions on the matrix of interaction (which contains a heterogenous diagonal) to obtain a unique and interior solution exists. We show indeed that row-stochasticity of the matrix of exchanges plus diagonal dominance of the matrix of interaction is sufficient condition.

Then, we address some comparative analysis with regard to the volume of transfers. We show that more revenue sharing enhances aggregate risk taking. This does not prevent *some* agents to decrease their risk as a response to more revenue sharing. Technically, our comparative statics is original. Indeed, the traditional exercise consists in exerting a perturbation that raises cross-effects in a context of symmetric interaction. In opposite, our perturbations, interpreted as an increase in the volume of transfers, are such that the diagonal elements of the interaction matrix vary in opposite direction with off-diagonal elements. To our knowledge, this paper is the first to undertake such a comparative static exercise under asymmetric interaction. To obtain our results, we use Farkas' lemma.

Last, we explore efficiency issue. Our game exhibits both positive and negative externalities. This arises from a simple tradeoff: when some agent increases investment in the risky technology, this raises both the expected return and the variance of the future transfer to her neighbors. We characterize the efficient risk profile as an affine function of a Bonacich measure defined over a network which aggregates all externalities. At equilibrium, agents may either under invest or over invest. We show that, when efficient allocation is positive, under-investment prevails on average. Then, we compare the sum of utilities of the efficient outcome to the sum of utilities at equilibrium, and we measure the difference between aggregate efficiency and equilibrium as the difference of aggregate respective allocations plus a quadratic term that takes into account correlations and transfers. Numerical simulations suggest that inefficiency decreases with correlation, and it appears to be nonnegligible for correlation roughly lower than one half.

We extend the analysis to more general sharing rules. A *group partial sharing rule* is built as follows. We consider a set of groups of neighbors, possibly overlapping. The rule expresses that all agents in a same group put the same fixed share of revenue in a common pool and that they share the pool equally. One major implication of this generalization is that own shares can be lower than one half (in fact own shares exceed the inverse of the size of the largest group). In this enlarged setting, the preceding results hold under the diagonal dominance of the linear first order conditions. Relaxing diagonal dominance has strong implication. First, multiple corner equilibria may arise. Second, average risk taking can be lower under increased volume of transfers, due to high intensity of interaction.[8] We insist

[6] In principle, contracts should take into account the realizations of third parties. Asymmetric information is then a matter.

[7] The Bonacich centrality measure, reminiscent of Katz (1953), was re-introduced in Bonacich (1987). Ballester et al. (2006) renewed the idea in the field of economics. Some additional insights in the theory are given in Ballester and Calvó-Armengol (forthcoming) and Bramoullé et al. (2010).

[8] That risk taking decisions can be lowered under increased insurance can also admit different explanations. For instance, Giné and Yang (2009) find a negative relationship between formal insurance contract and risk-taking, in the context of adoption of high-yield hybrid varieties of maize and groundnut among smallholder farmers in Malawi. The suggested explanation is that, absent formal insurance, there is an implicit insurance by the limited liability inherent to the loan contract, so that bundling a loan with formal insurance (for which an insurance premium is charged) is effectively an increase of the interest rate of the loan. Moreover, Fischer (2010) documents that microcredit, and especially joint liability, discourages risky investment choice, even under additional presence of informal risk-sharing.

that without heterogeneity in own shares, strategic interaction cannot produce such consequences.

1.1. Related literature

A recent theoretical literature about revenue sharing in developing economies examines the formation of risk-sharing networks. Bramoullé and Kranton (2006, 2007) examine the formation of risk-sharing networks under equal revenue sharing, and discuss stability/efficiency dilemma of the social network. Given that real world does not exhibit full equal sharing, some papers explain partial risk-sharing by self-enforcing mechanisms on networks (Ambrus et al., 2007; Bloch et al., 2008). These models consider contracts shaped by social norms. Hence, transfers are not optimal, and possibly heterogenous. These models relate the maximal volume of transfers that forbids hold up problems to network properties. The social network is then endogenous, but the rules that shape transfers on these links are kept exogenous. With regard to this literature, we let both the transfer rules and the social network exogenous and make revenues endogenous, by incorporating risk taking decisions.

The empirical literature on risk-sharing in village economies has emphasized some features of informal insurance. First, Townsend (1994) rejects the full equal sharing hypothesis in Indian villages.[9] Second, the importance of social networks as relevant channels for informal insurance has been attested, opening the scope for transfers' heterogeneity. Rosenzweig (1988) and Udry (1994) documented that the majority of transfers takes place only between neighbors and relatives. More recently, some works have confirmed this finding by collecting the whole social network in villages (Dercon and De Weerdt. 2006; De Weerdt and Fafchamps, 2007; Fafchamps and Gubert, 2007; Fafchamps and Lund, 2003). These works suggest that households share risk within confined networks of family and friends. Importantly, the identity based nature of networks of transfers indicates that they are presumably not formed for the unique objective of sharing revenue. Furthermore, social networks can be channels for information transmission on defectors of informal agreements (Bloch et al., 2008), or can be a conduct for social learning in adoption of innovation in developing countries (see Bandiera and Rasul, 2006; Conley and Udry, 2010; Duflo et al., 2006; Foster and Rosenzweig, 1995; Munshi, 2004). In a word, these networks are at least partially exogenous to optimal contracting decisions.[10] Coherent with the recent empirical literature, our model assumes that the network of transfers is exogenous to agents' decisions. To describe transfers, we present a simple linear sharing rule that incorporates transfers' heterogeneity.

This paper is also related to a literature on risk-taking. In many economic contexts, a redistribution of incomes in a society of risk averse agents enhances risk taking incentives. For instance, in labor markets, unemployment insurance encourages workers to seek higher productivity jobs because they are more willing to endure the possibility of unemployment (Acemoglu and Shimer, 1999, 2002). Similarly, redistributive taxation can enhance entrepreneurship (Boadway et al., 1991; Garcia-Penalosa and Wen, 2008; Kanbur, 1981; Mayshar, 1977; Sinn, 1996). The economic intuition behind this result is that redistribution reduces agents exposure to individual risk. Of particular interest is the recent paper of Angelucci et al. (2010). The authors document that in the context of village economies, having family ties guarantees more insurance and thus more investment (in terms of education of children). With regard to this literature, our model incorporates strategic interaction in risk-taking decisions.

Last, this paper is related to the theoretical literature on Bonacich centrality. This measure, reminiscent of Katz (1953), was re-introduced in Bonacich (1987). Ballester et al. (2006) renewed the idea in the field of economics. Some additional insights in the theory are given in Ballester and Calvó-Armengol (forthcoming) and Bramoullé et al. (2010). Technically, our paper is not a pure application of the existing theory, and we contribute twice. Our model is a game of strategic substitutes with asymmetric interaction and heterogenous constant. Concerning the characterization of equilibrium, we give new conditions under which there exists a unique and interior solution. Concerning comparative statics on the intensity of interaction, the traditional exercise consists in exerting a perturbation that raises cross-effects in a context of symmetric interaction. Our setting including asymmetric interaction, we use a tool adapted to this context, i.e. we use Farkas' lemma.

The article is organized as follows. Section 2 builds up a model of risk taking under informal revenue sharing. It introduces bilateral partial sharing rules. Section 3 studies the impact of revenue sharing on risk taking. We first focus attention on societies with homogenous own shares, and then we study societies with heterogenous own shares. We analyze Nash equilibria of the game, offer some comparative statics, and examine efficiency issue. Section 4 extends the analysis to group partial sharing rules. Section 4.3 concludes. All proofs are presented in the appendix.

2. The model

The society contains a finite set $N=\{1, 2, \cdots, n\}$ of risk averse agents. We consider a game in which, first, agents invest in risky projects, second, incomes are realized, and third, agents make transfers.

2.1. Investments and revenues before transfers

Each agent has one unit of a divisible resource, that she can invest in a combination of two technologies A and B. Technology B is risk-free and has a return normalized to 1. Technology A is more profitable but also more risky. In particular, the return y^A of technology A is random with expected mean $\mu>1$ and variance σ^2. Let $\rho\sigma^2$ be the covariance between the returns of two distinct projects that use technology A, with $\rho\in]0, 1[$. Here, correlations are related to systematic risks, which stem from the common factors shaping the return of the risky technology across projects. Let x_i be the amount of resource that agent i invests in the risky technology. We interpret x_i as the level of risk chosen by agent i. We do not allow for short-selling the risky technology, therefore $x_i\geq 0$. Further, we do not impose $x_i\leq 1$, allowing agents to borrow at zero interest rate if $x_i>1$.

For example, agents can be farmers, the resource would be land. We may think of technology B as the current technology (crop, fertilizer) of the farmer, while technology A may represent an innovation, like a new fertilizer or a new crop variety. Risk taking can then be interpreted as the rate of adoption of the new technology. Since land is divisible, this situation may fit with farmers experimenting the innovation on some sub-area of their land's surface.[11] In that situation, farmer allocate different plantations on a finite spatial resource, and we shall impose $x_i\in[0, 1]$. We will see later on that a sufficient condition for obtaining $x_i^*\leq 1$ is $\frac{\mu-1}{\sigma^2}\leq\kappa\rho$, i.e. the Sharp ratio is lower than the product of the correlation parameter and the coefficient of risk aversion.

[9] Some works have proposed as a possible explanation to this finding that limited commitment may be due to enforcement issues (Coate and Ravallion, 1993; Dubois et al., 2008; Ligon et al., 2001), or to moral hazard issues.

[10] However, many factors related to risk issue may explain why revenue sharing is heterogenous across households. To cite a few, self-enforcing mechanisms and trust (social sanctions may be heterogenous), heterogeneity in information flows, in income correlations, in geographic costs, increasing costs to group size (see Murgai et al., 2002).

[11] "Much agricultural technology is divisible. This is particularly true for much Green Revolution type technology, such as improved seeds, chemical fertilizer, and pesticides. This dramatically reduces the risk associated with farmer experimentation since it is fairly easy to try out a new technology on a small scale before adopting it on the whole farm." (Fafchamps, 2010)

For every agent i, let r_i^b denote the revenue of agent i before transfer. When agent i invests x_i in technology A and $1-x_i$ in technology B, her revenue before transfer is given by:

$$r_i^b(x_i) = (1-x_i)y^B + x_i y^A \tag{1}$$

The expected revenue of agent i before transfer is $E(r_i^b(x_i)) = (1-x_i) + x_i\mu$, while the variance of agent i's revenue is $Var(r_i^b(x_i)) = x_i^2\sigma^2$.

2.2. Sharing risks on networks

To reduce income fluctuations, agents share part of their realized incomes with their neighbors.[12] As said in the Introduction, the empirical literature on risk sharing in village economies has stressed the importance of social networks as relevant channels for informal insurance has been attested, opening the scope for transfers' heterogeneity. These networks are at least partially exogenous to optimal contracting decisions.

Coherent with this recent empirical literature, our model assumes that the social network that supports transfers is exogenous to agents' decisions. Now, for a given network of neighbors, the absence of some links between agents is hardly compatible with optimal contracts, which require verification of all realizations. Typically, agents may observe the realizations of their neighbors, but not the ones of the neighbors of their neighbors. This may create asymmetric information problems, preventing potentially optimal contracts. We therefore opt for a simple approach, in which transfers are state-contingent and possibly heterogeneous, but do not depend on the realizations of third parties. We consider a simple set of bilateral risk sharing rules adapted to transfers on social networks, that we call *bilateral partial sharing rules*. Considering any pair of neighbors, such a rule expresses that both agents put the same fixed share of revenue in a bilateral common pool and then they share the pool equally.

Formally, consider an exogenous network of social neighbors. Each pair of neighbors (i,j) puts the same share $\alpha_{ij}\in[0,1]$ of her revenue in a bilateral common pool and both agents share the pool equally. Two agents i, j that are not neighbors cannot share revenue (meaning $\alpha_{ij}=0$). Let r_i^a represent agent i's revenue after the realization of transfers. After-transfer revenues are written:

$$r_i^a = \left(1-\sum_{j\neq i}\alpha_{ij}\right)r_i^b + \sum_{j\neq i}\alpha_{ij}\cdot\left(\frac{r_i^b + r_j^b}{2}\right)$$

under the condition that $\sum_{j\neq i}\alpha_{ij}\leq 1$. Rearranging, we obtain

$$r_i^a = \lambda_{ii}r_i^b + \sum_{j\neq i}\lambda_{ij}r_j^b \tag{2}$$

with $\lambda_{ii} = \left(\frac{1}{2} + \frac{1-\sum_{j\neq i}\alpha_{ij}}{2}\right)$ and $\lambda_{ij} = \frac{\alpha_{ij}}{2}$. Hence, the rule is equivalent to considering linear transfers between two neighbors i, j. A bilateral partial sharing rule can be seen as an exchange, between each pair of neighbors i, j, of a share λ_{ij} of their revenue, and can be represented by a symmetric *matrix of exchanges* $\Lambda = [\lambda_{ij}]$. The term $\lambda_{ii} = 1-\sum_{j\neq i}\lambda_{ij}$ represents agent i's *own share*, i.e. the share of her revenue that she holds after transfer. By construction, $\lambda_{ii} + \sum_{j\neq i}\lambda_{ji} = 1$ for all i; then symmetry implies that matrix Λ is bi-stochastic. Since $\lambda_{ii} = \frac{1}{2} + \frac{1-\sum_{j\neq i}\alpha_{ij}}{2}$, basically $\lambda_{ii}\geq\frac{1}{2}$ for all i (we explore in Section 4 sharing rules that encompass lower own shares). Note that, defining $t = \frac{\alpha_{ij}\left(r_i^b - r_j^b\right)}{2}$, if $t>0$, agent i transfers quantity t to agent j, otherwise she receives quantity $-t$ from agent i.

To illustrate, Fig. 1– left presents a three-agent star network. In this example, there are two bilateral common pools. Agents 1 and 2 put $\alpha_{12} = \frac{1}{5}$ in a bilateral common pool, and agents 2 and 3 put $\alpha_{23} = \frac{2}{5}$ in another bilateral common pool. Agents 1 and 3 are not neighbors, therefore they do not share revenue. This generates the matrix of exchanges described in the right figure (Fig. 1– right).

Let $X = (x_1, x_2, \cdots, x_n)$ be the profile of risk, and $x = \sum_i x_i$ the aggregate risk level. Taking into account that before-transfer revenues are shaped by risks as in Eq. (1), and considering the transfers generated by the bilateral partial sharing rule as in Eq. (2), after-transfer revenues are expressed in terms of the profile of risk choice:

$$r_i^a(X) = \lambda_{ii}r_i^b(x_i) + \sum_{j\neq i}\lambda_{ij}r_j^b\left(x_j\right) \tag{3}$$

2.3. Ex ante utilities

Agents are risk averse. Let r be the uncertain individual income of agent i. We consider mean–variance utility[13]:

$$u_i(r) = E(r) - \frac{\kappa}{2}Var(r) \tag{4}$$

with $\kappa>0$ denoting the coefficient of individual risk aversion. Plugging after-transfer revenue as described in Eqs. (3) in (4), individual expected utility writes as:

$$u_i(X) = \sum_{j=1}^{n}\lambda_{ij}E(r_j^b(x_j)) - \frac{\kappa}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{ij}\lambda_{ik}cov(r_j^b(x_j), r_k^b(x_k))$$

with, letting symbol I stand for the indicator function, $cov\left(r_j\left(x_j\right), r_k(x_k)\right) = \sigma^2\cdot x_j x_k\left(I_{\{j=k\}} + \rho\cdot I_{\{j\neq k\}}\right)$. That is,

$$u_i(X) = 1 + (\mu-1)\sum_{j=1}^{n}\lambda_{ij}x_j - \frac{\kappa\sigma^2}{2}\sum_{j=1}^{n}\lambda_{ij}^2x_j^2 - \frac{\kappa\rho\sigma^2}{2}\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}\lambda_{ij}\lambda_{ik}x_jx_k.$$

Assuming that both investment decisions and individual realizations are observable by neighbors, we analyze Nash equilibria. Formally, a profile X^* is a (pure) Nash equilibrium if it satisfies that, for all i, for all $x_i\geq 0$, $u_i(x_i^*, x_{-i}^*; \Lambda)\geq u_i(x_i, x_{-i}^*; \Lambda)$. Let $h = \frac{\mu-1}{\kappa\sigma^2}$. This quantity corresponds to the equilibrium level of risk taken by agent in autarky. We assume that $h\leq 1$.[14]

The system of first order equations is written:

$$\begin{cases} \lambda_{ii}x_i^* + \rho\sum_{j\neq i}\lambda_{ij}x_j^* = h & \text{if } h\geq\rho\sum_{j\neq i}\lambda_{ij}x_j^* \\ x_i^* = 0 & \text{if } h<\rho\sum_{j\neq i}\lambda_{ij}x_j^* \end{cases} \tag{5}$$

We define the matrix Λ_ρ, with diagonal elements λ_{ii} and off-diagonal elements $\rho\lambda_{ij}$, as the matrix of interaction. Indeed, the first equation in system (5), written $\Lambda_\rho X = h\mathbf{1}$, shows that strategic interaction emerges from correlations between projects. Moreover, since $\rho>0$, individual risk levels are *strategic substitutes*, i.e. the marginal return of an increase of the level of risk is decreasing in the level of risk chosen by neighbors. The following potential function is associated with our game (see Monderer and Shapley, 1996):

$$F(X) = hX^T\mathbf{1} - \frac{1}{2}X^T\Lambda_\rho X. \tag{6}$$

[12] In risk management or in presence of moral hazard, agents would incur a personal cost of effort, while sharing benefits with others. As a consequence, transfers would reduce incentives to produce effort. In contrast, in our model the cost of risk taking (a higher variance) is shared with neighbors through transfers.

[13] Mean–variance utility generates linear best-responses. This formulation is crucial for the characterization of equilibrium risks in terms of Bonacich measure.

[14] "Partial adoption of a new crop or technology would also make sense from a diversification point of view: even though a new crop or technology may be more risky than an existing one, combining both may nevertheless reduce risk relative to the old technology alone. For this reason, one would expect risk averse farmers to keenly adopt new divisible technologies, but only partially." (Fafchamps, 2010)

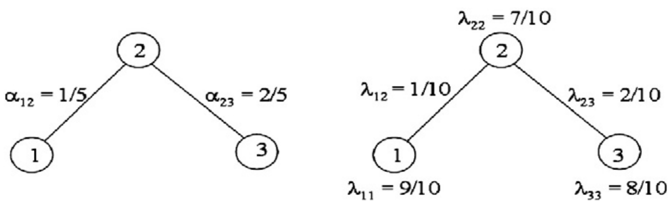

Fig. 1. Heterogenous sharing on the two-link network.

When matrix Λ_ρ is positive definite, the potential function is strictly concave, which guarantees uniqueness of equilibrium.[15] Since $\lambda_{ii} \geq \frac{1}{2}$ for all i, the matrix Λ_ρ turns out to be positive definite for all $\rho \in]0, 1[$.[16] Hence,

Preliminary result. *In a society with bilateral partial sharing rule, there is a unique risk taking equilibrium profile.*

Note that, under general concave utility function, strategic substitution between risk choices would also obtain, but not necessarily uniqueness of interior equilibrium.

2.4. Participation constraint

Although the history of social norms may have built the matrix of exchange, irrespective of contemporaneous individual incentives, we focus on circumstances in which individual utilities at equilibrium are higher under revenue sharing than under autarky. The issue is nontrivial. Basically, when risk levels are high, variances are high and utilities are possibly low. As we will see later on, it can be shown that, in a society with bilateral partial sharing rule, all agents' participation constraints hold (see Proposition 3 thereafter).

3. Bilateral partial sharing rule

In this section, we examine the impact of bilateral partial sharing rules on risk taking decisions. We will first analyze the case of homogenous own shares, which corresponds to $\lambda_{ii} = \lambda_0$ for all i. Second, we will study the case of heterogenous own shares.

3.1. Optimal risk taking under homogenous own shares

In the case of homogenous own shares, the linear system is easily solved. We obtain:

Proposition 1. *Under homogenous own shares, the equilibrium level of risk is unique and homogenous, and every individual risk level is given by*

$$x^{HOS}(\lambda_0) = \frac{h}{\lambda_0 + \rho(1-\lambda_0)}. \tag{7}$$

Proposition 1 states that the equilibrium level of risk is independent of the distribution of off-diagonal elements of the matrix of exchanges. Moreover, $x^{HOS} \in \left[h, \frac{h}{\rho}\right]$ and it is decreasing in λ_0.[17] Eq. (7) shows that individual risk level is decreasing in the value of own share. On the one hand, lowering λ_0 reduces exposure to own project (first term in the denominator), which pushes agents to take more risk. On the other hand, lowering λ_0 enhances strategic interaction (second term in the denominator), which reduces incentives to take risk. Eq. (7) shows that the first effect dominates. Hence, for societies with homogenous own shares, more revenue sharing enhances risk taking.

While societies with homogenous own shares generate homogenous risk levels, individual utilities depend on the whole distribution of transfers. In particular, at equilibrium, agents obtain the same expected revenue, but variances are differentiated and related to the distribution of exchanged shares between neighbors: the variance of revenue is indeed written as $(x^{HOS})^2 \cdot \left[\rho + (1-\rho)\sum_{j=1}^{n}\lambda_{ij}^2\right]$. Therefore, consider two matrices of exchanges Λ, Λ' with same homogenous own share. If $\sum_{j\neq i}\lambda_{ij}^2 < \sum_{j\neq i}\lambda_{ij}'^2$, then agent i's equilibrium utility is larger under matrix Λ than under matrix Λ'.

3.2. Optimal risk taking under heterogenous own shares

We have shown in the previous section that in case of homogenous own shares, risk levels were not differentiated, irrespective of the composition of exchanges. However, risk sharing often takes place in historical social networks, at least partly exogenous and formed with relatives or friends. In this respect, the structure of the risk sharing networks can be highly heterogenous. We will see now that the introduction of heterogenous own shares generates differentiated risk levels.

3.2.1. Characterization of equilibria

We will relate risk taking to a Bonacich measure of the network of transfers. This network will be represented by the $n \times n$ matrix $\Gamma = [\gamma_{ij}]$, with $\gamma_{ii} = 0$ for all i, and $\gamma_{ij} = \frac{\lambda_{ij}}{\lambda_{ii}}$ for all $i, j \neq i$. The element γ_{ij} is equal to the ratio of the share that agent j gives to agent i over agent i's own share. Note that the matrix Γ is neither symmetric, nor bi-stochastic.

We define now the Bonacich measure that will shape risk levels. We start with a rapid description of standard Bonacich centrality. Consider a $n \times n$ matrix M with null diagonal and off-diagonal elements $m_{ij} \in \{0, 1\}$, and let $\mathbf{1}$ denote the column vector of ones. Consider a scalar $\alpha \in \mathbb{R}+$. When the spectral radius of matrix M is smaller than $\frac{1}{\alpha}$, the matrix $(I - \alpha M)^{-1}$ exists and its solution, that we denote $B(M; \alpha)$, can be written as

$$B(M;\alpha) = \sum_{k=0}^{\infty} (\alpha M)^k \mathbf{1}. \tag{8}$$

The quantity $B_i(M; \alpha)$ measures the weighted (by decay factor α^k) sum over all integers k of the number of paths of length k from agent i to others through the network. This measure (actually, a slightly modified version) was introduced in Bonacich (1987). This standard centrality concept is easily extended to real-valued matrix terms, i.e. $m_{ij} \in [0, 1]$ (like in Ballester et al., 2006). In this case, the value of a path is the product of link strengths on the path, and the centrality index aggregates, over all path length k, no more the sum of all paths of length k to others, but the total weight of all paths of length k to others. Our setting corresponds to this latter point, since links, that represent shares of revenue, are real valued. However, our approach differs in the sense that our case corresponds to $\alpha < 0$. The series given in Eq. (8) converges under same condition, but the contribution of the network to the measure is ambiguous: odd paths contribute negatively to the measure, even paths contribute positively. To avoid confusion, we shall speak about *Bonacich measure*, without reference to centrality. When $\alpha < 0$, Bonacich measure can also be

[15] Bramoullé et al. (2010) use a similar potential function to characterize uniqueness in terms of the minimal modulus of eigenvalues of the network of interaction.

[16] When $\lambda_{ii} \geq \frac{1}{2}$ for all i, the matrix Λ_ρ is diagonal dominant. Combined with the fact that its diagonal is positive, this implies that all eigenvalues are positive.

[17] When the economic application requires $x^{HOS}(\lambda_0) \leq 1$, a sufficient condition is written $hb\frac{1+\rho}{2}$.

written as a function of the Bonacich centralities associated with matrix M^2 and decay factor α^2.[18]

In a society with bilateral partial sharing rule, own share exceeds one half. This basically guarantees the diagonal-dominance of the matrix Λ_ρ for all values of ρ, which induces uniqueness of equilibrium. Next theorem shows that the equilibrium risk profile is interior (meaning $x_i > 0$), and related to the Bonacich measure associated with the network represented by matrix Γ:

Theorem 1. In a society with bilateral partial sharing rule, the (unique) equilibrium is interior, and given by

$$x_i^* = \frac{h}{\rho}(1-(1-\rho)B_i(\Gamma;-\rho)) \tag{9}$$

where, for all $i \in N$, $B_i(\Gamma;-\rho) \in]0,1[$.

Theorem 1 guarantees an interior solution for all correlation parameter ρ. It shows that risk levels are differentiated when own shares are heterogenous, and shaped by the structure of transfers. Moreover, similar to homogenous own shares, risk levels are included in the interval $h,\frac{h}{\rho}[$. Indeed, since own shares exceed one half, the level of heterogeneity is low enough to maintain Bonacich measures in the interval $]0, 1[$.[19] Expressed differently, with regard to autarkic society, heterogeneity of risk-sharing makes risk-taking decisions differentiated, but does not affect the upper and lower bounds of risk levels.[20]

We illustrate Theorem 1 in the example of farmers. When farmers plant different new crop varieties or when farmers are geographically distant from each other, correlation in the risky technology across projects is close to zero. Then, risk levels are mainly shaped by own shares ($x_i^* \simeq \frac{1}{\lambda_{ii}}$), i.e. by exposure to own risk: a lower exposure to own risk leads to higher risk level. In opposite, when common factors drive returns of farmers' plantations, like weather, geographical proximity, or when crops are of similar varieties, the correlation between plantations can be very high. Then, risk levels are almost identical and close to the risk level taken in isolation ($x_i^* \simeq h$). This is true whatever the structure of exchanges; i.e. the diversification effect, which tends to higher risk levels, is almost deterred by the correlation between projects, which lowers risk-taking. In-between, Bonacich measures shape risk levels, and risk differentiation results from both the structure of the matrix of exchanges and correlation parameter.

From a pure technical perspective, Theorem 1 is original. It is well known that any diagonal dominant matrix with positive diagonal is positive definite. Given that the matrix of interaction Λ_ρ is diagonal dominant, uniqueness is then guaranteed. We show here that as, in addition, the matrix of exchanges Λ is row-stochastic, the equilibrium is interior.

Example (**continued**). To pursue with the example depicted in Fig. 1, consider for instance $h = \rho = .5$. Then, Bonacich measures are roughly given by $B_1(\Gamma, -.5) \simeq .95$, $B_2(\Gamma, -.5) \simeq .80$, $B_3(\Gamma, -.5) \simeq .89$. Agent 2, the one with the lowest own share, has the lowest index. The lower the measure, the higher the risk level. And indeed, $x_1^* \simeq .52$, $x_2^* \simeq .59$, $x_3^* \simeq .55$. Basically, the network provides agent 2 with more informal insurance than her neighbors, this encourages agent 2 to take more risk.

3.2.2. Comparative statics with respect to the volume of transfers

In this section, we undertake some comparative statics with respect to the volume of transfers. This can be useful, for instance, to compare the average risk taking of two villages with respect to the volume of transfers. We generalize the idea of 'more revenue sharing' as follows. Starting from any society, revenue sharing increases when own shares are decreased and other shares are increased, in a way that preserves both symmetry and bi-stochasticity. Formally:

> *Definition* [*more revenue sharing*]. *Consider one matrix of exchange* Λ, *and let* $\tilde{\Lambda} = \Lambda + \Theta$ *with* $\theta_{ii} = -\sum_{j \neq i}\theta_{ij}$ *for all i, and* $\theta_{ij} = \theta_{ji}$ *for all i, j. There is more revenue sharing in* $\tilde{\Lambda}$ *than in* Λ *if for all i,* $\theta_{ii} \le 0$ *and for all i,* $j \neq i$, $\theta_{ij} \ge 0$.

It is worth emphasizing that the set of perturbations that we consider departs from previous literature in two respects. First, usual comparative statics on interaction parameters consists in lowering cross-effects (see Ballester et al., 2006 under low interaction; Bramoullé et al., 2010 under large interaction). Translated in our context, this means an increase of matrix Λ_ρ, including possibly own shares. In contrast, our perturbation preserving bi-stochasticity, on-diagonal and off-diagonal elements move in opposite directions. This creates some complication to the exercise since lowering own shares increases diversification effect and thus risk levels, while enhancing exchanged shares increases strategic interaction. Second, our comparative statics encompasses perturbations that affect own shares asymmetrically, which is also a novelty. We obtain:

Theorem 2. *Consider two societies with bilateral partial sharing rule, say* Λ *and* $\tilde{\Lambda}$, *such that there is more revenue sharing in* $\tilde{\Lambda}$ *than in* Λ *Then, for any correlation parameter* ρ, $\tilde{x}^* \ge x^*$.

Theorem 2 allows to compare the aggregate risk level of two societies or villages. Recall that on the one hand, lowering own shares reduces exposure to own project, creating incentives to take more risk; on the other hand, lowering own shares increases strategic interaction, which in total reduces incentives to take risk. Theorem 2 shows that the first effect prevails on average, i.e. agents take higher risk on average, in the society with the higher volume of exchanges. As a direct application of Theorem 2, and denoting $\underline{\lambda} = \min_i \lambda_{ii}$, $\overline{\lambda} = \max_i \lambda_{ii}$, $\underline{x} = x^*(\underline{\lambda})$ and $\overline{x} = x^*(\overline{\lambda})$, the average level of risk belongs to the interval $[\overline{x}, \underline{x}]$.

However, more revenue sharing does not guarantee an increase of *all* individual risk levels. To illustrate, let $\tilde{\Lambda} = \Lambda + \Theta$ be such that there is more revenue sharing in $\tilde{\Lambda}$ than in Λ and such that some agent, say agent 1, is unaffected by the modification Θ ($\theta_{1j} = \theta_{j1} = 0$ for all j). Then, there exists one agent, say i_0, eventually distinct from agent 1, such that $\tilde{x}_{i_0}^* < x_{i_0}^*$ (this can be seen from the first order condition of agent i_0; see agent 3 in the following example).

Example (**continued**). In the example depicted in Fig. 1, and presented again in Fig. 2– left, consider again $h = \rho = .5$. Then, remind that $x_1^* \simeq .52$, $x_2^* \simeq .59$, $x_3^* \simeq .55$, and thus aggregate equilibrium risk is

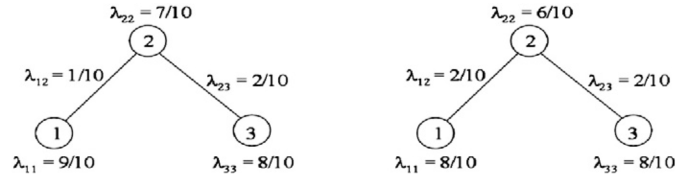

Fig. 2. Left: the initial configuration; left: the configuration with more exchanges.

[18] When $\alpha < 0$, $B(M;\alpha) = (I - |\alpha| M) \cdot B(M^2;\alpha^2)$. This individual Bonacich measure can also be written as the weighted sum of the Bonacich *centralities* of neighbors, i.e. $B_i(M;\alpha) = B_i(M^2;\alpha^2) - |\alpha| \sum_j m_{ij} B_j(M^2;\alpha^2)$. Under additional restrictions, equilibrium risks can be expressed as the Bonacich centrality associated with a transformed game with complementarities (see Ballester and Calvó-Armengol, forthcoming; Ballester et al., 2006).

[19] The diagonal dominance of the matrix $(I + \rho\Gamma)$ guarantees that $B_i(\Gamma;-\rho) \in]0,1[$.

[20] See Section 4 for an extension in which risk levels are not included in the interval $\left[h,\frac{h}{\rho}\right[$.

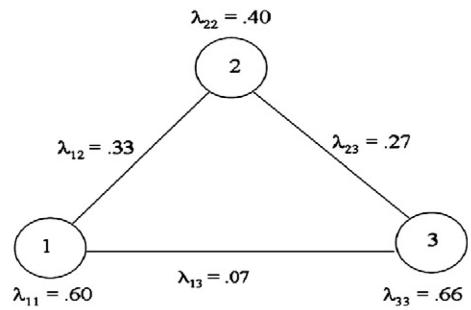

Fig. 3. The network of exchanges Λ_0.

$x^* \simeq 1.67$. Suppose that agents 1 and 2 increase the share they put in the bilateral common pool to $\alpha_{12} = \alpha_{23} = \frac{2}{5}$ in the new configuration depicted in Fig. 2– right. In this new configuration, there is more exchange than in the configuration depicted in Fig. 2– left. Risk levels are given by $\tilde{x}_1^* \simeq .54, \tilde{x}_2^* \simeq .65, \tilde{x}_3^* \simeq .54$. Here agents 1 and 2 increase their risk because they are more insured, while agent 3 decreases risk because of the strategic interaction with agent 2. In the end, as the intensity of interaction is low, aggregate risk is now higher, $\tilde{x}^* \simeq 1.73$.

3.2.3. Efficient allocation of risks

In this model, the sign of externalities is endogenous to the risk chosen by agents. Indeed, an increase of the level of risk of an agent induces both higher expected return and higher variance for her neighbors. In consequence, whether agents over or under invest in the risky technology is ambiguous. In what follows, we first characterize the efficient allocation, second we give sufficient conditions under which the efficient and equilibrium risk profiles can be compared on average, and third we compare the sum of utilities of the efficient outcome to the sum of utilities at equilibrium.

3.2.3.1. The efficient allocation. Let $u(X) = \sum_i u_i(X)$ denote the sum of utilities in the society, given risk level profile X. An efficient risk profile $\hat{X}$ maximizes the sum of utilities in the society. We define $\hat{u} = \sum_i u_i\left(\hat{X}\right)$.

Let $\Psi = \Lambda^2$. Then, matrix Ψ is both symmetric and bi-stochastic. For any risk profile X, and any matrix of exchanges Λ, the sum of utilities can be written as follows:

$$u(X) = n + \kappa\sigma^2\left[hx - \frac{1}{2}X^T\Psi_\rho X\right] \tag{10}$$

The efficient allocation satisfies the following system:

$$\begin{cases} \psi_{ii}\,\hat{x}_i + \rho\sum_{j\neq i}\psi_{ij}\,\hat{x}_j = h & \text{if } \mathrm{h} \geq \rho\sum_{j\neq i}\psi_{ij}\,\hat{x}_j \\ \hat{x}_i = 0 & \textit{if } \mathrm{h}\boldsymbol{b}\rho\sum_{j\neq i}\psi_{ij}\,\hat{x}_j. \end{cases} \tag{11}$$

It is easily shown that a unique efficient allocation obtains.[21] Define matrix Ψ_ρ, with diagonal elements ψ_{ii} and off-diagonal elements $\rho\psi_{ij}$. Any interior efficient allocation satisfies

$$\Psi_\rho\hat{X} = h \tag{12}$$

We remark that matrix Ψ can be interpreted as the matrix of exchanges of another game. Define the $n \times n$ matrix $\Phi = [\phi_{ij}]$ with $\phi_{ii} = 0$ for all i, $\phi_{ij} = \frac{\psi_{ij}}{\psi_{ii}}$ for all $i, j \neq qi$. From Theorem 1, we deduce the following characterization of the efficient risk profile. If $\psi_{ii} \geq \frac{1}{2}$ for all i, an efficient risk profile $\hat{X}$ is interior for all ρ, and it is written as

$$\hat{x}_i = \frac{h}{\rho}(1-(1-\rho)B_i(\Phi; -\rho)) \tag{13}$$

for all $i \in N$, where $B_i(\Phi;\ -\rho)$ is given by Eq. (8), with $\hat{x}_i \in \left[h, \frac{h}{\rho}\right[$. Hence, the efficient risk allocation is given by the Bonacich measure of an appropriate interaction matrix. However, the diagonal elements of matrix Ψ are not necessarily greater than one half for every matrix of exchanges Λ associated with a bilateral partial sharing rule. One sufficient condition is that $\lambda_{ii} \geq \frac{1}{\sqrt{2}}$ for all i.[22]

3.2.3.2. Over/under investment with regard to the efficient allocation. The efficient risk profile corresponds to the matrix of exchanges of a modified game. If the efficient allocation is positive,[23] we can compare the efficient allocation of risks and the optimal one, in the spirit of Theorem 2. Intuitively, if agents exchange a small share with the society, externalities are mainly mold by the return effect, i.e. they are positive. Symmetrically, if agents exchange too much with the society, the variance effect dominates in the shaping of externalities.

To sign the variation of aggregate risk, the following lemma is useful:

Lemma 1. *In a society with bilateral partial sharing rule, matrix Ψ is written $\Lambda + \Theta$, where Θ is a perturbation that entails more revenue sharing.*

Lemma 1 establishes that, for any society with bilateral partial sharing rule, the efficient allocation corresponds to a modified game in which there is more revenue sharing than in the original game. We can therefore use Farkas' lemma to compare aggregate risk levels, in the spirit of the Proof of theorem 2. This is shown in the following proposition:

Proposition 2. *For any society with bilateral partial sharing rule and positive efficient allocation, there is always under-investment with regard to efficient outcome.*

Proposition 2 shows that the return effect dominates the variance effect in the shaping of externalities, and provides a possible explanation of the lack of investment in risky innovations in developing villages.[24] Note that Proposition 2 does not prevent some agent to over-invest in the risky technology.

Example (**continued**). In the example depicted in Fig. 1, consider again $h = \rho = .5$. Then, the efficient risk profile is $\hat{x}_1 \simeq .53, \hat{x}_2 \simeq .68, \hat{x}_3 \simeq .57$, while the equilibrium profile is $x_1^* \simeq .52, x_2^* \simeq .59, x_3^* \simeq .55$. Note that, in the efficient allocation, agent 2's risk choice is much increased with regard to equilibrium risk. That way, agent 2 delivers large level of externality to others. In this configuration, every coordinate of the efficient allocation dominates the corresponding coordinate of the equilibrium risk, and under-investment prevails.

[21] Since matrix Ψ is basically positive definite, the matrix Ψ_ρ is also definite positive for all $\rho \in [0, 1[$. Indeed, for all $X \neq 0$, $\sum_i \psi_{ii} x_i^2 + \sum_{j\neq i}\psi_{ij}x_ix_j \geq 0$. If $\sum_{j\neq i}\psi_{ij}x_ix_j \geq 0$, clearly $\sum_i \psi_{ii}x_i^2 + \rho\sum_{j\neq i}\psi_{ij}x_ix_j \geq 0$ for all $\rho \in [0, 1[$. If $\sum_{j\neq i}\psi_{ij}x_ix_j < 0$, we have $\rho\sum_{j\neq i}\psi_{ij}x_ix_j < \sum_{j\neq i}\psi_{ij}x_ix_j$, thus $\sum_i \psi_{ii}x_i^2 + \rho\sum_{j\neq i}\psi_{ij}x_ix_j \geq 0$, and we are done.

[22] Basically, $\psi_{ii} \geq \lambda_i^2 + \frac{(1-\lambda_i)^2}{n-1}$. Denoting $\lambda(n) = \frac{1 + \sqrt{\frac{(n-1)(n-2)}{2}}}{n}$, basic computation shows that $\psi_{ii} \geq \frac{1}{2}$ for all i if $\lambda_{ii} \geq \lambda(n)$ for all i. But $\lambda(n) < \frac{1}{\sqrt{2}}$ for all n.

[23] Two simple (independent) conditions ensuring diagonal dominance of matrix Ψ_ρ are (i) $\lambda_{ii} > \frac{1}{\sqrt{2}}$ for all i, and (ii) $\rho \leq \frac{1}{3}$. These conditions guarantee a unique efficient allocation, interior, and written as a Bonacich measure.

[24] See Valente, 1997).

3.2.3.3. Aggregate utilities. Now, we focus on interior efficient allocations until the end of the subsection (these conditions are less restrictive than imposing $\psi_{ii} \geq \frac{1}{2}$ for all i). We compute the aggregate efficient utility. At efficient outcome, risk levels are given by Eq. (12). Inserting it in Eq. (10), we obtain:

$$\hat{u} = n + \frac{h\kappa\sigma^2}{2}\hat{x}. \tag{14}$$

That is, the efficient aggregate utility is an affine function of the aggregate efficient risk. It is possible to express the difference between aggregate efficient utilities $\hat{u}$, and aggregate equilibrium utilities, u^*, in a simple formula. Applying Eq. (10) to u^* and $\hat{u}$ and exploiting the fact that both $\Psi_\rho \hat{X} = h$ and $\Lambda_\rho X^* = h$, the difference between the sum of efficient utilities and equilibrium utilities is written as:

$$\hat{u} - u^* = \frac{\kappa\sigma^2}{2}\left[h\left(\hat{x} - x^*\right) + X^{*T}\left(\Psi_\rho - \Lambda_\rho\right)X^*\right]. \tag{15}$$

Hence, the difference between efficient and equilibrium utilities can be decomposed in two components: a factor proportional to the difference of aggregate efficient risk and aggregate equilibrium risk, plus a quadratic component related to the difference $\Psi_\rho - \Lambda_\rho$. For high correlation, both efficient and equilibrium risks are close to h, and inefficiency is small. When correlation decreases, inefficiency increases. Simulations indicate that under-investment prevails, and that both the inefficiency, measured as $\hat{u} - u^*$, and the rate of inefficiency, measured as the ratio $\frac{|\hat{u} - u^*|}{\hat{u}}$, are decreasing in parameter ρ (and nonnegligible for ρ lower than one half approximately).

4. Group partial sharing rule

So far, we have considered a social network as a set of pairs of neighbors, and we have described sharing rules such that any two neighbors put the same fixed share of revenue in a bilateral common pool and then share the pool equally. In this section, we extend the analysis to groups of neighbors (not only pairs), and we introduce the set of *group partial sharing rules*, in which groups of agents put the same fixed share of revenue in a common pool and then share the pool equally. Formally, consider some agent i, and without loss of generality suppose that agent i belongs to m distinct groups of agents, $N_1, N_2, \cdots, N_m$, or respective sizes $n_1, n_2, \cdots, n_m$, with $n_1 \geq n_2 \geq \cdots \geq n_m$. Suppose that, in each group k, agents put the same share $\alpha_k \in [0,1]$ of their revenues in a common pool. Given before-transfer revenue profile $(r_1^b, r_2^b, \cdots, r_n^b)$, agent i's after-transfer revenue is written as

$$r_i^a = \left(1 - \sum_{k=1}^m \alpha_k\right) r_i^b + \sum_{k=1}^m \alpha_k \cdot \left(\frac{\sum_{j \in N_k} r_j^b}{n_k}\right) \tag{16}$$

under the condition that $\sum_{k=1}^m \alpha_k \leq 1$. That is, denoting $\lambda_{ii} = 1 - \sum_{k=1}^m \alpha_k + \sum_{k=1}^m \frac{\alpha_k}{n_k}$,

$$r_i^a = \lambda_{ii} r_i^b + \sum_{k=1}^m \alpha_k \cdot \left(\frac{\sum_{j \in N_k \setminus \{i\}} r_j^b}{n_k}\right). \tag{17}$$

Denote, for all $k = 1, 2, \cdots, m$, $n_{-k} = n_1 n_2 \cdots n_{k-1} n_{k+1} \cdots n_m$. Then we obtain after rearrangement

$$\lambda_{ii} = \frac{1}{n_1} + \left(\frac{n_1 - 1}{n_1}\right)\left(1 - \sum_{k=1}^m \alpha_k\right) + \left(\frac{1}{n_1 n_2 \cdots n_m}\right)\sum_{k=2}^m \alpha_k (n_{-k} - n_{-1}) \tag{18}$$

Since $n_1 \geq n_2 \geq \cdots \geq n_m$, we have $n_{-k} \geq n_{-1}$ for all $k \geq 2$. Hence, $\lambda_{ii} \geq \frac{1}{n_1}$. That is, own shares exceed the inverse of the size of the largest group. Note also that $\lambda_{ij} = \sum_{k/j \in N_k} \frac{\alpha_k}{n_k}$.

4.1. An illustration: sharing with the whole society ($m = 1$)

This sharing rule can be represented by the matrix of exchanges $\Lambda^{PES}(\lambda_0)$ such that $\lambda_{ii}^{PES} = \lambda_0$ for all i, and $\lambda_{ij}^{PES} = \frac{1-\lambda_0}{n-1}$ for all $i, j \neq i$. To illustrate, suppose that a fixed proportion, say τ_0, of incomes is collected and equally redistributed. Then, agent i receives $\left(1 - \frac{n-1}{n}\tau_0\right) r_i^b + \frac{\tau_0}{n}\sum_{j \neq i} r_j^b$. Denoting $\lambda_0 = 1 - \frac{n-1}{n}\tau_0$, the equilibrium level of risk is increasing in the taxation rate τ_0 (less however than in the absence of strategic interaction). Such a society has homogenous own shares. When $\lambda_0 > \frac{1}{n}$, it is easily established that such a society always exhibits under-investment.

In this class of rules, of particular interest is full equal sharing, meaning $\lambda_{ij} = \frac{1}{n}$ for all i, j. This society exhibits maximal diversification. It is easily shown that this is the unique configuration such that the equilibrium risk profile coincides with the efficient allocation.[25] Furthermore, it can be shown that the matrix of full equal sharing guarantees the highest equilibrium utility for every agent. This usual result expresses that if agents were able to coordinate (at no cost) in order to collectively implement full equal sharing, every agent would be better off.

4.2. Participation constraint

When group sizes are high, the participation constraint is a matter. In which circumstances individual utilities are higher under transfers than under autarky is a nontrivial issue. When risk levels are high, variances are high and utilities are possibly low. Does the benefit from sharing revenues out-weighs the negative externalities that agents' choices may generate on others?

For any matrix of exchanges Λ, the individual participation constraint is satisfied if individual utility is greater than utility under autarky. If the solution X to system (5) is interior (which is guaranteed by the diagonal dominance of Λ_ρ), the participation constraint is satisfied if and only if

$$\lambda_{ii}^2 x_i^2 + \rho \sum_{j \neq i} \lambda_{ij}^2 x_j^2 \leq h^2. \tag{19}$$

In particular,

Proposition 3. *The participation constraint is satisfied if for all j, $\max_{i \neq j} \lambda_{ij} \leq \lambda_{jj}$; that is, each own share exceeds each share she gives to neighbors.*

The condition that every own share exceeds each share she gives to neighbors is rather mild, and plausible in many economic contexts. Note that the condition holds if every own share exceeds one half, i.e. groups in which agents share revenues in groups of size 2.

4.3. Validity of Theorems 1 and 2

We have conducted our analysis under bilateral partial sharing rules. Therefore, we restricted attention to own shares greater than one half, which allowed us to present results for every $\rho \in]0,1[$. Actually, Theorems 1 and 2 hold under diagonal dominance of the

[25] This result rests on the fact that $\Psi = \Lambda^2$. Hence, $\Psi = \Lambda$ means that the matrix Λ is idempotent. The matrix of full equal sharing is the unique to be both irreductible and idempotent (see Schwarz, 1967, Lemma 2, p. 309).

matrix Λ_ρ. Given the row-stochasticity of the matrix Λ, the condition is given by the following assumption:

Assumption 1. For all i, $\lambda_{ii}\in\left]\frac{\rho}{1+\rho},1\right]$.

Assumption 1 does not imply nor is implied by the condition on participation constraint. However, both are met when own shares exceed one half. Assumption 1 guarantees that the equilibrium risk profile is unique and interior. More, the same characterization in terms of a Bonacich measure holds, and more generally Theorem 1, Theorem 2 and Proposition 2 stay valid.

4.4. Relaxing Assumption 1

If Assumption 1 does not hold, strategic interaction is sufficiently high to generate strong implications. First, if the matrix of interaction Λ_ρ is positive definite, uniqueness holds, otherwise multiple equilibria may occur. Second, risk levels may not be restricted to the interval $\left]h,\frac{h}{\rho}\right[$, and more revenue sharing may reduce risk taking (violating Theorem 2). The next example illustrates the last point (Fig. 3). Suppose $h\rho=.9$ and consider the following matrices:

$$\Lambda_0=\begin{pmatrix}.60 & .33 & .07\\ .33 & .40 & .27\\ .07 & .27 & .66\end{pmatrix},\tilde{\Lambda}_0=\begin{pmatrix}.59890 & .33001 & .07218\\ .33001 & .39800 & .27199\\ .07218 & .27199 & .65583\end{pmatrix}.$$

Then matrix $\tilde{\Lambda}_0$ is obtained from matrix Λ_0 by adding a small perturbation that increases the volume of exchanges. Note that both matrices Λ_0 and $\tilde{\Lambda}_0$ satisfy the participation constraint. Neither matrix Λ_0 nor matrix $\tilde{\Lambda}_0$ satisfy Assumption 1, while both admit a unique interior equilibrium,[26] and $X_0^*\simeq(0.985h,\ 1.165h,\ 0.992h)$, $\tilde{X}_0^*\simeq(0.981h, 1.170h, 0.990h)$. In both equilibria, no risk level belongs to $\left]h,\frac{h}{\rho}\right[$. Here, agent 2 is more insured than the other agents, and more revenue sharing enhances her risk level. Then, strategic interaction induces a decrease of both agent 1 and agent 3 risk levels. Since the intensity of interaction is high, the aggregate risk levels is decreased ($x_0^*\simeq3.143h$, $\tilde{x}_0^*\simeq3.142h$).

5. Conclusion

This paper has considered a model of risk taking behaviors of agents in developing countries, like the experimentation of a new fertilizer or a new crop. We considered agents with same initial wealth and same risk aversion, and we examined the impact of the heterogeneity of transfers, inherent to the risk sharing network, on individual risk-taking, in presence of systematic risk. In this context, risk taking behaviors are strategic substitutes. Hence, risk taking behaviors are not only affected by the characteristics of the technologies, but also by the structure of the risk sharing network.

We found that if agents exchange a distinct proportion of their revenues with neighbors, risk choices are differentiated and shaped by the position of agents in the risk sharing network. Furthermore, when agents exchange less than half their revenue with others, increasing the volume of risk sharing fosters aggregate risk-taking, and agents under-invest with regard to the risk allocation that maximizes the sum of utilities, irrespective of the detail of the risk sharing network.

It would be interesting to test some empirical implications of the analysis, in particular Theorems 1 and 2. Second, our model examined the impact of heterogeneity of transfers on risk taking, but it disregarded that the rules of transfers may themselves be related to the level of risk in the society. It would be challenging to propose a set up with endogenous sharing rules. Last, in the context of demand for innovation, this simple model may be usefully augmented with other determinants of adoption of innovation in developing countries, like access to credit, savings, extra-earning jobs, etc.

Appendix

Lemma 2. Consider two parameters $\delta\in]0,1[$, $\alpha\in\mathbb{R}_+^*$. Consider also a square row-stochastic matrix $A=[a_{ij}]$. Define $E=[e_{ij}]$, with $e_{ii}=0$ for all i and $e_{ij}=\frac{a_{ij}}{a_{ii}}$ for all $i,j\neq i$. The system of equations such that, for all i,

$$\begin{cases} a_{ii}x_i+\delta\sum_{j\neq i}a_{ij}x_j=\alpha & \text{if } \alpha\geq\delta\sum_{j\neq i}a_{ij}x_j\\ x_i=0 & \text{if } \alpha<\delta\sum_{j\neq i}a_{ij}x_j\end{cases} \tag{20}$$

admits a unique and positive solution if $a_{ii}>\frac{\delta}{1+\delta}$ for all i. This solution lies in $\left[\alpha,\frac{\alpha}{\delta}\right]$, and is written as:

$$x_i=\frac{\alpha}{\delta}-\alpha\left(\frac{1-\delta}{\delta}\right)B_i(E;-\delta) \tag{21}$$

with $B_i(E;-\delta)\in]0,1[$.

Proof of Lemma 2. *Part 1: the solution is unique*. We consider the following transformation:

$$v_i=\left(\frac{\delta}{\alpha(1-\delta)}\right)\left(\frac{\alpha}{\delta}-x_i\right) \tag{22}$$

The first equation of system (20) becomes:

$$a_{ii}\left(\frac{\alpha}{\delta}-\left(\alpha\frac{1-\delta}{\delta}\right)v_i\right)+\delta\sum_{j\neq i}a_{ij}\left(\frac{\alpha}{\delta}-\left(\alpha\frac{1-\delta}{\delta}\right)v_j\right)=\alpha. \tag{23}$$

Dividing all terms by a_{ii}, and taking account of $\sum_{j\neq i}a_{ij}=1-a_{ii}$, we obtain in matrix form $(I+\delta E)V=1$. Since $a_{ii}>\frac{\delta}{1+\delta}$ for all i, the matrix $I+\delta E$ is strictly diagonal dominant. We deduce that the solution to $(I+\delta E)X=1$ exists and is unique (strict diagonal dominance plus positive diagonal implies positive definiteness).

Part 2: the solution X lies in $\left[\alpha,\frac{\alpha}{\delta}\right]$, *and written as a Bonacich*. The solution V of the transformed system can be written as $B(E;-\delta)=\sum_{k=0}^{\infty}(-\delta)^kE^k1\in\left]0,1\right[$. Indeed, diagonal dominance of matrix $I+\delta E$ implies that the product of parameter δ by the spectral radius of matrix E is lower than 1, which makes the series convergent. Factorizing the series, we obtain:

$$B(E;-\delta)=\left(\sum_{k=0}^{\infty}\delta^{2k}E^{2k}\right)\cdot(I-\delta E)1. \tag{24}$$

Notice that $\delta^{2k}[E^{2k}]_{ij}>0$ for all k,i,j. Further, as $a_{ii}>\frac{\delta}{1+\delta}$ for all i, the vector $(I-\delta E)1>0$. Hence, $B(E;-\delta)>0$. Moreover, a solution of $(I+\delta E)V=1$ also writes $V=1-\delta EV$. That is, if $V>0$, clearly $V<1$.

Last, that $\alpha>0$ and $V\in[0,1]$ imply that $x_i\in\left[\alpha,\frac{\alpha}{\delta}\right]$. □

Proof of theorem 1. It is well known that the system $\Lambda_\rho X=h1$ admits a unique solution if it is diagonal dominant (due to positive diagonal of matrix Λ_ρ). Given the row-stochasticity of matrix Λ, that own shares exceed one half ensures diagonal dominance for all $\rho\in[0,1[$. Since matrix Λ is row-stochastic, we apply Lemma 2 with $\delta=\rho$, $\alpha=h$, $A=\Lambda$ ($\alpha>0$ means $h>0$, which is valid). Note that by Lemma 2 the solution is interior. □

[26] Indeed, both matrices are positive definite. Hence both equilibria are stable.

Proof of theorem 2. We use the following lemma:

Lemma 3 (Adapted from Farkas' lemma). Let Q be an $n\times n$ matrix. If the equation $Q^T X=1$ admits a positive solution, then for all $Y\in\mathbb{R}^n$ such that $QY\geq 0$, we have $\sum_i y_i\geq 0$.

We will see that the conditions of Lemma 3 hold if we fix $Q=\Lambda_\rho$ and $Y=\tilde{X}^*-X^*$:

First, we show that there exists a positive solution to $(\Lambda_\rho)^T X=1$. Since $\Lambda^T=\Lambda$, we can apply Lemma 2 with $\delta=\rho, \alpha=1, A=\Lambda^T$, and we conclude that there is a positive solution to the system $(\Lambda_\rho)^T X=1$.

Second, we see that $\Lambda_\rho\left(\tilde{X}^*-X^*\right)\geq 0$. Remind that $\Lambda+\Theta=\tilde{\Lambda}$, and let Θ_ρ denote the matrix with diagonal θ_{ii} and off-diagonal ρa_{ij}. We observe that $\Lambda_\rho X^*=\tilde{\Lambda}_\rho\tilde{X}^*=h1$, and that $\Lambda_\rho=\tilde{\Lambda}_\rho-\Theta_\rho$. Hence, $\Lambda_\rho\left(\tilde{X}^*-X^*\right)=-\Theta_\rho\tilde{X}^*$, that is:

$$\left[\Lambda_\rho\left(\tilde{X}^*-X^*\right)\right]_i=-\left(\theta_{ii}\tilde{x}_i^*+\rho\sum_{j\neq i}\theta_{ij}\tilde{x}_j^*\right). \tag{25}$$

Now, since matrix $\tilde{\Lambda}$ lies in the class of bilateral partial sharing rules, Theorem 1 implies that $\tilde{x}_i^*\in\left[h,\frac{h}{\rho}\right[\Big[$ for all i. Then, given that $-\theta_{ii}\geq 0$ and $\theta_{ij}\geq 0$ for all i, j:

$$\left[\Lambda_\rho\left(\tilde{X}^*-X^*\right)\right]_i\geq(-\theta_{ii})h-\rho\sum_{j\neq i}\theta_{ij}\left(\frac{h}{\rho}\right). \tag{26}$$

That is, recalling that $\theta_{ii}=-\sum_{j\neq i}\theta_{ij}$, we obtain that $-\Theta\tilde{X}^*\geq 0$, which means that $\Lambda_\rho\left(\tilde{X}^*-X^*\right)\geq 0$.

Applying Lemma 3, we conclude that $\sum_i\left(\tilde{x}_i^*-x_i^*\right)\geq 0$, which proves the theorem. □

Proof of Lemma 1. We check that matrix Ψ, which is bi-stochastic by construction, can be deduced from matrix Λ by adding a perturbation that increases the volume of exchanges (decreasing own shares, increasing exchanged shares).

First, we check that $\Psi_{ii}\leq\lambda_{ii}$ for all i

$$\Psi_{ii}=\lambda_{ii}^2+\sum_{j\neq i}\lambda_{ij}^2. \tag{27}$$

Then, given that $\sum_{j\neq i}\lambda_{ij}=1-\lambda_{ii}$, clearly

$$\Psi_{ii}\leq\lambda_{ii}^2+(1-\lambda_{ii})^2. \tag{28}$$

But,

$$\lambda_{ii}^2+(1-\lambda_{ii})^2\leq\lambda_{ii} \tag{29}$$

means

$$(1-\lambda_{ii})(1-2\lambda_{ii})\leq 0 \tag{30}$$

which holds when own shares exceed one half. Thus, $\Psi_{ii}\leq\lambda_{ii}$.

Second, we check that $\Psi_{ij}\geq\lambda_{ij}$ for all $i, j\neq i$.

$$\Psi_{ij}=\left(\lambda_{ii}+\lambda_{jj}\right)\lambda_{ij}+\sum_{k\neq i,j}\lambda_{ki}\lambda_{kj} \tag{31}$$

Thus, $\Psi_{ij}\geq\lambda_{ij}$ if and only if

$$\sum_{k\neq i,j}\lambda_{ki}\lambda_{kj}\geq\left(1-\lambda_{ii}-\lambda_{jj}\right)\lambda_{ij} \tag{32}$$

which holds when own shares exceed one half. Thus, $\Psi_{ij}\geq\lambda_{ij}$. □

Proof of the Proposition 2. We replicate the Proof of Theorem 2. We apply Lemma 3 with $Q=\Psi_\rho$, and $Y=\tilde{X}-X^*$. When the efficient solution is positive, and given that the matrix is symmetric, we have $(\Psi_\rho^T)^{-1}1\geq 0$. Then $\tilde{x}\geq x^*$ obtains if $\Psi_\rho\left(\tilde{X}-X^*\right)\geq 0$. But writing $\Theta_\rho=\Psi_\rho-\Lambda_\rho$, we find $\Psi_\rho\left(\tilde{X}-X^*\right)=-\Theta_\rho X^*$. Since $X^*\in\left[h,\frac{h}{\rho}\right[\Big[$, and exploiting the fact that $\sum_{j=1}^n\theta_{ij}=0$ for all i, we conclude that $-\Theta_\rho X^*\geq 0$, and we are done. □

Proof of the Proposition 3. In general, utility is written:

$$u_i(X)=\sum_j\lambda_{ij}\left(\mu x_j+1-x_j\right)-\frac{\kappa\sigma^2}{2}\sum_j\lambda_{ij}^2x_j^2-\frac{\kappa\rho\sigma^2}{2}\left[\left(\sum_j\lambda_{ij}x_j\right)^2-\sum_j\lambda_{ij}^2x_j^2\right]. \tag{33}$$

That is,

$$u_i(X)=1+\left(\sum_j\lambda_{ij}x_j\right)\left[\mu-1-\frac{\kappa\rho\sigma^2}{2}\sum_j\lambda_{ij}x_j\right]-\frac{\kappa(1-\rho)\sigma^2}{2}\sum_j\lambda_{ij}^2x_j^2. \tag{34}$$

The FOC can be written as:

$$\sum_j\lambda_{ij}x_j=\frac{1}{\rho}\cdot(h-(1-\rho)\lambda_{ii}x_i) \tag{35}$$

while equilibrium utility in isolation writes $1+\frac{h}{2}(\mu-1)$. Thus, little calculation indicates that agent i's utility exceeds profit in isolation if:

$$\lambda_{ii}^2x_i^2+\rho\sum_{j\neq i}\lambda_{ij}^2x_j^2\leq h^2. \tag{36}$$

Or equivalently

$$\left(\frac{\lambda_{ii}x_i}{h}\right)^2+\rho\sum_{j\neq i}\left(\frac{\lambda_{ij}x_j}{h}\right)^2\leq 1. \tag{37}$$

If X is an interior solution to system (5), we have $\lambda_{ii}x_i\leq h$. Further, assuming that $max_{i\neq j}\lambda_{ij}\leq\lambda_{jj}$ for all j, and using the system (5) again, we obtain that $\lambda_{ij}x_j\leq h$. Hence, the inequality (37) is implied by:

$$\frac{\lambda_{ii}x_i}{h}+\rho\sum_{j\neq i}\frac{\lambda_{ij}x_j}{h}\leq 1 \tag{38}$$

which is true since equality holds from FOCs. □

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