Intergenerational Correlation and Social Interactions in Education*

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Abstract

We propose a dynastic model where individuals are born into an educated or uneducated environment that they inherit from their parents. We study the impact of social interactions on the correlation in parent-child educational status, independently of any parent-child interaction. When the level of social interactions is decided by a social planner, we show that the correlation in education status between generations decreases very fast as social interactions increase. In turn, when the level of social interactions is decided by the individuals themselves, we show that the intergenerational correlation still decreases, although less rapidly than with exogenous social interactions.

Key words: Social mobility, strong and weak ties, intergenerational correlation, education.

JEL Classification: I24, J13, Z13.

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1 Introduction

Explaining the educational outcomes of children is one of the greatest challenges for economists. Most studies have found that school quality (e.g., Card and Krueger, 1992) and family background (e.g., Ermisch and Francesconi, 2001, Plug and Vijverberg, 2003) have a significant and positive impact on children's level of education. Parents obviously influence their children's school performance by transmitting their genes, but also influence their children directly, via, for example, their parenting practices and the type of schools to which they send them (Björklund et al., 2006; Björklund and Salvanes, 2011). Neighborhood and peer effects also have an important impact on the educational outcomes of children (Durlauf, 2004; Ioannides and Topa, 2010; Sacerdote 2010; Patacchini and Zenou, 2011; Topa and Zenou, 2015).

We study the intergenerational relationship between parents' and offspring's long-run educational outcomes. It is well-documented that students' educational achievement is positively correlated with their parents' education, or with other indicators of their parents' socio-economic status (Björklund and Salvanes, 2011). In the present paper, we illustrate how this correlation can result from peer effects, abstracting from any direct parental influence. We also analyze how variations in social interactions translate into variations in intergenerational correlation.

There have been many attempts in the literature to analyze how the intergenerational correlation could be reduced by focusing on the direct influence that parents have on their children (Björklund and Salvanes, 2011). However, Calvó-Armengol and Jackson (2009) show that the correlation between a parent's and a child's outcomes can be explained without invoking any direct influence, but rather by virtue of the fact that they share a common environment, which affects their decisions. This provides us with a new channel of intervention that reduces the parent-child correlation, which this paper aims to examine.

We develop a dynastic model where, at each period of time, with some probability, a person (the parent) dies and is simultaneously replaced by a newborn (the child). The child thus never interacts with his parent and does not inherit any of his idiosyncratic characteristics. Instead, newborns inherit the environment (local community) where their parents lived. In our paper, the environment is modeled as a dyad in which the newborn interacts with a partner, called his *strong tie*. This strong tie represents the environment with which the parent interacted before he died. Using the language of the cultural transmission literature (Bisin and Verdier, 2000, 2001), in our model, there is no vertical transmission (i.e. socialization inside the family), but only horizontal transmission (i.e. socialization outside the family).

We start with a benchmark model where newborns only interact with their strong tie. When they are born, they discover the environment they are in, i.e. the educational status of their strong tie, and then decide whether to get educated or not. We show that, even if a parent never interacts with his child, there is still a significant positive correlation between the educational achievement of the parent and the child. A parent has a higher (lower) probability of being educated if he lives in a favorable (unfavorable) environment. Because the child shares the same environment, the probability that the child will get educated is also higher (lower). This benchmark model allows us to derive simple expressions for both the average level of education at steady state and the level of intergenerational correlation.

We then extend this model to introduce the opportunity for individuals to interact with peers outside their local community (*weak ties*). Since individuals start interacting with weak ties, strong ties have less influence on their education, and this mechanically decreases the parent-child correlation. We model social interactions as the fraction of time a newborn spends with weak ties and consider two cases: in the first, the level of social interactions is exogenously fixed by a social planer, and in the second, it is decided by the newborns themselves. In both cases, we show that peer effects, defined here as interactions with both types of ties, have major implications for public policies aimed at reducing social inertia.

Indeed, when the socializing decisions are exogenously made by the social planner, we show that the correlation in education status between generations decreases very fast as social interactions increase. Actually, the decrease in correlation is a power four of the increase in social interactions. Hence, a social planner promoting social mobility will force individuals to interact as much as possible, in which case we also show that the average level of education does not change compared to the benchmark case. However, this policy decreases social welfare. This is because, while individuals born into an unfavorable environment benefit from such a policy, those born into a favorable environment are penalized, as they now interact with potentially uneducated individuals. The net effect turns out to be negative, the losses of the former not being fully compensated by the gains of the latter. This illustrates a common social planner's trade-off between equity (i.e. decreasing intergenerational correlation) and efficiency.

When the socializing decisions are made by the individuals, those with uneducated strong ties always want to meet weak ties, while the reverse applies to individuals with educated strong ties. This is simply because the former will, in the worst case, meet another uneducated person and, at best, meet someone educated, and the reverse will happen for the latter. We show that when individuals can escape their inherited environment, the intergenerational correlation still decreases with respect to the benchmark case, but the extent to which it decreases depends on the average level of education in the population. The higher the proportion of educated individuals, the higher the impact of social interactions.

Finally, we show that when socializing decisions are made by the individuals, the average education level at steady state increases, contrary to the case where interaction choices are exogenous. Therefore, a social planner who lets individuals choose their own interaction levels should be promoting education. This in fact has two effects: the direct effect of increasing the education level in the population, and the indirect effect of decreasing the intergenerational correlation through the first effect.

The rest of the paper unfolds as follows. In the next section, we relate our model to the relevant theoretical literatures. Section 3 presents the model without social interactions while Section 4 focuses on exogenous levels of social interactions. In Section 5, we examine endogenous levels of social interactions, since individuals choose how much time they spend with weak and strong ties. In Section 6, we provide empirical evidence of the results obtained in Section 5.[1] Section 7 concludes. All proofs of propositions, lemmas and remarks can be found in Appendix 1.

[1]This paper is clearly a theoretical contribution. The empirical evidence should only be understood as anecdotal, and not considered a full-fledged empirical validation of our model.

2 Related literature

Apart from the paper by Calvó-Armengol and Jackson (2009),[2] our model is related to different literatures. First, it is related to the literature on peer effects in education. De Bartolome (1990) and Benabou (1993) are the standard references for peer and neighborhood effects in education. In this multi-community approach, individuals can acquire high or low skills or be unemployed. The costs of acquiring skills are decreasing in the proportion of the community that is highly skilled, and the higher the skills acquired, the greater the decrease in costs. This leads tosorting, although ex ante all individuals are identical. While there is an extensive empirical literature on the intergenerational transmission of income and education that focuses on the correlation of parents' and children's permanent income or education (Björklund and Jäntti, 2009; Black and Devereux, 2011; Björklund and Salvanes, 2011), there are very few theoretical models exploring this issue. Ioannides (2002, 2003) analyzes the intergenerational transmission of human capital by explicitly developing a dynamic model of human capital formation with neighborhood selection. The idea here is to study the impact of both parental education and the distribution of educational attainment within a relevant neighborhood on child educational attainment. From a theoretical viewpoint, Ioannides obtains a complete characterization of the properties of the intertemporal evolution of human capital. From an empirical viewpoint, he finds that there are strong neighboring effects in the transmission of human capital and that parents' education and neighbors' education have non-linear effects that are consistent with the theory.

Using a cultural transmission model à la Bisin and Verdier (2000, 2001), Patacchini and Zenou (2011) analyze the intergenerational transmission of education, focusing on the interplay between family and neighborhood effects. They develop a theoretical model suggesting that both neighborhood quality and parental effort are of importance for the education attained by children. Their model proposes a mechanism explaining why and how they are of importance, distinguishing between highly and less educated parents. Empirically, they find that the better the quality of the neighborhood, the higher the parents' involvement in their

[2]Contrary to their paper, we consider the impact of social interactions through weak and strong ties on the intergenerational correlation in education.

children's education.[3]

Second, our paper is also related to the social network literature. There is a growing interest in theoretical models of peer effects and social networks (see e.g., Ballester et al., 2006; Calvó-Armengol et al., 2009; Jackson, 2008; Jackson and Zenou, 2015; Ioannides, 2012). To the best of our knowledge, there is no theoretical model that looks at the impact of social networks on the intergenerational transmission of education. In the present paper, we model the network as the interaction between strong and weak ties. In his seminal contributions, Granovetter (1973, 1974, 1983) defines *weak ties* in terms of lack of overlap in personal networks between any two agents, i.e. weak ties refer to a network of acquaintances who are less likely to be socially involved with one another. Formally, two agents A and B have a weak tie if there is little or no overlap between their respective personal networks. Vice versa, the tie is *strong* if most of A's contacts also appear in B's network. In this context, Granovetter (1973, 1974, 1983) develops the idea that weak ties are superior to strong ties for providing support in getting a job.[4] In our model, we stress the role of strong ties as an important means of transmission of education. In other words, even though there is no direct influence from the parents, their indirect influence through the inheritance of strong ties positively affects the correlation between parent and child.

3 The benchmark model without social interactions

3.1 Model

There are n individuals in the economy.[5] We assume that individuals belong to mutually exclusive two-person groups, referred to as *dyads*. We say that two individuals belonging

[3]Both the models of Ioannides (2002, 2003) and Patacchini and Zenou (2011) highlight mechanisms that differ from ours. In their models, the main prediction is that the intergenerational transmission of education depends on the effort of parents and on the average education level in the neighborhood. In our model, parents play no role and individuals can socialize outside their neighborhood.

[4]For other models on weak and strong ties, see Montgomery (1994), Calvó-Armengol et al. (2007), Sato and Zenou (2015) and Zenou (2013, 2015).

[5]We assume throughout that n is large, and all the propositions in the paper should be understood as limiting propositions.

to the same dyad hold a *strong tie* with each other. We assume that dyad members do not change over time unless one of them dies. A strong tie is created once and for all and can never be broken. Thus, we can think of strong ties as links between members of the same family, or between very close friends. In this section, and here alone, we assume that different dyads do not interact.

We consider a dynamic model where, at each period, each individual in the dyad can die with probability $1/n$. When a person dies, he is automatically replaced by a newborn who is his child. The child is then paired with the individual who was previously in the same dyad (strong tie) as his parent. The only aspect that the son inherits from his father is his father's social environment or local community, here the father's strong tie. There is no other interaction between the father and the son. In particular, the father and the son never live at the same time. This is because we are seeking to analyze the effect of the environment (peer effects) on the child's education outcomes, independent of any parent-child interaction.

When individual i is born, he discovers the education type of his strong tie: $j = 0$ (uneducated) or $j = 1$ (educated). He also discovers his own idiosyncratic *ability* for education, given by some λ_i randomly drawn from the uniform distribution on $[0, 1]$. Education is costly and the return on education effort depends both on the individual's ability to learn and on the education type of the strong tie. The utility U_{ij} of individual i with strong tie j exerting effort e_{ij} is given by:

$$U_{i0}\left(\lambda_i\right) = \lambda_i e_{i0} - \frac{1}{2}e_{i0}^2 - \alpha\, e_{i0}$$

$$U_{i1}\left(\lambda_i\right) = \lambda_i e_{i1} - \frac{1}{2}e_{i1}^2$$

where $\alpha > 0$ is the penalty incurred when living in an unfavorable environment (i.e. being born with a less-educated strong tie). This cost captures the idea that uneducated role models can distract individuals from educating themselves by, for example, proposing activities that are not related to education (like watching TV, going to the movies, etc.).[6]

Each newborn decides how much effort he devotes to education. Individuals who do not get educated are guaranteed a minimum wage from which they derive utility $\overline{U}$. First-order

[6]We could set a cost of $\alpha_1 > 0$ when interacting with uneducated strong ties and α_2 (possibly negative) when interacting with educated strong ties, with $\alpha_1 > \alpha_2$. Here, without loss of generality, we have $\alpha_1 = \alpha > 0$ and normalize α_2 such that $\alpha_2 = 0$.

conditions yield:

$$e_{i0} = \max\{0, \lambda_i - \alpha\}$$

$$e_{i1} = \lambda_i$$

Quite naturally, individual i will be educated if $U_{ij}(\lambda_i) > \overline{U}$. This provides us with a threshold level $\widetilde{\lambda}_0$ (resp. $\widetilde{\lambda}_1$), such that individuals with an uneducated (resp. educated) strong tie and with ability above $\widetilde{\lambda}_0$ (resp. $\widetilde{\lambda}_1$) will get educated, while those with ability below $\widetilde{\lambda}_0$ (resp. $\widetilde{\lambda}_1$) will not. These two thresholds are defined as:

$$\widetilde{\lambda}_0 = \sqrt{2\overline{U}} + \alpha$$

$$\widetilde{\lambda}_1 = \sqrt{2\overline{U}}$$

We assume that $\overline{U}$ and α are such that

$$0 \leq \sqrt{2\overline{U}} < \sqrt{2\overline{U}} + \alpha \leq 1$$

Plugging each effort into each utility function, we obtain:

$$U_{i0}^{*}\left(\lambda_i\right) = \max\left\{\overline{U}, \frac{\left(\lambda_i - \alpha\right)^2}{2}\right\}$$

$$U_{i1}^{*}\left(\lambda_i\right) = \max\left\{\overline{U}, \frac{\lambda_i^2}{2}\right\}$$

As a result, the probability p_0 (resp. p_1) that an individual with an uneducated (resp. educated) strong tie will be educated is given by:

$$p_0 = 1 - \widetilde{\lambda}_0 = 1 - \sqrt{2\overline{U}} - \alpha \tag{1}$$

$$p_1 = 1 - \widetilde{\lambda}_1 = 1 - \sqrt{2\overline{U}} \tag{2}$$

These probabilities can be understood as the proportion of individuals that will be educated. Figure 1 summarizes how education choices are made.

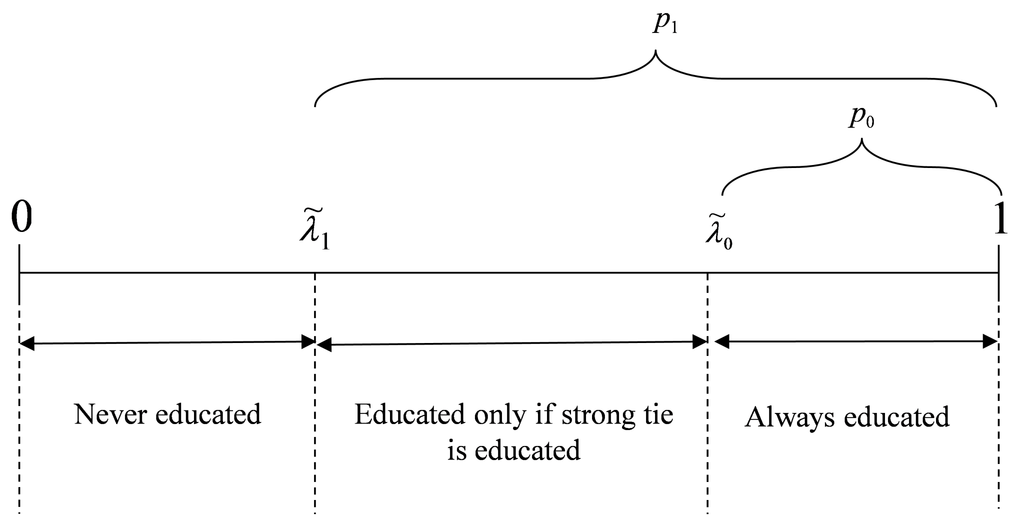

Figure 1: The different probabilities of being educated, without social interactions

3.2 Steady-state equilibrium

So far, we have described what happens within a period. Let us now explain the dynamics of the model and determine the steady-state equilibrium. In equilibrium, the share η_1 of educated individuals is given by:

$$\eta_1 = 1 - \widetilde{\lambda}_0 + (\widetilde{\lambda}_0 - \widetilde{\lambda}_1)\eta_1 = p_0 + (p_1 - p_0)\eta_1$$

Indeed, the fraction of educated individuals is given by either those whose ability lies between $\widetilde{\lambda}_0$ and 1, since they will be educated whatever the status of their partner (see Figure 1), or those whose ability lies between $\widetilde{\lambda}_1$ and $\widetilde{\lambda}_0$ and who are paired with an educated partner (this happens with probability η_1). Rearranging this expression, we obtain:

$$\eta_1 = \frac{p_0}{1 + p_0 - p_1}$$

Using (1) and (2), we get:

$$\eta_1^{dyad} \equiv \eta_1 = \frac{1 - \sqrt{2\overline{U}} - \alpha}{1 - \alpha} \tag{3}$$

When $\overline{U} = 0$, i.e. there is no outside option, in steady state, everybody will be educated ($\eta_1 = 1$). This is because $\widetilde{\lambda}_1 = 0$ and thus every newborn with an educated strong tie will be educated ($p_1 = 1$), while some positive fraction of those with uneducated strong ties will also be educated ($p_0 = 1 - \alpha$). The overall proportion of educated individuals thus increases and tends to 1. On the contrary, if $\sqrt{2\overline{U}} = 1 - \alpha$, i.e. when the outside option is high, then in steady state, no one will be educated ($\eta_1 = 0$), for exactly the opposite reasons.

3.3 The correlation in education between parents and children

We would now like to calculate the *intergenerational correlation in education* between parents and children. Though they do not interact with each other, there is a correlation ensuing from the social network (i.e. strong tie) the parent "transmits" to his child. Let X refer to the educational status of the parent and Y to the educational status of the child. The *intergenerational correlation* is given by:

$$Cor(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$$

where $Cov(X,Y)$ is the *covariance* between the educational status of the parent and the child while $Var(X)$ and $Var(Y)$ are the *variances* of the statuses of the parent and the child. We have:

$$Cov(X,Y)_{_{dyad}} = \mathbb{E}\left[(X)(Y)\right] - \left[\mathbb{E}(X)\right]\left[\mathbb{E}(Y)\right] = \eta_{11} - \eta_1^2$$

$$Var(X)_{_{dyad}} = \mathbb{E}\left[(X - \mathbb{E}(X))^2\right] = \eta_1 - \eta_1^2$$

where η_1 is the *marginal probability* that an individual chooses state 1, i.e. the probability of being educated in steady state (it is given by (3)), and η_{11}, the *joint probability* that an individual is in state 1 and that his father was in state 1.

Individuals can be in either of two different states: educated (state 1) and uneducated (state 0). Dyads, which consist of paired individuals, are, in steady state, in one of three different states:

(i) both members are educated (11);

(ii) one member is educated and the other is uneducated (01) or (10);

(iii) both members are uneducated (00).

The steady state distribution of dyads is given by $\mu = \{\mu_{00}, \mu_{10}, \mu_{01}, \mu_{11}\}$, where μ_{ij} stands for the *fraction of dyads* in state (ij).[7] Obviously, by symmetry, $\mu_{10} = \mu_{01}$. We obtain the following result:

Proposition 1 *When dyads do not interact with each other, the parent-child correlation is equal to:*

$$Cor_{dyad} = \alpha^2 \tag{4}$$

Thus, the correlation between a parent's and a child's educational status is *positive*, even though they never interact with each other. The intuition is simple: when the strong tie is educated, the parent is more likely to be educated. This is also true for the child who benefits from a favorable environment and, as a result, has greater chances of being educated.

In our setting, the correlation (4) increases with α, the cost of interacting with an uneducated strong tie. Indeed, the difference in individual effort $(e_{i1} - e_{i0})$ between meeting an educated and an uneducated strong tie is equal to α. If this difference is small, it is almost the same to be paired with an educated or an uneducated partner, and newborns decide whether to become educated or not independently of the status of their strong tie. If this difference is large, individuals' decisions strongly depend on the status of their partner.

Observe that the quadratic form of the correlation is due to the pattern of influences between parents and children, which transit through the community. In some sense, α measures the intensity of the peer effects. In order for the correlation to exist, the parent has to be subject to this peer effect, and the child too has to be subject to this peer effect. This "two-step" mechanism explains why α appears in a square in (4).

Observe also that $\alpha = p_1 - p_0$ so that $Cor_{dyad} = (p_1 - p_0)^2$. In terms of interpretation, p_1 can be seen as the probability of acting in the *same* way as an educated dyad partner, while p_0 is the probability of acting in the *opposite* way from an uneducated dyad partner. For instance, if $p_1 = 1$ and $p_0 = 0$, which means that individuals always act in accordance with their strong tie, then a parent and a child sharing the same strong tie will necessarily act the same way (in terms of education) and $Cor_{dyad} = 1$. Conversely, if $p_1 = p_0$, then individuals act just as often in the same way and in the opposite way as their strong tie, so that there is no parent-child correlation, i.e. $Cor_{dyad} = 0$.

[7] Alternatively, μ_{ij} can be interpreted as the *fraction of time* a typical dyad spends in state (ij)

This benchmark case illustrates, in a very simple model, how positive correlation can appear as a result of indirect transmission of behavior through peers, as pointed out by Calvó-Armengol and Jackson (2009). However, individuals usually interact with people outside their local community, which might have a significant impact on this correlation. We explore this in the next section by examining how weak ties impact the parent-child correlation.

4 Exogenous social interactions

We assume that individuals are exogenously forced to interact with people outside their own local community. A newborn will spend a fraction ω of his time with *weak ties* and a fraction $1-\omega$ of his time with his *strong tie*.[8]

4.1 Model

The utility of an individual i whose strong tie is uneducated is now given by:[9]

$$\begin{aligned} U_{i0}\left(\lambda_i\right) &= \lambda_i e_{i0} - \frac{1}{2}e_{i0}^2 - \omega\left(1-\eta_1\right)\alpha e_{i0} - \left(1-\omega\right)\alpha e_{i0} \\ &= \lambda_i e_{i0} - \frac{1}{2}e_{i0}^2 - (1-\omega\eta_1)\alpha e_{i0} \end{aligned}$$

where η_1 is the share of educated weak ties in steady state. Individual i, who is born with an uneducated strong tie, spends a fraction ω of his time with a weak tie. This weak tie can either be uneducated (with probability $1-\eta_1$), in which case he bears a penalty of α per unit of effort, or educated (with probability η_1), in which case he does not suffer from any negative peer effect. The rest of his time $(1-\omega)$ is spent with the uneducated strong tie.

As in the previous section, we normalize the cost of interacting with an educated (strong or weak) tie to 0, while the cost of interacting with an uneducated (strong or weak) tie is set to $\alpha > 0$. Therefore, the cost of education depends on the education type of both ties and on the fraction of time the newborn spends with each of them.

[8]Observe that strong ties and weak ties are assumed to be substitutes, i.e. the more time someone spends with weak ties, the less time he has to spend with his strong tie.

[9]The following expressions should be understood as *expected* utilities.

The utility of an individual i who is born with an educated strong tie can be written as:

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2}e_{i1}^2 - \omega(1-\eta_1)\alpha e_{i1}$$

Proceeding as in the previous section, in Appendix 1, we show that

$$p_0 = 1 - \sqrt{2\overline{U}} - (1-\omega\eta_1)\alpha \tag{5}$$

$$p_1 = 1 - \sqrt{2\overline{U}} - (1-\eta_1)\omega\alpha \tag{6}$$

and

$$\eta_1 = \frac{p_0}{1+p_0-p_1} = \frac{1-\alpha-\sqrt{2\overline{U}}}{1-\alpha} \tag{7}$$

Thus, $0 \leq \eta_1 \leq 1$ if

$$0 \leq \sqrt{2\overline{U}} \leq 1-\alpha \tag{8}$$

which also guarantees that p_0 and p_1 are between 0 and 1.

Looking at (7), it is easily verified that the individual probability of being educated, η_1, is increasing in both p_0 and p_1 and decreasing in α. Furthermore, p_0 is increasing in the time spent with weak ties, ω, while p_1 is decreasing with ω. Finally, η_1, p_0 and p_1 are all decreasing in $\overline{U}$.

Note that η_1, the average level of education in the population, is the same as in the previous section and thus given by η_1^{dyad} (see (3)). A larger share of those individuals born in an unfavorable environment will become educated, because the cost of their education is lower due to the time spent outside their own community. But conversely, a smaller share of those individuals born in a favorable environment will be educated, because they will meet uneducated weak ties. The two effects cancel out.

4.2 Steady-state equilibrium and intergenerational correlation

We are now able to determine the intergenerational correlation. We have:

Proposition 2 *Assume (8). With exogenous social mixing, the parent-child correlation is equal to:*

$$Cor_{exo} = (1-\omega)^4\alpha^2 \tag{9}$$

Observe that the correlation (9) can also be expressed as $(1-\omega)^2(p_1-p_0)^2$, where p_1-p_0 can be written as $(p_1-\eta_1)-(p_0-\eta_1)$. As a result, the correlation measures the *bias* in the probability of being educated induced by the chance of having an educated strong tie. Said differently, it is the difference between the conditional probability and the overall probability of being educated. Contrary to the previous section, these biases only appear insofar as individuals interact within their dyad, which happens in proportion $1-\omega$.

The quadratic form in (9) appears for the same reason as before: the parents are influenced by their environment, which in turn influences the children. Note, however, that social interactions act through two channels. First, for a given α, they decrease the difference between p_1 and p_0 by a factor of $1-\omega$ because education decisions depend less on the status of the strong tie. Second, the impact of the first channel only occurs for a fraction of $1-\omega$. The overall effect of social mixing is multiplicative in the effects of both these channels, hence the factor $(1-\omega)^4$.

When α increases, because the influence of a potentially uneducated strong tie is higher, this induces an increase in the correlation, which takes a quadratic form. Conversely, when ω increases, the correlation is reduced because individuals are more influenced by their weak ties than by their strong ties.

This is an interesting result from a policy viewpoint, as the planner can either decrease α or increase ω. Decreasing α has a positive impact on the average education level η_1 and a moderate impact on social mobility. On the other hand, increasing ω has no impact on average education, but has a strong positive impact on social mobility. While individuals with uneducated strong ties are favored by such policies,[10] those with educated strong ties suffer, as they are obliged to spend time outside their advantaged community. This is related to the standard policy debate on desegregation in schools (Guryan, 2004; Rivkin and Welch, 2006), where mixing students from different backgrounds has positive effects on the

[10]This is well-documented. For example, the Moving to Opportunity (MTO) programs in the United States, by giving housing assistance to low-income families, help them relocate to better and richer neighborhoods. The results of most MTO programs (in particular for Baltimore, Boston, Chicago, Los Angeles and New York) show a clear improvement in the education and well-being of participants and better labor market outcomes (see, in particular, Ladd and Ludwig, 2001, Katz et al., 2001, Rosenbaum and Harris, 2001, Chetty et al., 2016).

disadvantaged students but may have a negative effect on the advantaged students.[11] In fact, some studies find that mixing low- and high-ability students can be detrimental for the high-ability students. For example, using data from English secondary schools, Lavy et al. (2012) show that having a large proportion of "bad" peers, identified as students in the bottom 5% of the ability distribution, negatively and significantly affects the cognitive performance of their schoolmates, especially higher-ability students. Similarly, using data from the Longitudinal Study of Young People in England (LSYPE), Mendolia et al. (2016) found that being in a school with a large proportion of low-quality peers can have a significantly detrimental effect on individual achievement, especially for high-quality students.

4.3 Welfare analysis

As stated above, an increase in ω has a positive effect on less educated individuals but can be harmful to highly educated individuals. Because of this trade-off, we now address the welfare consequences of this effect. The total welfare is equal to

$$\mathcal{W} = \int_0^{\widetilde{\lambda}_1} \overline{U} d\lambda + \int_{\widetilde{\lambda}_1}^{\widetilde{\lambda}_0} (1-\eta_1)\overline{U} d\lambda + \int_{\widetilde{\lambda}_1}^{1} \eta_1 U_{i1}\left(\lambda\right) d\lambda + \int_{\widetilde{\lambda}_0}^{1} \left(1-\eta_1\right) U_{i0}\left(\lambda\right) d\lambda \tag{10}$$

The social planner can have two objectives. First, he may want to maximize the sum of utilities (10) of all agents. Second, he may want to minimize the impact of family background on the child's educational attainment, a policy that has been adopted by most democratic societies (Björklund and Salvanes, 2011). However, these two objectives are contradictory.

Proposition 3

(*i*) *If the objective of the planner is to maximize total welfare (10), then it is optimal to set the time spent with weak ties to* $\omega^o = 0$.

(*ii*) *If the objective is to minimize the intergenerational correlation, then it is optimal to set the time spent with weak ties to* $\omega^o = 1$.

[11] See Sáez-Martí and Zenou (2012) who obtain a similar result in a different context of affirmative action at the workplace.

This proposition shows that, depending on the objective function, the efficient outcome may be very different. Indeed, when maximizing total welfare (case (i) in the proposition), the social planner accounts for both the positive effect on individuals from unfavorable environments and the negative effect on students from favorable environments. Because the latter effect is weaker than the former, the loss of utility for a person with an educated strong tie meeting uneducated weak ties is not sufficiently compensated by the gain of utility for a person with an uneducated strong tie meeting educated weak ties. As a result, the planner finds it optimal to set $\omega^o = 0$. When the planner wants to minimize the intergenerational correlation (case (ii)), he does not want people to be stuck in their initial environment of strong ties and thus chooses $\omega^o = 1$.

Remark 1 *Both the aggregate welfare and the correlation are decreasing and convex. Thus, increasing ω decreases both the correlation and the welfare very quickly. There is no room for "intermediate" policies.*

5 Endogenous social interactions

We now endogenize ω so that individuals choose both educational effort and time spent with their strong (or weak) tie. The timing is now as follows. At each period of time, a person (the parent) chosen at random dies and is replaced by a newborn (the child) who takes his place in the dyad. The child then discovers the education type of his strong tie (educated or uneducated), as well as his λ_i. He then optimally decides ω_{ij}, the time spent with weak and strong ties and then e_{ij}, the optimal education effort level. As usual, we solve the model backward.

5.1 Model

The utility of individual i who chooses ω_{ij} and e_{ij} is now given by:

$$U_{i0}\left(\lambda_i\right) = \lambda_i e_{i0} - \frac{1}{2}e_{i0}^2 - (1-\omega_{i0}\eta_1)\alpha e_{i0}$$

$$U_{i1}\left(\lambda_i\right) = \lambda_i e_{i1} - \frac{1}{2}e_{i1}^2 - \omega_{i1}\left(1-\eta_1\right)\alpha e_{i1}$$

Denote:

$$\widetilde{\lambda}_0 \equiv \sqrt{2\overline{U}} + (1-\eta_1)\,\alpha \quad \text{and} \quad \widetilde{\lambda}_1 \equiv \sqrt{2\overline{U}} \tag{11}$$

Proposition 4

(i) *For individuals who inherited an* ***uneducated strong tie*** *from their parent, their choice of meeting weak ties depends on their initial ability λ_i. If $\lambda_i < \widetilde{\lambda}_0$, they choose to never meet weak ties, i.e. $\omega_{i0}^* = 0$, while those for which $\lambda_i \geq \widetilde{\lambda}_0$ always want to meet weak ties $\omega_{i0}^* = 1$.*

(ii) *Individuals who inherited an* ***educated strong tie*** *from their parent never want to meet weak ties, i.e. $\omega_{i1}^* = 0$.*

This result is quite intuitive and is in line with what should be expected: individuals born in a favorable environment do not want to interact with weak ties who are potentially uneducated. Conversely, individuals born in an unfavorable environment, with an initial ability that is high enough, want to spend as much time as possible outside their community to avoid the penalty α. Observe that, if we introduce a cost of socialization, $-\frac{1}{2}\omega^2$, into the utility function, then we will obtain interior instead of $(0,1)$ solutions for the choice of weak ties; however the results and intuitions are overall unchanged (see Appendix 2).

5.2 Steady-state equilibrium and intergenerational correlation

Proceeding as in the previous sections, we show in Appendix 1 that η_1 is the solution to

$$F(\eta_1) \equiv \alpha\eta_1^2 + \eta_1\,(1-2\alpha) - 1 + \sqrt{2\overline{U}} + \alpha = 0 \tag{12}$$

Proposition 5 *If $\sqrt{2\overline{U}} < 1-\alpha$, there exists a unique solution $\eta_1^* \in [0,1]$ to (12). It is such that*

$$\eta_1^* > \eta_1^{dyad} = \frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}$$

Furthermore,

$$\frac{\partial \eta_1^*}{\partial \overline{U}} < 0 \quad \textit{and} \quad \frac{\partial \eta_1^*}{\partial \alpha} < 0$$

The average level of education in the population is higher in the model with endogenous choices of interaction than in the previous models, where $\eta_1 \equiv \eta_1^{dyad}$ was defined by (3). It is higher than in the model without social interactions (benchmark model of Section 3) because the individuals with uneducated strong ties can get out and reduce the penalty α from the negative peer effect. It is also higher than in the model with exogenous social interactions (Section 4) because the individuals with educated strong ties do not suffer from the negative peer effect imposed by the planner.

Proposition 6 *Assume $\sqrt{2\overline{U}} < 1-\alpha$. Then the correlation between the educational statuses of the parent and the child is equal to:*

$$Cor_{net} = \left(1 - \eta_1^*\right)^2 \alpha^2 \tag{13}$$

Furthermore,

$$\frac{\partial Cor_{net}}{\partial \overline{U}} > 0 \quad \textit{and} \quad \frac{\partial Cor_{net}}{\partial \alpha} > 0$$

Contrary to the previous sections, the correlation Cor_{net} (positively) depends on $\overline{U}$. This is due to the fact that a change in $\overline{U}$ affects both p_0 and p_1. In Sections 3 and 4, both probabilities were affected in the same way and cancelled out in the quantity $p_1 - p_0$. Here, this is no longer the case because of the asymmetry in the behavior of individuals, depending on the status of their strong tie.

When interactions are chosen by the individuals, there is also a reduction in intergenerational correlation, but it is not as great as when social mixing is imposed. However, it is worth noting that when the planner implements a policy reducing the cost α of unfavorable environments, this has two effects. First, it directly reduces the correlation through the term α^2. Second, it also reduces the correlation indirectly through the increase in η_1^* that it triggers.

6 Empirical evidence

Although our paper focuses on intergenerational correlation in education, our model delivers some interesting empirical implications regarding socialization. We have shown that when individuals are allowed to choose ω, the level of social interactions with weak ties, only people who inherited an uneducated strong tie and are sufficiently gifted will want to meet weak ties.

While the result is extreme (either full socialization or no socialization), we wish to explore the following predictions: (i) socialization effort should be higher for people from disadvantaged backgrounds than for people from advantaged backgrounds and (ii) among the disadvantaged, socialization effort should be increasing in ability.

We stress that this exercise is not a formal test of our model. Rather, it aims at providing evidence to support our model.

Prediction 1: socialization effort is higher for people from disadvantaged backgrounds.

There are different ways of defining people from disadvantaged backgrounds, but an obvious one is to define them as ethnic minorities. Indeed, in most countries, ethnic minorities are considered as having disadvantaged backgrounds since they usually have a lower education level and experience higher unemployment rates (see e.g. Neil, 2006, or Hanushek, 2016, for the United States). There is, unfortunately, very little evidence on the relationship between socialization and individuals from different ethnic backgrounds. We therefore provide our own evidence on this relationship.

We use the National Longitudinal Survey of Adolescent Health (AddHealth).[12] The AddHealth survey was designed to study the impact of the social environment (i.e. friends, family, neighborhood and school) on adolescents' behavior in the United States by collecting data on students in grades 7-12 from a nationally representative sample of roughly 130

[12]This research uses data from AddHealth, a project directed by Kathleen Mullan Harris and designed by J. Richard Udry, Peter S. Bearman, and Kathleen Mullan Harris at the University of North Carolina at Chapel Hill, and funded by grant P01-HD31921 from the Eunice Kennedy Shriver National Institute of Child Health and Human Development, with cooperative funding from 23 other federal agencies and foundations. Special acknowledgment to Ronald R. Rindfuss and Barbara Entwisle for assistance in the original design. Information on how to obtain the Add Health data files is available on the AddHealth website (http://www.cpc.unc.edu/addhealth). No direct support was received from grant P01-HD31921 for this analysis.

private and public schools in the years 1994-95.

To measure the socialization effort of each student, we define ω_i as the sum of *after-school activities* of student i that require interactions with others. Each of these after-school activities takes a value of 1 if the student performs the activity, and 0 otherwise. The activities are: dance, music, any kind of sports, writing or editing the school newspaper, honors club, foreign language clubs, participating in the school council, other clubs. The activities are listed in Table A1 (in Appendix 3) together with the other variables used. Table A1 also provides some descriptive statistics for the different variables.

In Table 1, we run a simple OLS regression where the endogenous variable is ω_i, as defined above, and the main explanatory variable is a dummy variable, which is equal to 1 if the student is non-white (black or Hispanic) and 0 if the student is white. We present the results without controls, as well as results where we control for the grade the student is in and the gender of the student. We chose these particular controls among many possible others, because students are most likely to interact with students in the same grade and with students of the same gender.

As predicted by our model, we find that students from disadvantaged backgrounds (non-white students) socialize much more than students from advantaged backgrounds (white students).

[*Insert Table* 1 *here*]

Prediction 2: Among the disadvantaged, socialization effort is increasing in ability.

To check the second relationship between socialization effort and ability among disadvantaged students, we again use the AddHealth data and measure ability by their grade in Mathematics, as well as their grade in other subjects such as English, Science and History.

Table 2 reports the OLS regression for the non-white students, where the endogenous variable is ω_i, as defined above, and the main explanatory variable is the grade obtained in Mathematics, English, Science, History. More precisely, in columns (1), (2), (3) and (4), we list the grade for each subject separately while, in column (5), we combine all the grades.

We control for gender and the grade the student is in.[13]

[*Insert Table* 2 *here*]

As predicted by our model, we find that, among the students from disadvantaged backgrounds (non-white students), there is a significant and positive correlation between ability and socialization effort. Indeed, in Table 2 we see that students with higher grades (in any topic) socialize significantly more than those with lower grades. We are fully aware that what we obtain are only correlations, and we do not claim that the effect is causal.

7 Conclusion

In this paper, we developed a dynastic model where, at each period of time, a person (the parent) dies and is replaced by a newborn (the child). The newborn takes exactly the same position as the parent in the dyad and thus interacts with the same person (strong tie), i.e. the local community of his parent. There is therefore no vertical transmission but only horizontal transmission via peer and neighborhood effects.

We show that there is a substantial intergenerational correlation between parent's and child's outcomes that transits through the environment that parent and child share. While policies aimed at increasing social mobility usually rely on individuals' idiosyncratic characteristics, this model provides us with an alternative channel for public interventions.

In this very simple framework, we analyze the impact of social interactions on the intergenerational correlation in education, and find that it is a very powerful tool for promoting social mobility. When the level of social interactions is centrally decided, we show that the correlation decreases very fast, while the average education level remains constant. The price to pay for this very rapid decrease is that welfare also goes down fast when the level of social interactions increases. When the level of social interactions is decided individually, the correlation decreases, although less rapidly than with exogenous social interactions. In turn, the average education level increases.

[13] Observe that, in Table 1, the number of students was equal to 88,808 while, in Table 2, this number reduced to between 4,717 and 6,141 (depending on the subject) because, in Table 2, we only consider non-white students.

We believe that this paper sheds some light on the effect of the inherited neighborhood and peers on children's education outcomes. There is a small, but growing, literature that considers the impact of 'initial conditions' in determining labor market outcomes (see e.g. Åslund and Rooth, 2007; Almond and Currie, 2011). Recent research has also shown the importance of the birthplace for long-run outcomes (Bosquet and Overman, 2016) and puts forward the role of the geography of intergenerational mobility (Chetty et al., 2016; Chetty and Hendren, 2015; Del Bello et al., 2015). It does not, however, distinguish between direct parental and social influences on education. By ignoring the former and focusing solely on the latter, our model provides predictions that help us understand the impacts of a change in social environment on education.

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APPENDIX 1: Proofs

Proof of Proposition 1: Since η_1 is determined by (3), we need to derive the joint probability η_{11} . We get

$$\eta_{11} = p_1\mu_{11} + \frac{1}{2}p_0\mu_{01} + \frac{1}{2}p_0\mu_{10}$$

Indeed, in order to have both a newborn and his father in state 1, it is necessary that the father was in state 1 in his dyad. This happens with probability 1 if the individual randomly chosen to die was in a dyad in state (11), and there is a proportion μ_{11} of these dyads, or with probability 1/2 if the individual was in a (01) or (10) dyad. These dyads are in proportion μ_{01} and μ_{10}.

In the first case, the son inherits an educated strong tie, he then gets educated with probability p_1. In the second case, the son inherits an uneducated strong tie, he then gets educated with probability p_0.

Next, observe that

$$\mu_{11} = p_1\eta_1$$

Indeed, for a new dyad to be in state (11), it has to be that a newborn is born in a dyad with an educated strong tie (with probability η_1) and gets educated (with probability p_1). Using a similar argument, we also have:

$$\mu_{10} = \mu_{01} = \frac{1}{2}(1-p_1)\eta_1 + \frac{1}{2}p_0(1-\eta_1) = (1-p_1)\eta_1$$

and

$$\mu_{00} = (1-p_0)(1-\eta_1)$$

From these three expressions, we obtain:

$$\begin{aligned} \eta_{11} &= p_1\mu_{11} + p_0\mu_{01} \\ &= \eta_1\left[p_1^2 + p_0(1-p_1)\right] \end{aligned}$$

Finally, we have:

$$Cor(X,Y)_{dyad} = \frac{\eta_{11}-\eta_1^2}{\eta_1(1-\eta_1)} = \frac{p_1^2 + p_0(1-p_1) - \eta_1}{1-\eta_1}$$

which, after some manipulations, leads to (4). ∎

Derivation of p_1 and p_0 given by (5) and (6) and of η_1 given by (7): The utility functions are given by:

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2}e_{i0}^2 - (1-\omega\eta_1)\alpha e_{i0}$$

and

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2}e_{i1}^2 - \omega(1-\eta_1)\alpha e_{i1}$$

The first-order conditions give:

$$e_{i0} = \max\{0, \lambda_i - (1-\omega\eta_1)\alpha\} \tag{14}$$

$$e_{i1} = \max\{0, \lambda_i - (1-\eta_1)\omega\alpha\} \tag{15}$$

Plugging back e_{ij} in the utility function and accounting for the outside option $\overline{U}$ yields:

$$U_{i0}(\lambda_i) = \max\left\{\overline{U}, \frac{[\lambda_i - (1-\omega\eta_1)\alpha]^2}{2}\right\} \tag{16}$$

$$U_{i1}(\lambda_i) = \max\left\{\overline{U}, \frac{[\lambda_i - (1-\eta_1)\omega\alpha]^2}{2}\right\} \tag{17}$$

We can determine the threshold values $\widetilde{\lambda}_0$ and $\widetilde{\lambda}_1$ as follows:

$$\widetilde{\lambda}_0 = \sqrt{2\overline{U}} + (1-\omega\eta_1)\alpha \tag{18}$$

$$\widetilde{\lambda}_1 = \sqrt{2\overline{U}} + (1-\eta_1)\omega\alpha \tag{19}$$

The probability p_0 that an individual with an uneducated strong tie will get educated and the probability p_1 that an individual with an educated strong tie will get educated are given by:

$$p_0 = 1 - \widetilde{\lambda}_0 = 1 - \sqrt{2\overline{U}} - (1-\omega\eta_1)\alpha$$

$$p_1 = 1 - \widetilde{\lambda}_1 = 1 - \sqrt{2\overline{U}} - (1-\eta_1)\omega\alpha$$

which are (5) and (6). In order to close the model, we determine the value of η_1 as follows:

$$
\begin{aligned}
\eta_1 &= \eta_1 \{(1-\omega)p_1 + \omega [\eta_1 p_1 + (1-\eta_1)p_0]\} \\
&\quad +(1-\eta_1)\{(1-\omega)p_0 + \omega [\eta_1 p_1 + (1-\eta_1)p_0]\}
\end{aligned}
$$

Indeed, in equilibrium, a newborn gets educated if either (i) he meets an educated strong tie (probability η_1), spends a fraction $1-\omega$ of his time with this strong tie and gets educated (probability p_1) and spends a fraction ω of his time with a weak tie who can be either educated and the newborn gets educated (probability $\eta_1 p_1$) or who can be uneducated and the newborn gets educated (probability $(1-\eta_1)p_0$) or (ii) he meets an uneducated strong tie (probability $1-\eta_1$), spends a fraction $1-\omega$ of his time with this strong tie and gets educated with probability p_0 and spend a fraction ω of his time with a weak tie who can be either educated and the newborn gets educated (probability $\eta_1 p_1$) or who can be uneducated and the newborn gets educated (probability $(1-\eta_1)p_0$).

This expression can be simplified and we easily obtain:

$$
\eta_1 = \frac{p_0}{1+p_0-p_1} \tag{20}
$$

By replacing p_0 and p_1 by their values in (5) and (6), and solving in η_1, we easily obtain (7). ∎

Proof of Proposition 2: The joint probability to have both a newborn and his father educated, η_{11}, is given by

$$
\eta_{11} = (1-\omega)(\mu_{11}p_1 + \mu_{10}p_0) + \omega(\mu_{11}+\mu_{10})[\eta_1 p_1 + (1-\eta_1)p_0]
$$

Indeed, in order to have both a newborn and his father in state 1, there are two possibilities:

(i) either the son interacts within his dyad (probability $(1-\omega)$). In that case, the father has to be in state 1, which is the case with probability 1 if it is a (11) dyad (μ_{11}) and with probability 1/2 if it is a (10) or (01) dyad (μ_{10} or μ_{01}). The son will get educated with probability p_1 if the father was in the dyad 11 and with probability p_0 if the father was in a dyad (10) or (01).

(ii) or the son interacts with a weak tie (with probability ω). In that case, the father has to be educated (with probability $\mu_{11}+\frac{1}{2}(\mu_{10}+\mu_{01})$) and then the son gets educated with probability p_1 if he meets an educated individual (with probability η_1) and with probability p_0 if he meets an uneducated individual (with probability $(1-\eta_1)$).

In this framework, μ_{11} is given by

$$\mu_{11} = \eta_1[(1-\omega)p_1 + \omega(\eta_1 p_1 + (1-\eta_1)p_0)]$$

and μ_{10} is given by

$$\begin{aligned}\mu_{10} &= \frac{1}{2}(1-\eta_1)\left\{(1-\omega)p_0 + \omega\left[\eta_1 p_1 + (1-\eta_1)p_0\right]\right\} \\ &+\frac{1}{2}\eta_1\left\{(1-\omega)(1-p_1) + \omega\left[\eta_1(1-p_1) + (1-\eta_1)(1-p_0)\right]\right\}\end{aligned}$$

Indeed, for a (10) dyad to form, either an individual meets a type-0 individual (with probability $(1-\eta_1)$) and gets educated (either by staying within the dyad ($(1-\omega)p_0$) or outside the dyad ($\omega(\eta_1 p_1 + (1-\eta_1)p_0)$), or an individual meets a type-1 individual (with probability η_1) and decides not to educate (either by staying within the dyad ($(1-\omega)(1-p_1)$) or outside the dyad ($\omega(\eta_1(1-p_1) + (1-\eta_1)(1-p_0))$)).

Observing that $\eta_1 p_1 + (1-\eta_1)p_0 = \eta_1$, that $\eta_1(1-p_1) = (1-\eta_1)p_0$ and that $\eta_1(1-p_1) + (1-\eta_1)(1-p_0) = 1-\eta_1$, we have:

$$\mu_{11} = \eta_1[(1-\omega)p_1 + \omega\eta_1]$$

$$\mu_{10} = \frac{1}{2}(1-\eta_1)[(1-\omega)p_0 + \omega\eta_1] + \frac{1}{2}\eta_1[(1-\omega)(1-p_1) + \omega(1-\eta_1)]$$

$$\mu_{10} = \frac{1}{2}(1-\eta_1)(2\omega\eta_1) + \frac{1}{2}(1-\omega)[\eta_1(1-p_1) + (1-\eta_1)p_0]$$

$$\mu_{10} = \omega(1-\eta_1)\eta_1 + (1-\omega)\eta_1(1-p_1)$$

Furthermore, we have:

$$\mu_{11}p_1 + \mu_{10}p_0 = \eta_1[(1-\omega)(p_1^2 + p_0(1-p_1)) + \omega(p_1\eta_1 + p_0(1-\eta_1))]$$

$$\mu_{11}p_1 + \mu_{10}p_0 = \eta_1[(1-\omega)(p_1^2 + p_0(1-p_1)) + \omega\eta_1]$$

This implies that

$$\eta_{11} = (1-\omega)^2\eta_1(p_1^2 + p_0(1-p_1)) + \omega(1-\omega)\eta_1^2 + \omega\eta_1^2$$

and

$$
\begin{aligned}
\frac{\eta_{11}-\eta_1^2}{\eta_1} &= (1-\omega)^2\left[p_1^2+p_0(1-p_1)\right]+(2\omega-\omega^2-1)\eta_1 \\
&= (1-\omega)^2\left[p_1^2+p_0(1-p_1)-\eta_1\right]
\end{aligned}
$$

Finally

$$
\begin{aligned}
Cor_{exo} &= \frac{\eta_{11}-\eta_1^2}{\eta_1(1-\eta_1)} \\
&= (1-\omega)^2\frac{p_1^2+p_0(1-p_1)-\eta_1}{1-\eta_1} \\
&= (1-\omega)^2(p_1-p_0)^2 \\
&= (1-\omega)^4\alpha^2
\end{aligned}
$$

which is (9). ▮

Proof of Proposition 3: Let us first analyze (i). The total welfare is given by (10), which is

$$
\mathcal{W}=\int_0^{\widetilde{\lambda}_1}\overline{U}d\lambda+\int_{\widetilde{\lambda}_1}^{\widetilde{\lambda}_0}(1-\eta_1)\overline{U}d\lambda+\int_{\widetilde{\lambda}_1}^{1}\eta_1 U_{i1}(\lambda)\,d\lambda+\int_{\widetilde{\lambda}_0}^{1}(1-\eta_1)\,U_{i0}(\lambda)\,d\lambda
$$

The first two terms can be calculated and it is easily shown that:

$$
\int_0^{\widetilde{\lambda}_1}\overline{U}d\lambda+\int_{\widetilde{\lambda}_1}^{\widetilde{\lambda}_0}(1-\eta_1)\,\overline{U}d\lambda=\widetilde{\lambda}_0\overline{U}-\eta_1\overline{U}\left(\widetilde{\lambda}_0-\widetilde{\lambda}_1\right)
$$

which using (18) and (19) gives

$$
K\equiv\int_0^{\widetilde{\lambda}_1}\overline{U}d\lambda+\int_{\widetilde{\lambda}_1}^{\widetilde{\lambda}_0}(1-\eta_1)\,\overline{U}d\lambda=\overline{U}\left[\sqrt{2\overline{U}}+(1-\eta_1)\,\alpha\right]
$$

which is independent of ω (see (7)) and thus we can ignore these first two terms. So the planner maximizes

$$
\int_{\widetilde{\lambda}_1}^{1}\eta_1 U_{i1}(\lambda)\,d\lambda+\int_{\widetilde{\lambda}_0}^{1}(1-\eta_1)\,U_{i0}(\lambda)\,d\lambda
$$

Using (16) and (17), we have:

$$
\begin{aligned}
&\int_{\widetilde{\lambda}_1}^{1}\eta_1 U_{i1}(\lambda)\,d\lambda+\int_{\widetilde{\lambda}_0}^{1}(1-\eta_1)\,U_{i0}(\lambda)\,d\lambda \\
=\ &\frac{1}{6}\left\{\eta_1\left[(\lambda_i-\omega\alpha+\omega\eta_1\alpha)^3\right]_{\widetilde{\lambda}_1}^{1}+(1-\eta_1)\left[(\lambda_i-\alpha+\omega\eta_1\alpha)^3\right]_{\widetilde{\lambda}_0}^{1}\right\}
\end{aligned}
$$

Using (18) and (19), we see that

$$\widetilde{\lambda}_1 - \omega\alpha + \omega\eta_1\alpha = \sqrt{2\overline{U}} = \widetilde{\lambda}_0 - \alpha + \omega\eta_1\alpha$$

As a result, we obtain:

$$\begin{aligned}\mathcal{W} &= K - \frac{1}{6}\left(\sqrt{2\overline{U}}\right)^3 + \frac{1}{6}\left[\eta_1\left(1-\omega\alpha+\omega\eta_1\alpha\right)^3 + \left(1-\eta_1\right)\left(1-\alpha+\omega\eta_1\alpha\right)^3\right] \\ &= K' + \frac{1}{6}\left[\eta_1(1-\omega\alpha+\omega\eta_1\alpha)^3 + (1-\eta_1)(1-\alpha+\omega\eta_1\alpha)^3\right]\end{aligned}$$

where $K' \equiv K - \frac{1}{6}\left(\sqrt{2\overline{U}}\right)^3$. We have:

$$\begin{aligned}\frac{\partial\mathcal{W}}{\partial\omega} &= \frac{1}{2}\left(1-\eta_1\right)\alpha\eta_1\left[\left(1-\alpha+\omega\eta_1\alpha\right)^2 - \left(1-\omega\alpha+\omega\eta_1\alpha\right)^2\right] \\ &= -\frac{1}{2}\left(1-\eta_1\right)\alpha^2\eta_1\left(1-\omega\right)\left(2\omega\eta_1\alpha+1-\alpha+1-\omega\alpha\right)\end{aligned}$$

Since, according to (8), $\alpha < 1$, then

$$\frac{\partial\mathcal{W}}{\partial\omega} \leq 0$$

As a result, the optimal solution is $\omega^o = 0$.

Let us now analyze (ii). The correlation is given by (9), that is

$$Cor_{exo} = (1-\omega)^2(p_1-p_0)^2 = (1-\omega)^4\alpha^2$$

Since $\frac{\partial Cor_{exo}}{\partial\omega} < 0$, it is should be clear that if the planner wants to minimize the correlation, then the solution to this program is $\omega^* = 1$. ∎

Proof of Remark 1: We have shown in the proof of Proposition 3 that both $\frac{\partial\mathcal{W}}{\partial\omega} < 0$ and $\frac{\partial Cor_{exo}}{\partial\omega} < 0$. It is straightforward to verify that $\frac{\partial^2 Cor_{exo}}{\partial\omega^2} > 0$. For $\mathcal{W}$, we have:

$$\frac{\partial^2\mathcal{W}}{\partial\omega^2} = \frac{\eta_1\alpha^2(\eta_1-1)}{2}[-1+\eta_1\alpha-2\omega\eta_1\alpha+\omega\alpha]$$

which has a constant sign over $[0,1]$ and

$$\frac{\partial^2\mathcal{W}}{\partial\omega^2}|_{\omega=0} > 0$$

This proves the result. ∎

Proof of Proposition 4:

First-order conditions on efforts yield:

$$e_{i0}^{*}=\max\left\{0,\lambda_i-\left(1-\omega_{i0}\eta_1\right)\alpha\right\} \tag{21}$$

$$e_{i1}^{*}=\max\left\{0,\lambda_i-\left(1-\eta_1\right)\omega_{i1}\alpha\right\}$$

which imply:

$$U_{i0}^{*}\left(\lambda_i\right)=\frac{1}{2}\left(e_{i0}^{*}\right)^2=\max\left\{\overline{U},\frac{\left[\lambda_i-\left(1-\omega_{i0}\eta_1\right)\alpha\right]^2}{2}\right\} \tag{22}$$

$$U_{i1}^{*}\left(\lambda_i\right)=\frac{1}{2}\left(e_{i1}^{*}\right)^2=\max\left\{\overline{U},\frac{\left[\lambda_i-\omega_{i1}\left(1-\eta_1\right)\alpha\right]^2}{2}\right\} \tag{23}$$

Given these expressions, point (ii) is obvious. Let us show (i). Individuals can get $\overline{U}$ if they exert no effort (in which case we assume they set $\omega_{i0}^{*}=0$). Those who can get more than $\overline{U}$ by exerting an effort will get

$$U_{i0}^{*}\left(\lambda_i\right)=\frac{\left[\lambda_i-\left(1-\omega_{i0}\eta_1\right)\alpha\right]^2}{2},$$

which is maximized at $\omega_{i0}^{*}=1$.

These individuals get

$$U_{i0}^{*}\left(\lambda_i\right)=\frac{\left[\lambda_i-\left(1-\eta_1\right)\alpha\right]^2}{2},$$

which is greater than $\overline{U}$ only if $\lambda_i>\sqrt{2\overline{U}}+(1-\eta_1)\alpha\equiv\widetilde{\lambda}_0$. Therefore, $\omega_{i0}^{*}=1$ if $\lambda_i>\widetilde{\lambda}_0$, and $\omega_{i0}^{*}=0$ otherwise. ∎

Derivation of η_1 given by (12): We have:

$$p_0=1-\sqrt{2\overline{U}}-\left(1-\eta_1\right)\alpha \tag{24}$$

and

$$p_1=1-\sqrt{2\overline{U}} \tag{25}$$

We also have that:

$$U_{i0}\left(\lambda_i\right)=\max\left\{\overline{U},\frac{\left[\lambda_i-\left(1-\eta_1\right)\alpha\right]^2}{2}\right\}$$

$$U_{i1}(\lambda_i) = \max\left\{\overline{U}, \frac{\lambda_i^2}{2}\right\}$$

Let us compute the value of η_1. Again, individuals whose ability exceeds $\widetilde{\lambda}_0$ will get educated whatever the status of their strong tie. They represent a mass of size p_0. Those whose ability is lower than $\widetilde{\lambda}_1$ will never get educated while those such that $\widetilde{\lambda}_0 > \lambda_i > \widetilde{\lambda}_1$ will get educated only if they meet an educated strong tie. There is a mass $p_1 - p_0$ of these individuals. As a result,

$$\eta_1 = p_0 + (p_1 - p_0)\eta_1$$

Using the values of p_0 and p_1, we obtain:

$$F(\eta_1) \equiv \alpha\eta_1^2 + \eta_1(1-2\alpha) - 1 + \sqrt{2\overline{U}} + \alpha = 0$$

which is (12). ▮

Proof of Proposition 5: We have $F(0) = \sqrt{2\bar{U}} - 1 + \alpha < 0$ and $F(1) = \sqrt{2\bar{U}} > 0$, so there is one solution η_1^* between 0 and 1. Furthermore, for $\eta_1 \in [0,1]$, $F(\eta_1) < 0$ if and only if $\eta_1 < \eta_1^*$. It is then enough to check that $F(\frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}) < 0$. After some manipulations we get

$$Sgn\left[F(\frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha})\right] = Sgn\left[\alpha(\frac{\sqrt{2\overline{U}}}{1-\alpha} - 1)\right] < 0$$

Thus $\eta_1^* > \frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}$.

As for the comparative statics, we know that around η_1^*, $\frac{\partial F}{\partial \eta_1} > 0$. Furthermore, $\frac{\partial F}{\partial \overline{U}} > 0$, and $\frac{\partial F}{\partial \alpha} = (1-\eta_1)^2 > 0$, so we get the desired conclusion. ▮

Proof of Proposition 6: Let us calculate the correlation between the father and son. This correlation is given by:

$$Cor_{net} = \frac{\eta_{11} - \eta_1^2}{\eta_1(1-\eta_1)} \tag{26}$$

We have

$$\eta_1 = \eta_1 p_1 + (1-\eta_1)p_0$$

The steady-state distributions are given by

$$\mu_{11} = \eta_1 p_1$$

$$\mu_{10} = \frac{1}{2}\eta_1(1-p_1) + \frac{1}{2}(1-\eta_1)p_0$$

Indeed, for a μ_{11} dyad to be formed, it must be that a newborn meets an educated strong tie (probability η_1) and that he gets educated. But since $\omega_{i1}^* = 0$ in equilibrium, he only interacts with his strong tie and gets educated with probability p_1.

Accordingly, for a μ_{10} (or a μ_{01}) dyad to be formed, it must be that either a newborn meets an educated strong tie (he then sets $\omega_{i1} = 0$) and does not get educated, with probability $(1-p_1)$ or the newborn meets a non educated strong tie (probability $1-\eta_1$, he then sets $\omega_{i0} = 1$) and will get educated if $\lambda_i > \widetilde{\lambda}_0$. This happens with probability p_0.

Thus we obtain:

$$\begin{aligned}
\mu_{10} &= \frac{1}{2}\eta_1(1-p_1) + \frac{1}{2}(1-\eta_1)p_0 \\
&= \frac{p_0(1-p_1)}{2[1-(p_1-p_0)]} + \frac{(1-p_1)p_0}{2[1-(p_1-p_0)]} \\
&= \frac{p_0(1-p_1)}{[1-(p_1-p_0)]} \\
&= (1-p_1)\eta_1
\end{aligned}$$

In turn η_{11} is given by

$$\eta_{11} = \mu_{11}p_1 + \mu_{10}p_0$$

Indeed, for the father and the son to be both educated, it must be the case that the father was educated and that the son gets educated. Either the father was part of a μ_{11} dyad and then the son meets an educated strong tie, in which case he gets educated with probability p_1, or the father was in a μ_{10} dyad and then the son meets an uneducated strong tie, in which case he only interacts with weak ties and gets educated with probability p_0. Replacing for μ_{11} and μ_{10}, we obtain

$$\eta_{11} = \eta_1 p_1^2 + (1-p_1)p_0\eta_1$$

Hence

$$\begin{aligned}
Cor_{net} &= \frac{\eta_{11} - \eta_1^2}{\eta_1(1-\eta_1)} \\
&= \frac{\eta_1 p_1^2 + (1-p_1)p_0\eta_1 - \eta_1^2}{\eta_1(1-\eta_1)} \\
&= \frac{p_1^2 + (1-p_1)p_0 - \eta_1}{1-\eta_1}
\end{aligned}$$

Because

$$\eta_1 = \frac{p_0}{1-(p_1-p_0)}$$

we have

$$1-\eta_1 = \frac{1-p_1}{1-(p_1-p_0)}$$

Plugging these values in the expression above, we have:

$$\begin{aligned}
Cor_{net} &= \frac{p_1^2+(1-p_1)\,p_0-\eta_1}{1-\eta_1} \\
&= \frac{\left[p_1^2+(1-p_1)\,p_0\right]\left[1-(p_1-p_0)\right]-p_0}{1-p_1} \\
&= \frac{\left[p_1\,(p_1-p_0)+p_0\right]\left[1-(p_1-p_0)\right]-p_0}{1-p_1} \\
&= (p_1-p_0)\,\frac{p_1\left[1-(p_1-p_0)\right]-p_0}{1-p_1} \\
&= (p_1-p_0)^2
\end{aligned}$$

Using the values of p_1 and p_0, we finally obtain:

$$Cor_{net} = (1-\eta_1)^2\,\alpha^2$$

which is (13). ∎

APPENDIX 2: The model with socialization costs

Let us extend our model by introducing a socialization cost equal to $-\frac{1}{2}\omega_{ij}^2$ so that:

$$U_{i0}(\lambda_i) = \lambda_i e_{i0} - \frac{1}{2}e_{i0}^2 + \omega_{i0}\left[-(1-\eta_1)\alpha\right]e_{i0} - (1-\omega_{i0})\alpha e_{i0} - \frac{1}{2}\omega_{i0}^2$$

$$U_{i1}(\lambda_i) = \lambda_i e_{i1} - \frac{1}{2}e_{i1}^2 + \omega_{i1}\left[-(1-\eta_1)\alpha\right]e_{i1} - \frac{1}{2}\omega_{i1}^2$$

Let $\widetilde{\lambda}_0 \equiv \sqrt{2\overline{U}(1-\eta_1^2\alpha^2)} + \alpha$ and $\widetilde{\lambda}_1 \equiv \sqrt{2\overline{U}}$. Then

Proposition 7

(*i*) *For individuals who inherited an* ***uneducated strong tie*** *from their father, their choice of meeting weak ties depend on their initial ability* λ_i*. If* $\lambda_i < \widetilde{\lambda}_0$*, they choose to never meet weak ties, i.e.* $\omega_{i0}^* = 0$*, while, for those for which* $\lambda_i \geq \widetilde{\lambda}_0$*, they set*

$$\omega_{i0}^* = \eta_1\alpha\left[\frac{\lambda_i - \alpha}{1-\eta_1^2\alpha^2}\right] > 0$$

(*ii*) *Individuals who inherited an* ***educated strong tie*** *from their father never want to meet weak ties, i.e.* $\omega_{i1}^* = 0$*.*

The proof of this proposition is similar to that of Proposition 4 so we omit it. Compared to the result of Proposition 4 we see that the only difference is in case (i) when $\lambda_i \geq \widetilde{\lambda}_0$. Indeed, in that case, introducing a quadratic socialization cost changes ω_{i0}^* from one to an interior solution.

As above, the steady-state level of education is given by:

$$\eta_1 = \frac{p_0}{1-(p_1-p_0)}$$

where $p_0 = 1 - \widetilde{\lambda}_0$ and $p_1 = 1 - \widetilde{\lambda}_1$. We obtain η_1^* as a solution of:

$$F(\eta_1) = \eta_1(1 + \sqrt{2\bar{U}} - \alpha) + (1-\eta_1)\sqrt{2\bar{U}}\sqrt{1-\eta_1^2\alpha^2} - 1 + \alpha = 0 \qquad (27)$$

and obtain the following proposition.

Proposition 8 *If $\sqrt{2\overline{U}} < 1-\alpha$, there exists a unique solution $\eta_1^* \in [0,1]$ to (27). It is such that*

$$\eta_1^* > \frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}$$

Furthermore, we have:

$$\frac{\partial \eta_1^*}{\partial \overline{U}} < 0 \quad \textit{and} \quad \frac{\partial \eta_1^*}{\partial \alpha} < 0$$

We see that the results are qualitatively unchanged compared to the case when there is no socialization costs.

Proof: We have $F(0) = \sqrt{2\bar{U}} - 1 + \alpha < 0$ and $F(1) = \sqrt{2\bar{U}} > 0$, so there is at least one solution η_1^* between 0 and 1. Now,

$$\frac{\partial F}{\partial \eta_1} = 1 + \sqrt{2\bar{U}} - \alpha - \sqrt{2\bar{U}}\sqrt{1-\eta_1^2\alpha^2} + \sqrt{2\bar{U}}\eta_1\alpha^2\frac{\eta_1 - 1}{\sqrt{1-\eta_1^2\alpha^2}}$$

Because $\sqrt{2\overline{U}} < 1-\alpha$,

$$1 + \sqrt{2\bar{U}} - \alpha - \sqrt{2\bar{U}}\sqrt{1-\eta_1^2\alpha^2} > \sqrt{2\bar{U}}$$

and thus

$$\frac{\partial F}{\partial \eta_1} > \sqrt{2\bar{U}}[1 - \frac{(1-\eta_1)}{\sqrt{1-\eta_1\alpha}}\frac{\eta_1\alpha^2}{\sqrt{1+\eta_1\alpha}}]$$

Using

$$\sqrt{1-\eta_1} = \frac{1-\eta_1}{\sqrt{1-\eta_1}} > \frac{1-\eta_1}{\sqrt{1-\eta_1\alpha}}$$

we get

$$\frac{\partial F}{\partial \eta_1} > \sqrt{2\bar{U}}[1 - \underbrace{\sqrt{1-\eta_1}\frac{\eta_1\alpha^2}{\sqrt{1+\eta_1\alpha}}}_{<1}] > 0$$

, which proves the uniqueness of $\eta_1^* \in [0,1]$.

To show that $\eta_1^* > \frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}$, we use the fact that $\frac{\partial F}{\partial \eta_1} > 0$ and check that $F(\frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}) < 0$. Some manipulations lead to

$$F(\frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}) < 0 \Leftrightarrow 2\overline{U}\left(-1 + \sqrt{1 - \left(\frac{1-\sqrt{2\overline{U}}-\alpha}{1-\alpha}\right)^2\alpha^2}\right) < 0$$

and this inequality is always true.

As for the comparative statics, using $\frac{\partial F}{\partial \eta_1} > 0$ and $\frac{\partial F}{\partial \overline{U}} > 0$, we get $\frac{\partial \eta_1^*}{\partial \overline{U}} < 0$.

Next, to see that $\frac{\partial \eta_1^*}{\partial \alpha} < 0$, we need to show that $\frac{\partial F}{\partial \alpha} > 0$. We have

$$\frac{\partial F}{\partial \alpha} = (1 - \eta_1) \left[1 - \sqrt{2\overline{U}} \frac{\alpha \eta_1^2}{\sqrt{1 - \eta_1^2 \alpha^2}} \right]$$

We also have

$$\sqrt{2\overline{U}} \alpha \eta_1^2 < \sqrt{2\overline{U}} < 1 - \alpha < 1 - \eta_1 \alpha < 1 - \eta_1^2 \alpha^2 < \sqrt{1 - \eta_1^2 \alpha^2}$$

which proves that $\frac{\partial F}{\partial \alpha} > 0$. ∎

APPENDIX 3: Table A1

Table A1 – Descriptive statistics

VARIABLES	Description	(1) N	(2) mean	(3) sd	(4) min	(5) max
Non white	Dummy taking value 1 if the race is not white.	90,118	0.315	0.509	0	1
Female	Dummy taking value 1 female.	89,387	0.498	0.500	0	1
Grade	Respondent grade.	89,315	9.585	1.613	7	12
English	Grade A = 4, B = 3, C = 2, D or lower = 1 in English.	74,263	2.835	0.986	1	4
Mathematics	“” in mathematics.	72,624	2.740	1.027	1	4
History	“” in history.	67,406	2.869	1.008	1	4
Science	“” in science.	68,009	2.833	1.013	1	4
French	Dummy variable taking value 1 if participation to the club/activity.	90,118	0.0383	0.192	0	1
German	“”	90,118	0.0135	0.115	0	1
Latin	“”	90,118	0.0165	0.127	0	1
Spanish	“”	90,118	0.0739	0.262	0	1
Book	“”	90,118	0.0116	0.107	0	1
Computer	“”	90,118	0.0302	0.171	0	1
Debate	“”	90,118	0.0229	0.149	0	1
Drama	“”	90,118	0.0664	0.249	0	1
Future farm	“”	90,118	0.0185	0.135	0	1
History club	“”	90,118	0.0114	0.106	0	1
Math club	“”	90,118	0.0322	0.176	0	1
Science club	“”	90,118	0.0329	0.178	0	1
Band	“”	90,118	0.117	0.321	0	1
Cheer dance	“”	90,118	0.0805	0.272	0	1
Chorus	“”	90,118	0.0950	0.293	0	1
Orchestra	“”	90,118	0.0218	0.146	0	1
Other club	“”	90,118	0.174	0.379	0	1
Baseball or softball	“”	90,118	0.156	0.363	0	1
Basket	“”	90,118	0.182	0.386	0	1
Field hockey	“”	90,118	0.0120	0.109	0	1
Football	“”	90,118	0.126	0.332	0	1
Ice hockey	“”	90,118	0.0212	0.144	0	1
Soccer	“”	90,118	0.0826	0.275	0	1
Swim	“”	90,118	0.0519	0.222	0	1
Tennis	“”	90,118	0.0485	0.215	0	1
Track	“”	90,118	0.119	0.324	0	1
Volley	“”	90,118	0.0739	0.262	0	1
Wrestling	“”	90,118	0.0404	0.197	0	1
Other sport	“”	90,118	0.0890	0.285	0	1
Newspaper	“”	90,118	0.0436	0.204	0	1
Honor	“”	90,118	0.0917	0.289	0	1
Stud. counc	“”	90,118	0.0737	0.261	0	1
Yearbook	“”	90,118	0.0757	0.264	0	1
Activity sum	Sum of all the dummies.	90,118	2.144	2.640	0	33

Table 1- Socialization efforts for people from disadvantaged/advantaged backgrounds

VARIABLES	(1) Extra curriculum activity	(2) Extra curriculum activity	(3) Extra curriculum activity
Non white	0.1868***	0.1462***	0.1770***
	(0.0193)	(0.0188)	(0.0168)
Grade		-0.0822***	-0.0827***
		(0.0053)	(0.0053)
Female			0.2067***
			(0.0170)
Constant	2.0894***	2.8777***	2.7648***
	(0.0105)	(0.0523)	(0.0525)
Observations	90,118	89,315	88,808

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2 - Socialization efforts for people from disadvantage backgrounds and ability

VARIABLES	(1) Extra curriculum activity	(2) Extra curriculum activity	(3) Extra curriculum activity	(4) Extra curriculum activity	(5) Extra curriculum activity
Mathematics	0.2530***				0.1050**
	(0.0315)				(0.0412)
English		0.3177***			0.1028**
		(0.0330)			(0.0461)
Science			0.3237***		0.1488***
			(0.0323)		(0.0443)
History				0.3264***	0.1689***
				(0.0326)	(0.0448)
Grade	-0.0884***	-0.1247***	-0.1117***	-0.1191***	-0.1101***
	(0.0203)	(0.0199)	(0.0208)	(0.0202)	(0.0225)
Female	0.1512**	0.0925	0.1506**	0.1280*	0.1356*
	(0.0650)	(0.0653)	(0.0667)	(0.0667)	(0.0735)
Constant	2.2769***	2.4530***	2.2977***	2.3475***	1.7796***
	(0.2157)	(0.2119)	(0.2184)	(0.2124)	(0.2481)
Observations	6,012	6,141	5,612	5,633	4,717

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1