Doping and competition uncertainty

Web Appendix

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A Robustness

We discuss several variations of the test we report in Section 2 of the paper.

First, we account for the introduction of sprint races in the second half of the 1990s, which might have changed the incentives of some players. These races are shorter and thus involve different types of skills. The rankings in such races are by nature less correlated to the final ranking. Figure A1 depicts the evolution in the number of distance and sprint races. This may create a potential bias, as 1996 is also the first year where an upper limit on [Hb] was imposed. To account for that change, we replicate all our correlation computations considering only distance races. We then reconstruct a hypothetical final ranking where only points from distance races are counted and compare race-specific rankings to this modified final ranking. Removing sprint races from the sample leaves between 10 and 17 races each year, with an average of 13. We report the p-value computations based on distance races alone and the associated yearly distance rankings in Figure A2. Figure A2 displays qualitative results similar to Figure 3 in the main text. The p-values are high before and right after the 1990s and extremely low during the EPO years.

The emergence of sprint races may have induced athletes to stop specializing. When first introduced, sprint races represented a tiny share of all races and top athletes did not need to perform well in them. In the 2000s, however, they represented a third of races on average. Rational athletes seeking points for the final ranking now needed to train for both sprint and distance races. Top athletes being

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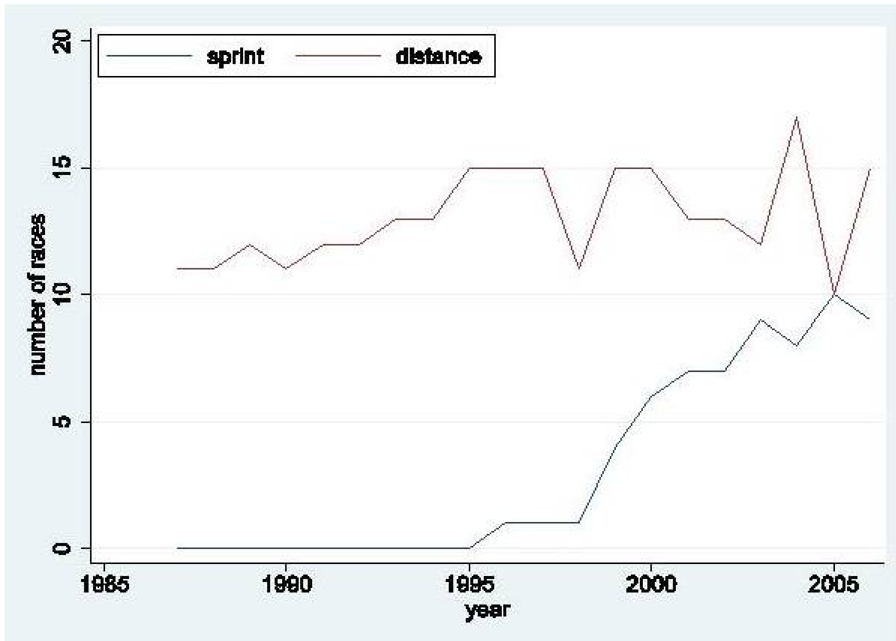

Figure A1: Number of distance and sprint races over time, 1987-2006

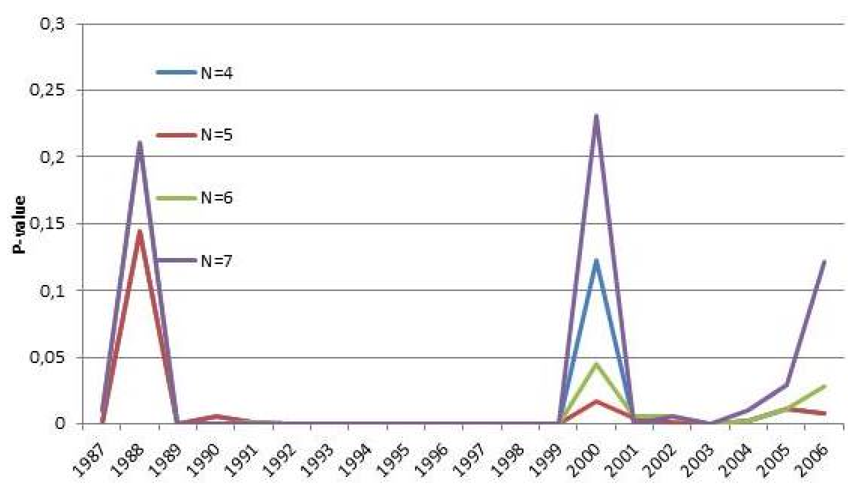

Figure A2: P-values of the zero-correlation test as a function of the minimum number of top-15 skiers, distance races only, 1987-2006. The null hypothesis of the test is $\bar{\rho}_t = 0$.

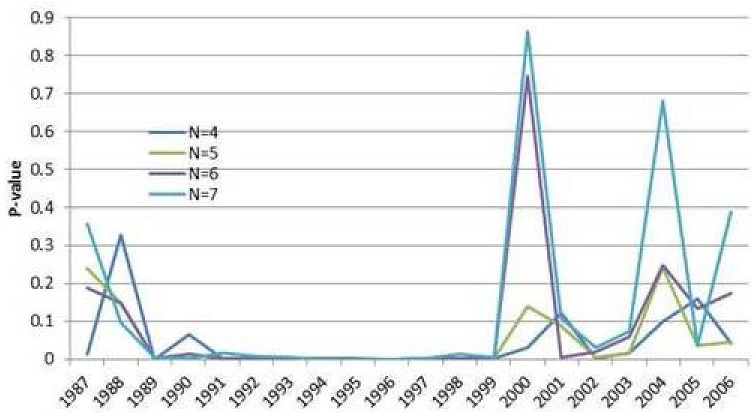

Figure A3: P-values of the zero-correlation test as a function of the minimum number of top-10 skiers, distance races only, 1987-2006. The null hypothesis of the test is $\bar{\rho}_t = 0$.

more all-round, they might have suffered of less consistent results in distance races. On the contrary, a glance at the most successful race winners in the 2000s shows clear specialization[1]. Petter Northug from Norway is actually the only skier who managed to win a significant number of races in both the sprint and the distance categories. One such athlete is not enough to impact our results.

Second, we focus on a smaller subset of top level athletes. Instead of looking at the top 15, we restrict to the top 10, because the less well-ranked skiers may not worry so much about their final ranking. To avoid this bias, Figure A3 focuses on a smaller group of top athletes, those in the top 10 distance skiers, and confirms the general pattern shown by Figures 3 from the main text and A2. Race-specific rankings are more correlated to the final ranking in the 1990s than in any other period.

Third, the pool of athletes changes over time. In particular, the entry of new skiers and the exit of older ones may affect pool composition. Irrespective of doping, the new athletes may perform more or less consistently than the others. This could affect the relevance of our computations, especially if the timing of such changes coincides with the overall doping pattern explained above.

The entry of new athletes is unlikely to play a key role because they do not aim for the top positions. The proportion of newcomers in the top 15 of the final ranking was about 6.5% in 1988 and 1989, and zero in the rest of the sample. Similarly, the proportion of newcomers in the top 10 was zero for the whole sample.

The exit of older skiers is negligible between 1987 and 2006 except for 1998 and 1999, when four of the top ten skiers exited the rankings. Two out of these four skiers belonged to the top 3, one exiting in 1998 and the other one in 1999. Both

[1]See the Wikipedia page http://en.wikipedia.org/wiki/FIS_Cross-Country_World_Cup

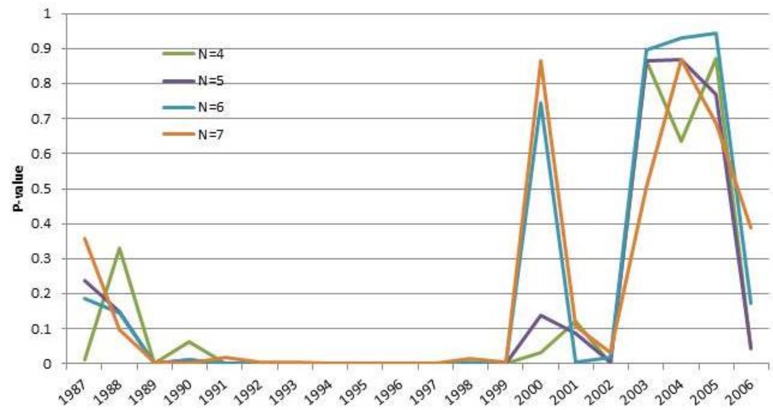

Figure A4: P-values as a function of the minimum number of top-10 skiers, without exiters, distance races only, 1987-2006. The null hypothesis of the test is $\bar{\rho}_t = 0$.

athletes were big stars, with a very long career and top positions in the CCS World Cup throughout the 1990s. The emergence of such skiers is a direct implication of our theory. According to our model, a fall in the cost of doping, such as experienced in the late 1980s, favors the best athletes. They choose a higher level of doping and as a consequence the underdogs lower theirs. The probability of top dog winning a competition increases greatly, consistent with what is observed here. However, there is an alternative explanation. These athletes could be genetic freaks who naturally achieved excellent performances. Because these genetic freaks may have contributed to the stability of rankings in the 1990s, we need a robustness check for our empirical analysis. How can we distinguish between the impact of stars and a general decline in the cost of doping? The tournament literature that we discuss in the Introduction of our paper has already examined the impact of stars on other players' efforts. It argues that the presence of such stars is detrimental to the others' performances through effort reduction. Thus, if ranking stability in the 1990s was only due to such stars, then the other players' rankings should have been very volatile.

To test this argument, we remove the four top 10 skiers who retired in 1998 and 1999 from the dataset. We then reconstruct the hypothetical rankings of each race as well as the yearly rankings. Finally we carry out the same computations as before. Figure A4 shows that removing the superstars from the dataset does not alter our conclusions. Namely, rankings become remarkably stable in the early 1990s, and correlations fall in 2000 and remain low in the 2000s.

Fourth, we account for the fact that our test statistics compare different skiers across different races. To address this issue, for each year we follow several specific competitors and see what happens each time they compete with each other. We consider two groups: a top group composed of the best athletes in the final distance ranking, and a lower-ranking group. We then form pairs of athletes by picking one

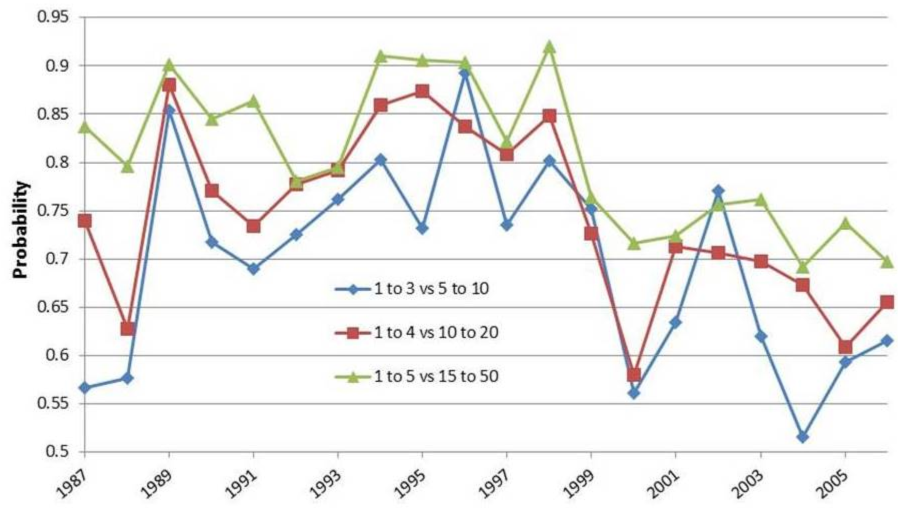

Figure A5: Probability of athletes from the top group ranking better than athletes from the weaker group in a given race, 1987-2006.

skier from each group. We form as many pairs as possible and count how many times the competitor from the top group ranks better than the competitor from the weaker group.

Figure A5 provides our results for three different athletes grouping. In the first one, we only consider athletes in the top 10. The top group is composed of the top 3, whereas the weaker group is composed of athletes ranked between 5 and 10. We choose to avoid the number 4 so as to ensure a gap between the two groups. In the second grouping, we oppose athletes in the top 4 to athletes ranked between 10 and 20. The third grouping opposes the top 5 to athletes ranked between 15 and 50.

The three curves confirm the general pattern previously identified. The probability of athletes from the top group winning is higher in the 1990s than in the rest of the sample, there is a sharp decline in the late 1990s-2000, immediately followed by a spike in 2002, and a steadier decline thereafter.

B The standard framework

In this Appendix, we analyze what we call the *standard model* of cheating and formally prove the statements made in Section 3.1 of the paper.

We assume there is no regularity effect, i.e., $h \equiv 0$. Therefore cheating only increases the maximum performance of each player. We consider several models, from the simplest to the most elaborate, and show that all of them imply that (i) all players are unhappy that cheating possibilities exist and (ii) the best players win less often with doping than without it. The main lesson of this analysis is that the standard model fails at explaining important features of cheating, not because of a particular specification of a model, but because the regularity effect is missing.

We examine the following variations of the standard model:

(a) homogeneous players ($\underline{a}_1 = \underline{a}_2$ and $\overline{a}_1 = \overline{a}_2$) and two doping levels, $d_i \in \{0, 1\}$

(b) homogeneous players ($\underline{a}_1 = \underline{a}_2$ and $\overline{a}_1 = \overline{a}_2$) and finite doping levels, $d_i \in \{0, d_1, \ldots, d_k\}$

(c) heterogeneous players ($\underline{a}_1 > \underline{a}_2$ and $\overline{a}_1 > \overline{a}_2$) and finite doping levels

(d) heterogeneous players and continuous doping levels $d_i \in [0, 1]$.

In all these models, the Nash equilibrium, whether in pure or in mixed strategies, is unique: players do not dope when the doping cost is too high; both players choose a positive doping level with positive probability otherwise. This allows us to analyze welfare: we call *welfare of player i* his payoff at equilibrium U_i^*.

The four models have significantly distinct features. In (a) the resulting 2×2 game is a classical prisoner's dilemma and the unique Nash equilibrium is in pure strategies where players dope. In (b) the game exhibits the same characteristics, the equilibrium is unique and symmetric, and the equilibrium doping level decreases when the cost of doping increases. This model is in line with standard views on doping. Berentsen (2002) shows that considering heterogeneity in players' type introduces drastic changes in the analysis. This is confirmed by model (c), where the only equilibrium is in mixed strategies, probabilities of doping are asymmetric, and the marginal cost of doping has asymmetric effects on the different players: as the cost increases, doping effort decreases for the top dog while it increases for the underdog. Finally, and more in line with our general model, when doping is a continuous variable as in (d), the unique Nash equilibrium is in mixed strategies and decreases for both players (in the sense of first-order stochastic dominance) when the cost of doping increases.

All these models share the following crucial features:

Proposition B.1 (Heuristics on the standard framework) *For the models described above:*

• *Relative to a world without doping, welfare is lower for the underdog ($U_2^* \leq U_2(0,0)$) and strictly lower for the top dog ($U_1^* < U_1(0,0)$)*
• *The top dog's winning probability is lower than in a world without doping.*

Proposition B.1 follows from Propositions B.2, B.3 and B.4 presented below.

• Models (a) and (b)
Two identical players face a binary choice $d = 0$ or $d = 1$. The game is summarized by the following payoff matrix.

	0	1
0	$\frac{1}{2}, \frac{1}{2}$	$\frac{3}{8}, \frac{5}{8} - c$
1	$\frac{5}{8} - c, \frac{3}{8}$	$\frac{1}{2} - c, \frac{1}{2} - c$

Payoffs in the diagonal cells are obvious. When one player dopes and the other does not, the doped player wins whenever he performs well (with prob. 1/2) and obtains half the prize when both players are in a bad state (with prob. $1/2 \times 1/2$). He also bears the cost of doping. This is a a typical prisoner's dilemma situation.

Proposition B.2 (Homogeneous players)

(i) *This game has a unique equilibrium:* $(0, 0)$ *if* $c > 1/8$ *and* $(1, 1)$ *if* $c < 1/8$

(ii) *If both players dope, the winning probabilities are the same as those without doping*

(iii) $U_i(1, 1) < U_i(0, 0)$

This easily extends when several doping levels are allowed, i.e., $d \in \{0, d_1, ..., d_k\}$. Whatever the value of c, there is always a unique and symmetric equilibrium. Further, any symmetric doping strategy can be an equilibrium for an appropriate value of c. As the cost increases, the equilibrium level of doping decreases. When players use the same doping strategy, the winning probabilities are unchanged. As a consequence for both players, $U_i(d^*, d^*) < U_i(0, 0)$ for every equilibrium $d^* > 0$.

• Model (c)
Two heterogeneous players face discrete doping possibilities. We assume $\overline{a}_1 > \overline{a}_2$, $\underline{a}_1 > \underline{a}_2$ and $a(1) > \overline{a}_1 - \overline{a}_2$. With player 1 standing for the top dog and player 2 for the underdog, the payment matrix is:

	0	$\overline{d}$
0	$\frac{3}{4}, \frac{1}{4}$	$\frac{1}{2}, \frac{1}{2} - c$
$\overline{d}$	$\frac{3}{4} - c, \frac{1}{4}$	$\frac{3}{4} - c, \frac{1}{4} - c$

Proposition B.3 (Player heterogeneity) *Let* $\overline{a}_1 > \overline{a}_2$, $\underline{a}_1 > \underline{a}_2$, *and* $a(1) > \overline{a}_1 - \overline{a}_2$. *We have*

(i) *If* $c > 1/4$, *then the only pure-strategy equilibrium is* $d_1^* = d_2^* = 0$;

(ii) *If* $c \leq 1/4$, *then there is a unique mixed-strategy equilibrium* (γ_1^*, γ_2^*), *where* $\gamma_i \in [0, 1]$ *stands for the probability that player* i *plays the doping strategy. It is such that*

- $\gamma_1^* = 1 - 4c$ *and* $\gamma_2^* = 4c$;
- $U_1(\gamma_1^*, \gamma_2^*) = 3/4 - c < U_1(0, 0) = 3/4$ *and* $U_2(\gamma_1^*, \gamma_2^*) = 1/4 = U_2(0, 0)$.

By elimination of strictly dominant strategies, $(0, 0)$ is the unique pure-strategy equilibrium if and only if $c > 1/4$. Otherwise, there is no pure-strategy equilibrium. The argument is simple: the underdog may choose to fill the natural performance gap by using PEDs. If this is profitable for him, then the other player will choose to dope as a response to the increase in his opponent's maximum performance. An arms race takes place until the disadvantaged player stops doping, because the quantity of PEDs required has become too high. The best player will respond by quitting doping too, and as they return to the $(0, 0)$ situation, the arms race starts all over again.

• Model (d)

Doping is a continuous variable and players are heterogeneous.

Proposition B.4 (Continuous doping efforts) *Assume that* $a(d) = d$ *and call* $\delta := \overline{a}_1 - \overline{a}_2$. *Then*

(i) *If* $4c\delta \geq 1$, $(0, 0)$ *is the only Nash equilibrium and the equilibrium payoff is* $(3/4, 1/4)$;

(ii) *If* $4c\delta < 1$ *there is a unique mixed Nash equilibrium* (μ_1^*, μ_2^*), *where the probability distributions* μ_1^* *and* μ_2^* *are given by*

$$\mu_1^*(0) = 4c\delta,\ \mu_1^*(]0, d_1]) = 4cd_1,\ \forall d_1 \in]0, \frac{1}{4c} - \delta];$$

$$\mu_2^*(0) = 4c\delta,\ \mu_2^*(]\delta, d_2]) = 4c(d_2 - \delta),\ \forall d_2 \in]\delta, \frac{1}{4c}].$$ [2]

and the equilibrium payoff is $(1/2 + c\delta, 1/4)$.

[2] In other words, μ_1^* and μ_2^* are the sum of a dirac distribution in 0 and a uniform distribution.

The payoff function of player 1 when agents are using pure strategies is simply

$$U_1(d_1, d_2) \begin{cases} -cd_1 + \frac{1}{2} & \text{if} \quad d_1 + \delta < d_2, \\ -cd_i + \frac{5}{8} & \text{if} \quad d_1 + \delta = d_2 \\ -cd_i + \frac{3}{4} & \text{if} \quad d_1 + \delta > d_2 \end{cases}$$

Thus if $\mu := (\mu_1, \mu_2)$ is a profile of mixed strategies such that $\mathbb{P}_\mu\left(d_i + \overline{a}_i = d_{-i} + \overline{a}_{-i}\right) = 0$ we have

$$\begin{aligned} U_1(\mu_1, \mu_2) &= -c\,\mathbb{E}_{\mu_1}(d_1) + \frac{1}{2} + \frac{1}{4}\mathbb{P}_\mu(d_1 - d_2 > -\delta), \\ U_2(\mu_1, \mu_2) &= -c\,\mathbb{E}_{\mu_2}(d_2) + \frac{1}{4} + \frac{1}{4}\mathbb{P}_\mu(d_2 - d_1 > \delta). \end{aligned}$$

Obviously if μ^* is a mixed Nash equilibrium then $\mathbb{P}_\mu\left(d_i + \overline{a}_i = d_{-i} + \overline{a}_{-i}\right) = 0$ and the last identity holds in μ^*. The unique pure strategy equilibrium occurs when either the cost is too high or when the differences in maximum performance levels are too large. In all other cases, there is no pure strategy equilibrium, because the arms race previsouly described takes place[3]. However, there is a unique mixed strategy equilibrium, in which the top dog has a lower payoff than in a world without doping.

Proof. Point (i) is obvious. We focus on proving (ii). We call $Supp(\mu)$ the support of μ, i.e.,

$$Supp(\mu) := \{d \in \mathbb{R}_+ : \ \mu(]d - \epsilon, d + \epsilon[) > 0 \ \forall \epsilon > 0\}$$

Recall that $Supp(\mu)$ is the smallest closed set F such that $\mu(F) = 1$.

Lemma 1 *Let $d \geq 0$. Then*

$$d \in Supp(\mu_1^*) \Longleftrightarrow d + \delta \in Supp(\mu_2^*).$$

Proof. Pick $d \notin Supp(\mu_1^*)$ and $\epsilon > 0$ such that $\mu_1^*(]d - \epsilon, d + \epsilon[) = 0$. Then we claim that $\mu_2^*(]d + \delta - \epsilon/2, d + \delta + \epsilon/2[) = 0$. If this were not the case, then player 2 could profitably deviate by transferring the weight that μ_2^* puts on $]d+\delta-\epsilon/2, d+\delta+\epsilon/2[$ onto $\{d + \delta - \epsilon\}$. Thus

$$d \notin Supp(\mu_1^*) \Rightarrow d + \delta \notin Supp(\mu_2^*).$$

A reverse argument in the previous step gives

$$d + \delta \notin Supp(\mu_2^*) \Rightarrow d \notin Supp(\mu_1^*). \blacksquare$$

[3]The threshold difference in maximum performances, when player 1 is better than player 2, is given by $a((4c)^{-1})$. Indeed, the highest possible doping level for player 2 is $\bar{d}$ such that $5/8 - c\bar{d} > 3/8$ so $\bar{d} < (4c)^{-1}$. When $a_1(0) - a_2(0) > a((4c)^{-1})$ then $(0, 0)$ is the unique equilibrium, otherwise there is no equilibrium.

Lemma 2 *We have*

$$\mu_1^*(d_1) > 0 \Rightarrow d_1 = 0; \;\; \mu_2^*(d_2) > 0 \Rightarrow d_2 = 0.$$

Proof. First we show the following result.

Let d_1 be such that $\mu_1^*(d_1) > 0$. Then

$$\exists \epsilon > 0 : \,]d_1 + \delta - \epsilon, d_1 + \delta[\subset (Supp(\mu_2^*))^c.$$

Similarly, let $d_2 > \delta$ be such that $\mu_2^*(d_2) > 0$. Then

$$\exists \epsilon > 0 : \,]d_2 - \delta - \epsilon, d_2 - \delta[\subset (Supp(\mu_1^*))^c.$$

Assume that $\mu_1^*(d_1) > 0$ and, for any $\epsilon > 0$, there exists $d(\epsilon) \in]d_1 + \delta - \epsilon, d_1 + \delta[\cap Supp(\mu_2^*)$. Since $\mu_1^*(d_1) > 0$ there exists ϵ small enough so that deviating from $d_1 + \delta - \epsilon$ to $d_1 + \delta$ guarantees a strictly higher payoff[4] to player 2. The second statement can be shown with similar arguments.

Now, assume that $d_1 > 0$ is such that $\mu_1^*(d_1) > 0$. Then, there exists $\epsilon > 0$ such that $\mu_2^*(]d_1 + \delta - \epsilon, d_1 + \delta[) = 0$. Consequently, player 1 can profitably deviate by transferring the weight that μ_1^* puts on d_1 to $d_1 - \epsilon/2$.

For player 2, the same argument states that, for any $d_2 > \delta$, we have $\mu_2^*(d_2) = 0$. Clearly, we have $\mu_2^*(]0, \delta[) = 0$. Consequently we just need to prove that $\mu_2^*(\delta) = 0$. Assume that $\mu_2^*(\delta) > 0$. If $\mu_1^*(0) > 0$ then player 2 can obtain a strictly better payoff by transferring the weight on δ to $\delta + \epsilon$. If $\mu_1^*(0) = 0$ then player 2 can profitably deviate by transferring the weight on δ to 0.■

Lemma 3 *We have* $Supp(\mu_1^*) = [0, b_1]$ *and* $\mu_1^*(0) > 0$. *Also we have* $Supp(\mu_2^*) = \{0\} \cup [\delta, b_1 + \delta]$.

Proof. First, we show that if $[a_1, b_1]$ is the smallest closed interval that contains $Supp(\mu_1^*)$, then $[a_1, b_1] = Supp(\mu_1^*)$. Similarly, if $[a_2, b_2]$ is the smallest closed interval that contains $Supp(\mu_2^*) \setminus \{0\}$, then $[a_2, b_2] = Supp(\mu_2^*) \setminus \{0\}$.

Since $Supp(\mu_1^*)$ is closed, we have $a_1, b_1 \in Supp(\mu_1^*)$. Assume that there exists $c_1 \in]a_1, b_1[$ such that $c_1 \notin Supp(\mu_1^*)$. Then $c_1 + \delta \notin Supp(\mu_2^*)$. Now call $\underline{c_1} := \inf\{d > c_1 : \; d \in Supp(\mu_1^*)\}$, (resp. $\overline{c_1} := \sup\{d < c_1 : \; d \in Supp(\mu_1^*)\}$. By a basic property of a Nash equilibrium, player 1 must be indifferent between $\underline{c_1}$ and $\overline{c_1}$, against μ_2^*. However, by Lemma 1, we have $\mu_2^*(]\underline{c_1} + \delta, \overline{c_1} + \delta[) = 0$. Hence player 1 has a strictly higher payoff when he plays $\underline{c_1}$ than when he plays $\overline{c_1}$, a contradiction. Exactly the same argument proves the assertion concerning player 2.

We also know, by Lemma 1, that $a_2 = a_1 + \delta$ and $b_2 = b_1 + \delta$.

Now, assume that $\mu_1^*(a_1) = 0$. Then we have $U_2(\mu_1^*, 0) > U_2(\mu_1^*, a_1 + \delta)$, and $a_1 + \delta \in Supp(\mu_2^*)$, which is a contradiction to the fact that μ^* is a Nash equilibrium.

[4]Deviating from $d(\epsilon)$ to $d_1 + \delta$ costs her (at most) ϵc, but increases her payoff by a positive quantity, which is independent of ϵ.

Consequently, $a_1 = 0$ and $\mu_1^*(a_1) > 0$.[5] This proves that $Supp(\mu_1^*) = [0, b_1]$ and $\mu_1^*(0) > 0$. Also we have $Supp(\mu_2^*) = \{0\} \cup [\delta, b_1 + \delta]$.■

We are ready to prove the proposition. First, notice that, since $U_1(0, \mu_2^*) = U_1(b_1, \mu_2^*)$, we have

$$\frac{3}{4}\mu_2^*(0) + \frac{1}{2}(1 - \mu_2^*(0)) = 3/4 - cb_1;$$

Hence $\frac{1}{4}(1 - \mu_2^*(0)) = cb_1$

On the other hand, $\lim_{\epsilon \to 0^+} U_2(\mu_1^*, \delta + \epsilon) = U_2(\mu_1^*, b_1 + \delta)$[6], i.e.,

$$-c\delta + \frac{1}{2}\mu_1^*(0) + \frac{1}{4}(1 - \mu_1^*(0)) = -c(b_1 + \delta) + \frac{1}{2},$$

which gives $cb_1 = \frac{1}{4}(1 - \mu_1^*(0))$. As a consequence, we have $\mu_2^*(0) = \mu_1^*(0) > 0$. Since $U_2(\mu_1^*, 0) = \frac{1}{4}$, we necessarily have $-c(\delta + b_1) + 1/2 = 1/4$, i.e. $b_1 = \frac{1}{4c} - \delta$ and $\mu_1^*(0) = \mu_2^*(0) = 4c\delta$.

To see why the distributions μ_1^* and μ_2^* are uniform respectively on $]0, b_1]$ and $[\delta, b_1 + \delta]$, note that, for player 2, we must have $U_2(\mu_1^*, 0) = U_2(\mu_1^*, d_2)$ for any $d_2 \in]\delta, b_1 + \delta]$. Hence

$$\frac{1}{4} = \frac{1}{4} + \frac{1}{4}\mu_1^*([0, d_2 - \delta]) - cd_2,$$

which means that $\mu_1^*(]0, d_2 - \delta]) = 4c(d_2 - \delta)$, for any $d_2 \in]\delta, b_1 + \delta]$ and μ_1^* is uniform on $]0, b_1]$. Analogously, μ_2^* is uniform on $[\delta, b_1 + \delta]$.

It is clear that no deviation is profitable for any player as, by construction of μ^*, we have

$$U_1(d_1, \mu_2^*) = c\delta + \frac{1}{2}, \forall d_1 \in [0, b_1]; \quad U_1(d_1, \mu_2^*) = \frac{3}{4} - cd_1 < \frac{1}{2} + c\delta, \ \forall d_1 > b_1.$$

Also

$$U_2(\mu_1^*, d_2) = \frac{1}{4}, \forall d_2 \in \{0\} \cup]\delta, d_1 + \delta]; \quad U_2(\mu_1^*, d_2) = 1/2 - cd_2 < 1/4, \ \forall d_2 > b_1 + \delta.$$

The proof is complete.■

[5] Recall that 0 is the only point μ_1^* can put a positive weight on.

[6] Here we need to take the limit because the payoff function of player 2 is discontinuous in δ when player 1 plays μ_1^*.