Addiction in networks*

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Abstract

Addiction rarely develops in isolation: social influence is a powerful driver of consumption, yet network effects remain largely unexplored in the economics of addiction. This paper develops a dynamic model of addiction on networks, where individuals' consumption evolves under peer influence. We characterize steady-state consumption as a function of both network position and forward-looking attitudes, comparing myopic, time-consistent, and present-biased consumers. We then evaluate the effectiveness of public policies aimed at curbing demand for addictive goods. In

*We acknowledge financial support from the French government under the "France 2030" investment plan managed by the French National Research Agency Grant ANR-17-EURE-0020, and by the Excellence Initiative of Aix-Marseille University - A*MIDEX. This work was also supported by French National Research Agency Grant ANR-18-CE26-0020-01.

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particular, we study a key-player policy—modeled as a targeted rehabilitation program—that strategically exploits network spillovers to maximize aggregate impact.

Keywords. Addiction; Peer Network; Rational Addiction, Time-Inconsistency; Key-Player Policy

JEL Classification. C72; D83; D85

1 Introduction

How does the behavior of friends and peers shape the demand for addictive goods? And how do patterns of social interactions amplify or mitigate these effects, as well as the effectiveness of public policies? These questions are particularly relevant in highly interconnected societies, especially among young people, given the widespread prevalence of addiction worldwide.[1] Additionally, many indicators point to a rise in illicit drug use over the past decade, especially in Europe, with particular increases in the use of cocaine and methamphetamine.[2] These alarming statistics highlight the importance of understanding addiction and its social dimensions. For instance, it is well-documented that the consumption of alcohol, tobacco, and drugs is influenced by peer behavior. Studies, such as those by Kremer and Levy (2008) for alcohol use, Clark and Lohéac (2007) for alcohol and drug use, Mir et al. (2011) for Marijuana use, have shown that peer consumption plays a significant role in fostering these behaviors. In a recent meta-analysis, Liu et al. (2017) show that adolescents exposed to smoking peers have about twice the relative odds of starting or continuing to smoke compared to those who are not exposed.

[1] According to a 2018 report from the World Health Organization (WHO), alcohol abuse contributed to over 3 million deaths in 2016, accounting for one in every 20 deaths globally. Similarly, WHO estimates that tobacco use led to more than 8 million deaths annually in 2019, including approximately 1.2 million deaths due to secondhand smoke.

[2] Recent data show that 29% of respondents to the 2024 European Web Survey on Drugs reported using cocaine in the past year, making it the third most consumed illicit substance after cannabis and MDMA/ecstasy. Moreover, an estimated 3.5 million adults used cocaine in 2023, while 2 million used amphetamines or methamphetamine, with a marked increase in methamphetamine availability and use in several Central and Eastern European countries.

While the general mechanism of peer influence on private consumption is well established, its implications for addictive goods raise additional questions. Addictive behaviors accumulate over time, and peer influence can therefore have lasting consequences on individual health. Yet despite these important implications, the literature has paid little attention to how the structure of social networks—that is, who interacts with whom—shapes addiction and its long-run dynamics.

This paper addresses this gap by proposing a dynamic model of addiction, in which individuals are influenced by the consumption behaviors of their peers. We introduce a social network, where individual consumption is reinforced by the consumption of friends. The contemporaneous influence of peers not only affects current consumption but also shapes future consumption patterns. The extent of this impact depends on whether consumers are myopic or whether they consider the long-term consequences of their current consumption on future addiction. We explore two key questions under various assumptions: how the structure of social networks influences the long-term demand for addictive goods, and how public policies can account for these network effects.

To explore these questions, we develop the first model of addictive consumption on networks, using a model featuring a single addictive good, where consumers are influenced by the aggregate consumption of their social contacts. We specify linear-quadratic utility functions[3] and a tendency to conform to peers' consumption. Addiction is modeled according to modern economic theories including rational addiction, with two key settings: one where consumers are myopic, and addiction functions as habit formation (Pollak (1970b)) —where past consumption increases the return on current consumption; and another in line with

[3] This is made in the literature on addiction; see for instance Becker et al. (1994).

the rational addiction model, where consumers account for the negative consequences of their current consumption on their future health (Becker and Murphy (1988)). Additionally, we incorporate time-inconsistency through hyperbolic discounting (O'Donoghue and Rabin (2001), Gruber and Köszegi (2001)).

We present our results in several stages. First, we establish the impact of network position on long-run consumption for three types of behaviors. We examine myopic consumers, then time-consistent consumers, and then present-biased (time-inconsistent) naive consumers who wrongly think to be time-consistent in the future, in the spirit of O'Donoghue and Rabin (2001).[4] Whatever the behavior of consumers, Bonacich centrality shapes steady state consumption. Through that centrality measure, a consumer is affected not only by her direct neighbors, but also indirectly by all others in the network, with influence traveling along the connections that link them. We also find that, as soon as future health damages are substantial, the long-run consumption of myopic consumers is larger than the long-run consumption of time-consistent consumers for all networks, while numerical computations suggest that the consumption of present-biased consumers lies in-between.

We then undertake some comparative statics related to addiction characteristics and peer influence that holds for both myopic and time-consistent behaviors. Noticeably, the statics with respect to the intensity of peer pressure aggregates two conflicting forces: on the one hand, more dependence to peers pushes consumption up by intertemporal complementarities; on the other hand, conformism mod-

[4] O'Donoghue and Rabin (2001) define two classes of present-biased consumers: naive and sophisticated. We do not analyze sophisticated consumers in the present paper for tractability issue. Introducing networks generates a system of non-linear Riccati equations governing long-run behaviors, that can be hardly solved analytically.

erates consumption. In total, increased preference for conformity fosters long-run consumption irrespective of the position on the network.

We then illustrate how network structure, together with consumers' forward-looking attitudes, jointly determine addictive consumption by performing simulations on specific network structures, for various parameters. Numerical computations suggest that the impact of peers, measured through the ratio of consumption under peer influence over consumption in isolation, can be substantial, and that the impact of peers is greater for more central agents. Simulations indicate that, under present-biased behavior, a stronger bias toward the present systematically raises addictive consumption. The intuition is that, as present bias increases, the weaker concern for future health costs dominates the consumer's mistaken belief that she will behave time-consistently in the future—so that overall consumption rises. However, the ratio of the long-run consumption with network effect over the long-run consumption without network effect, that measures how much the network amplifies or reduces consumption relative to the isolated case, can be either decreasing with present bias parameter, or non-monotonic and following a U-shaped pattern; in that latter case, the strength of peer influence is higher for very impatient or nearly rational consumers.

Last, we consider public policies, aiming at decreasing the aggregate consumption levels. We first consider an homogeneous price increase, typically in the context of legal drugs like cigarettes, and we find that more central agents are more responsive to price variation. This clear-cut result shows that social network structure in itself induces heterogeneous individual responses to price variation. We then consider a network-based policy, that can be oriented to legal or illegal drugs, by examining rehabilitation programs focused on a single consumer.

Assuming limited budget and myopic consumers, and given the networked influence, this results in a key-player type of policy,[5] in which the treated agent does not necessarily fully stop consuming. In particular, a rehabilitation program aiming at reducing the addiction characteristics of consumers leads to target an agent maximizing a specific centrality index; interestingly, this centrality index depends on the budget level, and it is highly sensitive to network structure.

Overall, our paper stresses how the structure of the network of peer influence affects addictive behaviors under both myopic and rational addict behaviors, and how policy intervention should take the network structure into account.

Literature. This paper contributes to the literature on the demand for addictive goods, focusing on how social networks and peer influences affect individual consumption decisions. Several studies have documented the role of peers in influencing addiction-related behaviors. For instance, Kremer and Levy (2008) find that students who are randomly assigned roommates who drank heavily before university significantly increase their own alcohol consumption. This effect persists even after the first year, suggesting that early exposure to heavy drinking can have long-lasting impacts on individual behavior. Similarly, Clark and Lohéac (2007) show that the probability of an adolescent starting to smoke increases significantly if their friends smoke. Their estimates suggest a peer elasticity of 0.5, meaning that a 10 percent increase in smoking within the peer group leads to a 5 percent increase in individual consumption. In the same vein, Mir et al. (2011) demonstrates that marijuana use among high school students is strongly influenced by their close friends, with network effects amplifying the spread of

[5] This type of network intervention was introduced by Ballester et al. (2006), who analyze how dropping specific agents from the network can generate large spillovers throughout the network.

risky behaviors.

Beyond the direct influence of peers, several economic models have examined how addiction is shaped by individual behavior over time, considering factors such as habits, rational addiction, and time-inconsistency. Pollak (1970a) introduces the concept of habit formation, modeling how consumption decisions are influenced not only by current preferences or income but also by past consumption. This work suggests that people develop habits, which in turn affect their demand for goods over time. Becker and Murphy (1988) develop the theory of rational addiction, where individuals make consumption decisions about addictive goods based on a forward-looking utility maximization framework. According to this model, individuals weigh both the immediate satisfaction and the future costs of their addictive behaviors. Gruber and Kőszegi (2001) further extend this idea by introducing time-inconsistent preferences into the model. They argue that individuals often underestimate the future costs of addictive behaviors, which has important implications for public policy, particularly concerning the taxation of addictive goods.[6] In the same vein, O'Donoghue and Rabin (2001) model addiction as a result of present bias. This bias leads to over-consumption of addictive goods, as people underestimate their future self-control struggles. The authors differentiate between sophisticated agents, who anticipate these challenges and may seek safeguards, and naive agents, who fail to foresee their lack of future restraint. The model explains how both types can fall into addictive patterns, driven by misaligned preferences over time.

[6] Becker et al. (1994) provide evidence in favor of rationality in the context of cigarette addiction, but Gruber and Kőszegi (2001) argue that it is difficult for empirical studies to distinguish rationality from alternative models such as hyperbolic discounting.

These models primarily focus on how inter-temporal decisions related to addiction are influenced by rational or forward-looking behavior. Our paper builds on this literature by incorporating social networks into the analysis of addiction. We introduce a detailed framework in which individuals' consumption decisions are influenced not only by their own preferences but also by the consumption behaviors of their peers, thereby capturing the social diffusion of addiction. This network-based approach complements existing models by adding a social dimension to the understanding of addiction.[7]

Another strand of literature addresses the impact of health policies on group behaviors.[8] For example, Cutler and Gleaser (2010) show that when tobacco taxes increase, consumption declines not only among the smokers directly affected but also among their non-smoking friends, indicating a social diffusion effect. This suggests that public policies targeting individual behaviors can have broader impacts on social networks. Building on this, our paper introduces a novel policy tool: key-player policies, which link the effectiveness of interventions to the structure of social networks.[9] By identifying key players within a network, policymakers can target individuals whose behavior will have the most significant ripple effect on others, thereby optimizing the impact of public health interventions.

Last, our paper adds to the literature on network games. That literature has

[7] Reif (2019) models group influence on addictive behavior with closely related modeling assumptions. The main differences with our setup are that consumers are influenced by the mean consumption of the addictive good by other consumers in her reference group; i.e. there is no network in their analysis.

[8] There is also a literature about taxes and advertising restriction; see Chaloupka (1991), Carpenter and Cook (2008), Cawley and Ruhm (2012).

[9] In the context of health economics, Barrenho et al. (2025) studies key-player in the diffusion of innovation among senior researchers.

been initiated by Ballester et al. (2006), and pursued by Bramoullé et al. (2014). In that literature, few papers are more closely related to ours. First, Boucher et al. (2024) consider network games applied to risky behaviors, including cigarette and alcohol goods, and explore non linear best-responses. With respect to their work we provide micro-foundations to addictive behaviors on networks, by bridging the literature on dynamic addiction and the literature on network games. Regarding key-player policies, Ballester et al. (2006) provide the first analysis, in which the target is dropped out of the network. Recently, Lee et al. (2021) present a methodology for empirically identifying the key player, whose removal from the network leads to the optimal change in aggregate activity level in equilibrium. Belhaj and Deroïan (2018) extend the analysis to a setup with contracts, and where the contract affects the action of the target without necessarily inducing a drop out.

The remainder of the paper is organized as follows. We introduce our framework in Section 2. The equilibrium in consumption for addictive good under myopia is analyzed in Section 3, while Section 4 focuses on forward-looking time-consistent consumers, and examines forward-looking present-biased consumers. We examine public interventions in Section 6. Section 7 concludes the paper. All proofs are relegated to Section A. Appendix B gives the conditions of global convergence under rational addiction with time-consistent behavior, and Appendix C presents tables comparing individual behaviors under various behavioral assumptions, networks and parameters.

2 Model

We introduce a network of peers in a standard model of addiction, in a model that integrates rational addiction and time-inconsistent preferences. We consider a dynamical setting with an infinite number of discrete periods $t \in \{0, 1, 2, \cdots\}$, where a society of infinitely lived agents choose an individual consumption level of an addictive good at each period, and maximize the flow of their instantaneous utilities over all periods. Social peers influence agent's consumption at every period through instantaneous utilities.

Networks. Let $\mathcal{N} = \{1, 2, \cdots, n\}$ represent a society with a finite number of agents. Agents interact through an undirected network. The network is defined by its agency matrix $\mathbf{G}$, a binary and symmetric matrix representing social relationships. That is, the entry $g_{ij} = g_{ji} = 1$ if agent i and j are connected in the network, and $g_{ij} = g_{ji} = 0$ if agent i and j are not connected. By convention $g_{ii} = 0$ for all i. We denote by $N_i = \{j : g_{ij} = 1\}$ the set of agents linked to i in network $\mathbf{G}$. To avoid trivialities, we assume that the network is connected (no agent is isolated).

Instantaneous utilities. Let $c_{i,t}$ represent the consumption of addictive good by agent i at time t. Agent i derives instantaneous utility from consuming the addictive good at time t. This individual utility depends on the present consumption of the good $c_{i,t}$, the stock of past consumption of the good $A_{i,t}$, and the consumption of the good by the neighbors in the network $\bar{c}_{i,t} = \sum\limits_{j \in \mathcal{N}} g_{ij} c_{j,t}$. The stock of past consumptions is given by

$$A_{i,t} = (1 - \gamma) A_{t-1} + c_{i,t-1}$$

Note that the stock of past consumption is equivalently written as the discounted sum of all past consumption of the good, i.e., $A_{i,t} = \sum_{s=1}^{t}(1-\gamma)^{s-1}c_{i,t-s}$. The discounting of the sum represents how consumption that is further in the past matter less for the present utility. If the discounting parameter $\gamma \in]0,1]$ is large, it means that the consumption of addictive good will matter less in the long run. For instance, in the case of smoking, a high γ implies that a cigarette consumed a year ago has negligible impact on current addiction, while one smoked yesterday carries much greater weight.

The instantaneous utility of agent i at time t can then be expressed as

$$u_{i,t} = u(c_{i,t}; A_{i,t}, \bar{c}_{i,t}) \tag{1}$$

Agents are influenced by the sum of consumption of her neighbors.[10] Following the addiction literature, we specify an instantaneous utility function of linear-quadratic form:[11]

$$u_{i,t}(c_{i,t}, A_{i,t}, \bar{c}_{i,t}) = \alpha_c c_{i,t} - \frac{1}{2}\alpha_{cc}c_{i,t}^2 + \alpha_{cA}c_{i,t}A_{i,t} - \frac{1}{2}\alpha_{AA}A_{i,t}^2 - \frac{1}{2}\alpha_p(c_{i,t} - \bar{c}_{i,t})^2 \tag{2}$$

with $\alpha_c > 0$, $\alpha_{cc} > 0$, $\alpha_{cA} > 0$, $\alpha_{AA} > 0$, $\alpha_p > 0$. This utility function can be decomposed into four parts. The first part is the utility directly associated with the consumption of the addictive good. It is increasing and concave. The second part represents the addictivity of the good. By $\alpha_{cA} > 0$, the more one has consumed the good in the past, the higher the marginal utility of the present consumption of the

[10] An alternative hypothesis is to consider that agents are rather influenced by the average neighbors' consumption. That is, assuming that no agent is isolated, this corresponds to supposing $\bar{c}_{i,t} = \frac{1}{d_i}\sum\limits_{j\in\mathcal{N}} g_{ij}c_{j,t}$. Technically, our analysis is easily extended to this alternative model, by replacing matrix $\mathbf{G}$ with matrix $\tilde{\mathbf{G}} = (\frac{g_{ij}}{d_i})$ in the analysis.

[11] See Becker and Murphy (1988), Becker et al. (1994), Chaloupka (1991), and Gruber and Köszegi (2001), Reif (2019).

good. The third part represents the disutility generated by the past consumption of the good. When the addictive good is harmful, if consumption has been high in the past, this can deteriorate health and then generate disutility.
Finally, the last part of the utility represents the peer effects. We use a conformity specification here, where consumer's utility depends negatively on the distance between their consumption and their neighbors' consumption.[12]

Dynamical problem. Consumers maximize the discounted sum of their utilities over time (as in Becker and Murphy (1988)), in a rational addiction setting encompassing time-inconsistency (as in Laibson (1997) or Gruber and Köszegi (2001)). Formally, let $\mathbf{c}_t = (c_{i,t})_{i \in \mathcal{N}}$ be the profile of agents' consumptions in period t. Let $C_{i,t} = (c_{i,t}, c_{i,t+1}, \cdots)$ be the profile of agent i's consumption at period t and all subsequent periods, and let $\mathbf{C}_t = (C_{i,t})_{i \in \mathcal{N}}$ be the profile of agents' strategies over all periods from period t. Then, agent i's stream of utility over time is given by

$$U_i^t(C_{i,t}, \{A_{i,t+s}\}_{s \geq 0}, \bar{C}_{i,t}) = u_{i,t}(c_{i,t}, A_{i,t}, \bar{c}_{i,t}) + \beta \sum_{s=1}^{\infty} \delta^s u_{i,t+s}(c_{i,t+s}, A_{i,t+s}, \bar{c}_{i,t+s})$$

Equivalently, $U_i^t = u_{i,t} + \beta\delta U_i^{t+1}$, meaning that the stream of utilities in period t is expressed as the sum of the instantaneous utility in period t plus a discounted stream of utility starting in period $t+1$. The sum of future utilities is discounted by two factors which play different roles. Parameter $\delta \in [0, 1[$ is the classical preference for the present. It is the relative preference between two consecutive

[12] An alternative specification would be to assume spillovers rather than conformism, in which peer effects take the following form: $+\alpha_p c_{i,t}\bar{c}_{i,t}$. That formulation entails close, but distinct, best-response consumption. In short, the model with synergies is more favorable to addictive consumption than the conformist model. Boucher et al. (2024) identify conformism specification in risky behaviors such as cigarettes or alcohol among teenagers.

periods in time. Parameter $\beta \in [0,1]$ is the bias toward the present period. It is the relative preference between the present period and any future period. Note that if $\beta = 0$ or $\delta = 0$ then consumers are myopic and only consider their current utility. If $\beta = 1$, then consumers are rational and their consumption choices in each period will be consistent with their past and future decisions. If $\beta \in]0,1[$, utilities exhibit present-biaised preferences (see Laibson (1997) or O'Donoghue and Rabin (1999)) inducing time-inconsistency.

In the paper, $\mathbf{c}^M, \mathbf{c}^{TC}, \mathbf{c}^N$ will represent respectively the consumption profiles of myopic, time-consistent, and present-biased naive consumers.

3 Myopic agents

In this section, we consider the myopic case, that is we assume $\beta = \delta = 0$. We will give a detailed analysis in this benchmark case, which will be compared to models with forward-looking behavior. A key implication of peer influence is to induce complementarities in contemporaneous addictive consumption. We present our main characterization result, by identifying the long run behaviors under networked peer influence. We also undertake comparative statics with respect to main parameters of the model.

Myopic agents maximize the instantaneous utility (2), given the stock of addiction and the interaction with peers. The first-order conditions under myopia give the following system of best-responses (to others' current consumption level, given own stock of addiction):

Proposition 1. *The best-response consumption of a myopic agent i in period t*

can be written as:

$$c_{i,t}^{BR,M}(\bar{c}_{i,t}, A_{i,t}) = \frac{1}{\alpha_{cc} + \alpha_p}\left(\alpha_c + \alpha_{cA}A_{i,t} + \alpha_p\bar{c}_{i,t}\right) \tag{3}$$

By Proposition 1, the best-response is increasing in past consumption through the addictive stock: a higher past consumption increases the addictive stock, which pushes toward enhanced current period consumption. Moreover, the best-response is increasing in neighbors' current consumption by conformism, i.e. the consumption of peers amplifies addiction.

This formulation has an exact counterpart on equilibrium path. Indeed, exploiting the linear relationship between current addictive stock and one-period lagged addictive stock, the best-response consumption of agent i in period t can be written as a function of current and preceding consumption profile as follows:

Corollary 1. *The best-response consumption of amyopic agent i in period t can be written as:*

$$c_{i,t}^{BR,M}(\bar{c}_{i,t}, c_{i,t-1}, \bar{c}_{i,t-1}) = \frac{\gamma\alpha_c}{\alpha_{cc} + \alpha_p} + \left(1 - \gamma + \frac{\alpha_{cA}}{\alpha_{cc} + \alpha_p}\right)c_{i,t-1} + \frac{\alpha_p}{\alpha_{cc} + \alpha_p}\left(\bar{c}_{i,t} - (1-\gamma)\bar{c}_{i,t-1}\right) \tag{4}$$

By Corollary 1, the entire history of past consumption levels condenses into the dependence of the last period's consumption profile on the best-response path. Not surprisingly, lagged consumption tends to foster current consumption through an additional induced addiction. Peer influence is now represented by the discounted difference between current and lagged peer consumption. One implication is that the relationship between peer consumption path and current consumption stays positive when all the consumption trends are increasing for every agent in the society, for all values of parameter γ.

Given the influence of peers, we define a Nash equilibrium in each period t as a function of past consumptions (due to addiction). Formally, setting $\mathbf{c}_t = (c_{i,t}, \mathbf{c}_{-i,t})$ for convenience, a Nash equilibrium induced by the myopic game is defined as follows:

Definition 1. *A Nash strategy profile for myopic agents in period t, $(\mathbf{c}_t^M)_{t\geq 1}$, is such that $u_{i,t}(c_{i,t}^M, A_{i,t}, \mathbf{c}_{-i,t}^M) \geq u_{i,t}(c_{i,t}, A_{i,t}, \mathbf{c}_{-i,t}^M)$ over all feasible consumption levels $c_{i,t}$.*

The linear system generates a unique Nash consumption profile in period t, that is found by inverting the above period-t consumption system as a function of lagged consumptions and primitives:

$$\mathbf{c}_t^M(\mathbf{c}_{t-1}, \bar{\mathbf{c}}_{t-1}; \mathbf{G}) \;\; = \tfrac{1}{\alpha_{cc}+\alpha_p}\big(\mathbf{I} - \tfrac{\alpha_p}{\alpha_{cc}+\alpha_p}\mathbf{G}\big)^{-1}\Bigg(\gamma\alpha_c\mathbf{1} + \Big(\alpha_{cA} + (1-\gamma)(\alpha_{cc}+\alpha_p)\Big)\mathbf{c}_{t-1} - \alpha_p(1-\gamma)\bar{\mathbf{c}}_{t-1}\Bigg)$$

By the uniqueness of the Nash equilibrium in any period t, the initial consumption profile determines a unique sequence of Nash equilibria across periods. Note that the adverse effect of the consumption of addictive good on current health, captured by parameter α_{AA}, does not harm the consumption of addictive good, due to myopia.

We then characterize the consumption of addictive good by a society of myopic consumers at the steady state, i.e. a situation where an agent's consumption (and any associated addictive stock) remains constant over time. Let $\lambda(\mathbf{G})$ denote the maximal eigenvalue of network $\mathbf{G}$.[13] The next assumption guarantees the convergence of the dynamical system under myopia:

Assumption 1. The greatest eigenvalue of network $\mathbf{G}$ satisfies:

$$\lambda(\mathbf{G}) < \frac{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}}{\alpha_p}$$

[13] This largest eigenvalue is a real number, given that the network is undirected.

Assumption 1 guarantees both uniqueness of the steady state and global convergence. The dynamical system generated by best-responses being a linear system of order-1, the contraction property guaranteeing global convergence is also the condition of the uniqueness of steady state equilibrium, whatever the initial consumption profile. The next proposition provides a characterization of steady-state consumption vector under myopic behavior in terms of Bonacich centrality index:

Proposition 2. *Consider myopic agents. Under Assumption* 1*, the vector of consumption of addictive good converges to*

$$\mathbf{c}_{\infty}^{M}(\mathbf{G}) = \kappa^{M}(\mathbf{I} - \mu^{M}\mathbf{G})^{-1}\mathbf{1} \tag{5}$$

where

$$\begin{cases} \kappa^{M} = \frac{\alpha_c}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}} \\ \\ \mu^{M} = \frac{\alpha_p}{\alpha_c} \cdot \kappa^{M} \end{cases}$$

Proposition 2 has several implications. First, consumption levels are related to a centrality index, $\mathbf{b}(\mathbf{G}, \mu^{M}) = (\mathbf{I} - \mu^{M}\mathbf{G})^{-1}\mathbf{1}$; this vector represents the Bonacich centrality associated with network $\mathbf{G}$ and decay parameter μ^{M}. The Bonacich centrality measures the influence of an agent in the network G by considering both direct and indirect connections. The parameter μ^{M} represents the relative weight given to indirect ties, higher μ^{M} emphasizes the influence of more distant nodes. More central agents in the sense of that centrality measure have a greater propensity to consume. This is due to networked complementarities in consumption. Second, the impact of peers on long run consumption is amplified by addiction through parameter γ. That parameter affects not only the constant of the long run interaction system, but also the intensity of interaction μ^M. Last, denser networks foster consumption, meaning that the impact of the network of peers is always positive with respect to the no-peer case.

What the network brings. To assess how much additional consumption is due to the network at the steady state, it is instructive to examine the ratio of consumption with network effect over consumption without network effect. Call the empty network $\mathbf{G}^e$. Given that the long-run consumption of an isolated agent is equal to κ^M, that ratio for agent i is equal to

$$\frac{c_{i,\infty}^M(\mathbf{G})}{c_{i,\infty}^M(\mathbf{G}^e)} = b_i(\mathbf{G}, \mu^M) \tag{6}$$

Hence, the impact of the network on addictive consumption is more pronounced for agents in more central position in the network.

Comparative statics. We undertake comparative statics with respect to parameters α_{cA}, γ, representing addictivity, and parameter α_p, representing the impact of peers. For any parameter of interest say α, the derivative of the steady state consumption profile with respect to a parameter α is given by

$$\frac{\partial \mathbf{c}_\infty^M}{\partial \alpha} = \frac{\partial \kappa^M}{\partial \alpha}\mathbf{b} + \kappa^M \frac{\partial \mu^M}{\partial \alpha}\frac{\partial \mathbf{b}}{\partial \mu^M} \tag{7}$$

The network affects the derivative of steady state consumption twice: first, centrality acts as a multiplier effect of the derivative of the constant of the linear interaction system; second the centrality is itself affected through the variation of the intensity of interaction. It is therefore crucial to understand how centralities vary with the intensity of interaction. Let $\mathbf{b_b} = (\mathbf{I} - \mu^M\mathbf{G})^{-1}\mathbf{b}$ represent the centrality weighted by the simple centrality vector.[14] We find:

[14] For a given vector $\mathbf{a}$, the weighted Bonacich centrality $\mathbf{b_a} = (\mathbf{I} - \mu^M\mathbf{G})^{-1})\mathbf{a}$.

Lemma 1.

$$\frac{\partial \mathbf{b}}{\partial \mu^M} = \frac{1}{\mu^M}(\mathbf{b}_{\mathbf{b}} - \mathbf{b}) \tag{8}$$

Lemma 1 provides a simple formulae that relates the marginal increase of Bonacich centrality with respect to the intensity of interaction to the difference between the weighted Bonacich centrality and the simple Bonacich centrality. Injecting (8) into (7), we get

$$\frac{\partial \mathbf{c}_\infty^M}{\partial \alpha} = \frac{\partial \kappa^M}{\partial \alpha}\mathbf{b} + \frac{\kappa^M}{\mu^M}\frac{\partial \mu^M}{\partial \alpha}(\mathbf{b}_{\mathbf{b}} - \mathbf{b}) \tag{9}$$

This allows us to sign the statics:

Proposition 3 (Comparative statics)**.** *On any network, the steady state addictive consumption is increasing in α_{cA}, decreasing in γ, and increasing in α_p.*

By construction, a more addictive good, through parameter α_{cA}, has a positive marginal effect on consumption. In addition, decreasing the depreciation of the stock of past consumption by lowering the parameter γ, increases its long-run impact, thus increasing the steady-state consumption. The comparative statics on peer pressure aggregates two conflicting forces: on the one hand, more dependence to peers pushes consumption up by intertemporal complementarities; on the other hand, conformism moderates consumption. In total, increased preference for conformity fosters long-run consumption irrespective of the position on the network.

4 Time-consistent rational addict consumers

In this section, we study time-consistent rational addict consumers, i.e. we consider $\beta = 1, \delta \in (0, 1]$. Consumers' current consumption is a function of lagged consumption, by addiction, and of future consumption, by rationality, but, in addition to Becker et al. (1994), peers also affects consumption. We essentially provide insights into the dynamics of behaviors and then we focus on long run behaviors.

This model is a dynamic game of infinite-horizon. An isolated consumer, not influenced by any peer, can solve this problem through an intertemporal optimization problem starting at the beginning of period 1. Indeed, by time-consistency, revisiting their choice in any further period replicates the initial optimal strategy; that is, commitment is not an issue for isolated time-consistent agents. Things are different with peers. We adapt the optimization to Nash equilibrium according to the following protocol. Consistent with the literature on addiction, we assume no commitment to announced plans over time.[15] At each date t, every consumer publicly announces an infinite sequence of consumptions $C_{i,t} = (c_{i,t}, c_{i,t+1}, \cdots)$. A Nash equilibrium induced by the game is defined as follows:

Definition 2. *A Nash strategy profile $(\mathbf{C}_t^{TC})_{t\geq 1}$ for time-consistent consumers is such that, for all $t \geq 1$ and all $i \in \mathcal{N}$, $U_i^t(C_{i,t}^{TC}, A_{i,t}, \mathbf{C}_{-i,t}^{TC}) \geq U_i^t(C_{i,t}, A_{i,t}, \mathbf{C}_{-i,t}^{TC})$ for all feasible strategies $C_{i,t}$.*

A Nash strategy should satisfy at each period t:

$$\frac{u_i^t(c_{i,t}^{TC}, A_{i,t}, \bar{c}_{i,t}) - u_i^t(c_{i,t}, A_{i,t}, \bar{c}_{i,t})}{\delta}$$

$$\geq U_i^{t+1}(\mathbf{C}_{i,t+1}^{TC}, (1-\gamma)A_{i,t} + c_{i,t}^{TC}, \mathbf{C}_{i,t+1}^{TC}) - U_i^{t+1}(\mathbf{C}_{i,t+1}^{TC}, (1-\gamma)A_{i,t} + c_{i,t}, \mathbf{C}_{i,t+1}^{TC})$$

[15] Under no commitment, the equilibrium concept differs from an open-loop equilibrium where agents establish an optimal consumption plan at date 0. Under conditions of global convergence that will be given thereafter, the commitment consideration does not affect the long-run dynamics.

That is, at every period t, agent i selects an optimal consumption $c_{i,t}^{TC}$ by trading her immediate gains against her future consumption stream and given the others' play.

In the present model, consumers best-respond, in period t, to others consumption strategies by taking into account both lagged-period and next-period consumptions as follows:

Proposition 4. *The best-response consumption of time-consistent consumer i in period t can be written as:*

$$c_{i,t}^{BR,TC}(c_{i,t-1}, c_{i,t+1}, \bar{c}_{i,t-1}, \bar{c}_{i,t+1}; \delta) = \theta^0 + \theta\, c_{i,t-1} + \delta\theta\, c_{i,t+1} + \tau\, \bar{c}_{i,t} + \tau^-\, \bar{c}_{i,t-1} + \delta\tau^-\, \bar{c}_{i,t+1} \tag{10}$$

with

$$\begin{cases} \theta^0 &= \frac{\gamma(1-\delta(1-\gamma))\alpha_c}{\Theta} \\ \theta &= \frac{\alpha_{cA}+(1-\gamma)(\alpha_{cc}+\alpha_p)}{\Theta} \\ \tau &= \frac{\alpha_p(1+\delta(1-\gamma)^2)}{\Theta} \\ \tau^- &= \frac{-\alpha_p(1-\gamma)}{\Theta} \\ \Theta &= 2\alpha_{cA}\delta(1-\gamma) + (\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1) + \delta\alpha_{AA} \end{cases}$$

Conform to the literature on forward-looking addiction, Proposition 4 shows current consumption is positively related to future consumption. A higher anticipated future consumption of the agent makes it optimal to raise current consumption as well. This reflects dynamic complementarity within the individual by which present and future consumption reinforce each other (through habits, addiction, or intertemporal synergy). Current consumption is negatively related to the future consumption of peers. Indeed, by complementarity higher future consumption of peers implies higher health damage through increased future own consumption, which can be tempered through a reduction of current consumption.[16]

In each period, and under condition of contraction of the linear system, i.e. $\tau\lambda(\mathbf{G}) < 1$, there is a unique Nash consumption profile for given announced future consumption plans:

$$\mathbf{c}_t^{TC}(\mathbf{c}_{t-1}, \mathbf{c}_{t+1}; \delta, \mathbf{G}) = \Big(\mathbf{I} - \tau\mathbf{G}\Big)^{-1}\Big(\theta^0\mathbf{1} + \theta\,\mathbf{c}_{t-1} + \delta\theta\,\mathbf{c}_{t+1} + \tau^-\,\bar{\mathbf{c}}_{t-1} + \delta\tau^-\,\bar{\mathbf{c}}_{t+1}\Big)$$

Note that, given that agents take future consumption paths as given when playing their current-period Nash strategy, this setup can lead to multiple equilibrium consumption paths in general.

We pursue with steady state characterization. The next assumption guarantees the global convergence of the dynamical system under forward looking behavior:

Let us define

$$\xi_i^0 = \frac{1}{\delta\theta} \cdot \frac{1 - \tau\lambda_i}{1 + \frac{\tau^-}{\theta}\lambda_i}, \xi_i^1 = \frac{-1}{\delta\theta} \cdot \frac{\theta + \tau^-\lambda_i}{1 + \frac{\tau^-}{\theta}\lambda_i}$$

Assumption 2. For all eigenvalue λ_i of matrix $\mathbf{G}$, the absolute value of the two roots of the polynomial $r^2 - \xi_i^0 r - \xi_i^1 = 0$ is strictly smaller than 1.

Given the linear second-order dynamical system, Assumption 2 requires that all the roots of the quadratic equations generating the dynamics strictly lie inside the unit circle, and gives a characterization in terms of the eigenvalues of network $\mathbf{G}$.

All equilibrium paths converge to a single steady state under Assumption 2. The next proposition provides a characterization of steady-state consumption vector under rational addiction with exponential discounting and local peer effects:

[16]This is an incentive effect, by which agents refrain in current period to avoid future misbehavior - see O'Donoghue and Rabin (2001).

Proposition 5. *Under Assumption 2, the steady state consumption profile of time-consistent consumers converges to:*

$$\mathbf{c}_{\infty}^{TC}(\delta, \mathbf{G}) = \kappa^{TC}(\mathbf{I} - \mu^{TC}\mathbf{G})^{-1}\mathbf{1} \tag{11}$$

with

$$\begin{cases} \kappa^{TC} = \frac{\alpha_c}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\delta}{\gamma}\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)}} \\ \\ \mu^{TC} = \frac{\alpha_p}{\alpha_c} \cdot \kappa^{TC} \end{cases}$$

Note that for $\delta = 0$ we go back to the long run consumption of myopic consumers. Like myopic behavior, the steady state is expressed as a Bonacich centrality that captures the long run network effects. More central agents, in that regard, consume more. However, because the ranking of Bonacich centrality depends on the magnitude of the decay parameter, the most central myopic agent needs no longer coincide with the most central time-consistent agent; That is, the forward-looking behavior alters the relative ranking of consumption across agents in the same network, so that the most central agent under myopic behavior may not coincide with the most central agent under time consistency.

Comparative statics. We explore the comparative statics with respect to forward-looking behavior, considering an increase in the time preference for the present δ. By this statics, we are able to compare rational addiction to myopic addiction.

Proposition 6. *The steady state time-consistent consumption is always monotonic in parameters δ. It is decreasing in parameters δ if and only if*

$$\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma \tag{12}$$

Hence, on a given network, if $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$ (resp. $\frac{\alpha_{AA}}{\alpha_{cA}} < \gamma$), time-consistent consumers consume less (resp. more) in the long run than myopic consumers.

By Proposition 6, the statics, taking parameters separately, are always monotonic. The sign of those statics does not depend on the position on the network. Regarding forward-looking behavior, when the ratio $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$, higher parameter δ, i.e. lower preference for the present, reduces the consumption of addictive good. In particular, the network characteristics do not affect the statics. Hence, forward looking behavior leads to lower long-run consumption than under myopia when inequality (12) holds, irrespective of the network structure.

Last, the impact of the network on addictive consumptions is easily comparable with the case of myopic agents. For consumer i,

$$\frac{c_{i,\infty}^{TC}(\mathbf{G})}{c_{i,\infty}^{TC}(\mathbf{G}^e)} = b_i(\mathbf{G}, \mu^{TC}) \tag{13}$$

Hence, whatever the network structure, the impact of the network on addictive consumption is more pronounced for myopic agents than time-consistent agents whenever $\mu^M > \mu^{TC}$, that is $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$.

5 Present-biased (naive) consumers

In this section, we study forward looking behaviors with a bias for the present, i.e. we allow $\beta > 0, \delta \in (0, 1]$. We assume naivete,[17] in that a consumer maximizes her intertemporal utility with a psychological attitude that renders her unaware of

[17] Naive present-bias may be linked to the internal management of emotions, and in particular the hot versus cold state issue. Loewenstein (2005) discusses prospective and interpersonal hot–cold empathy gaps for medical decision making, and shows that people who are in a 'cold' state may fail to fully appreciate the impact of 'hot' states on other people's behavior. In the specific context of addiction, prospective hot-cold empathy gap has been documented by Giordano et al. (2004).

the fact that her future selves will revise her plans.[18]

Importantly, not only is a consumer naive, but also she believes that others are naive. By naivete, all consumption plans are revised and re-announced at every period, and every consumer believes that all announced plans are time-consistent. To analyze the dynamic equilibrium in this game, we adapt the notion of perception-perfect strategies for naive agents—originally introduced in O'Donoghue and Rabin (2001)—to interactions with peers. Specifically, at any period t, we must account for two key (mis)perceptions held by a present-biased consumer: (i) her incorrect belief that her own future consumption path will follow that of a time-consistent consumer planning at $t+1$, where this plan is itself influenced by her consumption at period t; and (ii) her incorrect belief that her peers will behave as time-consistent agents from period $t+1$ onward; That is, denoting by $C^{TC}_{i,t+1}(\mathbf{A}_{i,t+1})$ the optimal consumption plan of a time-consistent consumer i at period $t+1$ given stock $\mathbf{A}_{i,t+1}$, with $\mathbf{A}_{i,t+1} = (1-\gamma)\mathbf{A}_{i,t} + c_{i,t}$, and denoting by $\overline{C}^{TC}_{i,t+1}(\mathbf{A}_{-i,t+1})$ the optimal consumption plans of time-consistent peers at period $t+1$, we get:

Definition 3. *A perception-perfect Nash strategy profile for naifs $(\mathbf{C}^N_t)_{t\geq 1}$ is such that, for every consumer i, for all t, for any profile of addictive stocks $(\mathbf{A}_{i,t})_{i\in\mathcal{N}}$, and for any feasible consumption $c_{i,t}$:*

$$\begin{aligned} & U^t_i(c^N_{i,t}, C^{TC}_{i,t+1}((1-\gamma)\mathbf{A}_{i,t} + c^N_{i,t}), \mathbf{A}_{i,t}, \overline{\mathbf{c}}^N_{i,t}, \overline{C}^{TC}_{i,t+1}(\mathbf{A}_{-i,t+1})) \\ \geq & U^t_i(c_{i,t}, C^{TC}_{i,t+1}((1-\gamma)\mathbf{A}_{i,t} + c_{i,t}), \mathbf{A}_{i,t}, \overline{\mathbf{c}}^N_{i,t}, \overline{C}^{TC}_{i,t+1}(\mathbf{A}_{-i,t+1})) \end{aligned}$$

[18] O'Donohue and Rabin (1999) and Gruber and Kőszegi (2001) distinguish between naive and sophisticated agents; The former is unaware of their future self-control problem, the latter is aware. As suggested in O'Donoghue and Rabin (2001), limited evidence suggest that people exhibit elements of both sophistication and naivete.

Individual best-responses in period t can be expressed as follows. Let $c_{i,t}^{TC}$ denote the consumption of a time-consistent consumer i (that is, of a consumer with $\beta = 1$) at time t.

Proposition 7. *The best-response consumption of present-biased and naive consumer i in period t can be written as:*

$$c_{i,t}^{N}(c_{i,t-1}, c_{i,t+1}^{TC}, \bar{c}_{i,t}, \bar{c}_{i,t-1}, \bar{c}_{i,t+1}^{TC}; \delta, \beta) = \theta_\beta^0 + \theta_\beta c_{i,t-1} + \delta\theta_\beta^+ c_{i,t+1}^{TC} + \tau_\beta \bar{c}_{i,t} + \tau_\beta^- \bar{c}_{i,t-1} + \delta\tau_\beta^- \bar{c}_{i,t+1}^{TC} \tag{14}$$

with

$$\begin{cases} \theta_\beta^0 &= \frac{\gamma(1-\delta(1-\gamma))\alpha_c}{\Theta_\beta} \\ \theta_\beta &= \frac{\alpha_{cA}+(1-\gamma)(\alpha_{cc}+\alpha_p)}{\Theta_\beta} \\ \theta_\beta^+ &= \frac{(1-\gamma)(\alpha_{cc}+\alpha_p)+\beta\alpha_{cA}}{\Theta_\beta} \\ \tau_\beta &= \frac{\alpha_p(1+\delta(1-\gamma)^2)}{\Theta_\beta} \\ \tau_\beta^- &= \frac{-\alpha_p(1-\gamma)}{\Theta_\beta} \\ \Theta_\beta &= \alpha_{cA}\delta(1-\gamma)(\beta+1) + (\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1) + \beta\delta\alpha_{AA} \end{cases}$$

Similar to myopic context, a unique Nash equilibrium arises in period t, for a given set of false inferences of agents regarding future consumptions. However, due to the multiplicity of equilibrium paths in the time-consistent setting, there are multiple equilibrium paths under naive setting.

In the context of naive present-biased addiction, the positive impact of the future time-consistent consumption reflects the agent's misguided belief that their future consumption will be optimally controlled. Because the agent is naive, they fail to anticipate their own future self-control problems. Instead, they overestimate their ability to moderate consumption in the future.

We characterize the consumption of addictive good by a society of naive consumers at the steady state. Convergence to a steady state requires both the convergence of time-consistent perceptions, i.e. that of the system of time-consistent consumers, and convergence of the system of linear best-response naive consumptions, which is given by next assumption.

Assumption 3. The greatest eigenvalue of network $\mathbf{G}$ satisfies:

$$\lambda(\mathbf{G}) < \frac{1-\theta_\beta}{\tau_\beta + \tau_\beta^-} = \frac{((1+\beta)\delta(1-\gamma)-1)\alpha_{cA} + (\gamma+\delta(1-\gamma)^2)(\alpha_p+\alpha_{cc}) + \beta\delta\alpha_{AA}}{\alpha_p(\gamma+\delta(1-\gamma)^2)}$$

We obtain:

Proposition 8. *Under Assumptions 2 and 3, the dynamical system converges globally to the steady state consumption profile of naive consumers, which is given by*

$$\mathbf{c}_\infty^N(\delta,\beta,\mathbf{G}) = \Big(\mathbf{I} - \frac{\tau_\beta+\tau_\beta^-}{1-\theta_\beta}\mathbf{G}\Big)^{-1}\left(\Big(\frac{\theta_\beta^0}{1-\theta_\beta}\Big)\mathbf{1} + \Big(\frac{\delta}{1-\theta_\beta}\Big)\Big(\theta_\beta^+\mathbf{I} + \tau_\beta^-\mathbf{G}\Big)\mathbf{c}_\infty^{TC}\right) \tag{15}$$

where $\mathbf{c}_\infty^{TC}$ *is given by equation (11).*

When present-bias preference intensifies (i.e. when parameter β increases), self-control concerns diminish, which tends to reduce long-run consumption under high health damage. However, this intensification also reinforces the consumer's erroneous belief that she will act in a time-consistent manner in future periods, thereby increasing current consumption. It is natural to ask which force dominates as a function of parameter β. The next corollary shows that the former force dominates the latter when consumers are isolated:

Corollary 2. *When* $\gamma\alpha_{cA} < \alpha_{AA}$*, the steady state consumption of an isolated naive consumer is decreasing in parameter* β*.*

Numerical computation suggest that this corollary should extend to any network structure.[19]

[19]We found no counter-examples in spite of intense numerical computations.

Comparison of behaviors. To illustrate how network structure and behaviors affect addictive consumption in the long-run, we performed simulations on specific network structures, for various parameters. A series of tables in Appendix C, that illustrate the impact of peers on individual steady state consumption on various network structures and parameters, by reporting the ratio of consumption over the consumption of an isolated consumer. We consider a regular network (where consumers have the same number of neighbors given by parameter k below), the star network (in the star network, a single agent is involved in all links). To isolate network structure, we assume that all networks have the same number of agents, and that all consumers have identical characteristics and thus only differentiate though their position on the network.

The general insights are as follows. First, myopic agents consume more than forward-looking agents, whatever the network; in that vein, a higher bias toward the present reduces addictive consumption. Second, in regular networks higher density favors consumption. The impact of peers, measured in terms of the ratio of consumption over the consumption of an isolated agent, can be substantial (exceeding possibly 5 in the presented numerical computation). Third, in the star network, the central agent consumes more than peripherals. Fourth, higher peer influence can have drastic effect on central's ratio in star, and induces an indirect effect on peripherals; that statement holds for both types of consumers with both low and high patience. The ratio attained by the central agent in the star for $\alpha_p = 0.4$ is greater than 9. Highering α_p from 0.1 to 0.4 multiplies the ratio of a consumer on a regular network by nearly two. Fifth, the ratio can be non-monotonic in present-bias parameter β, while absolute consumption levels cannot. Indeed, the ratio can decrease from $\beta = 0.1$ to $\beta = 0.5$, and then increase to $\beta = 1$ (which corresponds to a time-consistent consumer). That U-shaped pattern means that the strength of peer influence can be higher for very impatient or nearly time-consistent consumers.

6 Public policy

In the face of harmful addictions, public intervention is justified when it generates external costs.[20] When consumers are myopic, government intervention can for instance be justified because people misjudge the future consequence on their health. This is no longer the case under rational addiction, because consumers then fully internalize these consequences. However, time-inconsistency restores the interest of public intervention (Gruber and Köszegi (2001)). This being said, irrespective of whether consumers take the future into account in their choice, there are other forms of externalities. For instance, addiction affects relatives, friends, families (see Manning et al. (1991)). Moreover, the network of peers induces synergies in addiction, that can be an additional matter for the health of consumers. That is, the presence of peers in itself justifies public intervention.

In this section, we consider two policy interventions. We will assume throughout the section that $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$, meaning that the impact of the consumption of addictive good on health is a real concern leading the government to intervene in order to reduce the consumption of addictive good. We will first examine a variation of the price of a legal addictive good, like cigarette or alcohol. Then, we will explore a network-based key-player public policy, in the context of either legal

[20]Public intervention can also be justified under ignored internal costs associated with these behaviors.

or illegal addictive good, consisting in a rehabilitation program through adequate medicine.[21]

6.1 Price variation

Harmful addictions being sensitive to price, the government can discourage these behaviors by taxation. Suppose that the price of addictive good contains a tax component that is chosen by the government. We examine the impact of peer networks for legal addictive goods like cigarette, by studying how price variation affects the demand under forward-looking behavior. Define by p the unit price of the addictive good (considered fixed in time). The instantaneous individual utility then becomes

$$u_{i,t}(c_{i,t}, A_{i,t}, \bar{c}_{i,t}; p) = (\alpha_c - p)c_{i,t} - \frac{1}{2}\alpha_{cc}c_{i,t}^2 + \alpha_{cA}c_{i,t}A_{i,t} - \frac{1}{2}\alpha_{AA}A_{i,t}^2 - \frac{1}{2}\alpha_p(c_{i,t} - \bar{c}_{i,t})^2$$

That is, price affects the steady state consumption through the constant of the system of interaction. Hence, considering time-consistent consumers, under As-

[21] Assessing the performance of rehabilitation programs is a complex task. In that regard, relapse is often considered a part of the recovery process, and various factors, including the type of substance used, duration of use, and individual circumstances, can influence relapse rates. Comprehensive treatment programs that include medical, psychological, and social support components have been shown to improve outcomes and reduce the likelihood of relapse. There are few statistics on relapse rates and the effectiveness of rehabilitation programs. For instance, the National Institute on Drug Abuse (NIDA) reports that relapse rates for substance use disorders are between 40 percent and 60 percent, which is comparable to relapse rates for other chronic diseases such as hypertension or asthma. Additionally, McPheeters et al. (2023) found that approximately 50 percent of individuals with alcohol use disorders relapse within the first year following treatment. This study highlights the challenges in maintaining long-term sobriety and underscores the need for ongoing support and intervention.

sumption 2 the steady state profile of addictive good consumption is equal to

$$\mathbf{c}^{\infty}(\delta;p) = \kappa(\delta;p)\mathbf{b}(\delta)$$

with $\mathbf{b}(\delta) = (I - \mu(\delta)\mathbf{G})^{-1}\mathbf{1}$ the Bonacich centrality with decay parameter $\mu(\delta)$, and with

$$\begin{cases} \kappa(p,\delta) = \frac{\alpha_c - p}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\delta}{\gamma}\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)}} \\ \\ \mu(\delta) = \frac{\alpha_p}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\delta}{\gamma}\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)}} \end{cases}$$

The price affecting the constant κ and not the decay parameter μ, clearly increasing the unit price entails a decrease in all steady state consumption levels, and individual consumption change is proportional to the agent's Bonacich centrality. Therefore:

Proposition 9. *Consider a legal addictive good. More central agents, in the sense of Bonacich centrality measure, are more responsive to price variation.*

Proposition 9 proposes a peer-effect based micro-foundation to Chaloupka (1991), who estimates that more addicted (myopic) individuals are found to respond more to price, in the long run, than less addicted (myopic) individuals.

Proposition 9 has important practical insights regarding policy efficiency. First, not only more central agents consume more, but also they are more responsive to price increase. Thus, in opposite to possible alternative psychological mechanisms favoring the inertia of big consumers (such as habit formation, where repeated consumption can strengthen automaticity and reduce sensitivity to price changes), the message here is that the share of consumption driven by peer effects remains sensitive to price-based policies. Second, since steady state consumption is shaped by Bonacich centrality, the level of consumption predicts the sensitivity to price change. Hence, the policymaker does not need to observe the network to understand the sensitivity of a consumer to price change. Third, that simple proposition has potentially testable implication; roughly speaking, controlling for individual characteristics, a positive relationship between current consumption and consumption change may help tracking peer effects.

Last, myopic agents are more responsive to price variation than forward-looking agents.[22] Hence, to some extent, the response to price change can then serve as an indicator of consumer behaviors.

6.2 Key-player policy

We analyze a key-player policy consisting in reducing the consumption of a given consumer by an exogenous amount. For instance, this can be the result of a rehabilitation program. By the presence of peer effects, this will affect the consumption of other consumers. Given peer effects, reduced individual consumption entails a reduction in consumption of peers. The optimal targeting policy may thus be a function of the structure of the network of peers.

To have a clue on how peer networks affect targeting, let's consider the folloing key player policy. To simplify, we focus on myopic agents. A policymaker has a budget to spend in the rehabilitation of one consumer of its choice, in the aim of decreasing the aggregate addictive good consumption. Depending on the budget,

[22]This is immediate from observing that highering δ reduces the magnitude of κ, entailing a smaller response to price change. We also performed simulations confirming that the response of present-biased consumers is intermediate between myopic consumers and time-consistent consumers.

the reduction can be partial or total. We focus here on budget $\omega > 0$ spent to care about consumer i entailing partial rehabilitation for any treated consumer i. The rehabilitation program applied to agent i induces a reduction in her addiction parameter α_{cA} (a very similar exercise can be done with a decrease in parameter γ). We suppose that with budget ω the program leads to a reduction of addiction through a change of parameter α_{cA} to α'_{cA}, such that $\alpha'_{cA} = \alpha_{cA} - \Delta_i(\omega)$, with function Δ_i increasing in the budget, and assuming $\lim_{\omega\to\infty} \Delta_i(\omega) \leq \alpha_{cA}$; so that $\alpha'_{cA} \geq 0$.

Contemplating the system of linear interaction characterizing the steady state consumption, this entails a change in both constant and intensity of interaction in line i. The next proposition exploits this limited change. Let

$$f_i(\omega) = \frac{\alpha_p \Delta_i(\omega)}{\gamma} \cdot \frac{1}{(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma})(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\Delta_i(\omega)}{\gamma})}$$

Recall that $\mathbf{M} = (I - \mu \mathbf{G})^{-1}$, and call the diagonal entry (i,i) of matrix $\mathbf{M}$, m_{ii}, the self-loop centrality of agent i; The self-loop centrality captures agent i's direct and self-reinforcing influence on her own consumption, reflecting how her behavior is amplified by the network structure and her own feedback loop. Then:

Proposition 10. *Consider a policymaker undertaking a private addiction-oriented key-player policy, with a limited budget ω leading to change α_{cA} to $\alpha'_{cA} = \alpha_{cA} - \Delta_i(\omega) \geq 0$. The optimal key-player policy consists in choosing consumer i maximizing*

$$f_i(\omega) \cdot \frac{b_i^2}{\mu + f_i(\omega)(m_{ii} - 1)}$$

Proposition 10 shows that the optimal target maximizes an index depending on both Bonacich centrality, self-loop centrality and budget. When all consumers are equally sensitive to the rehabilitation program, i.e. $f_i(\omega) = f(\omega)$ for all i, the optimal target maximizes the index

$$\frac{b_i^2}{\mu + f(\omega)(m_{ii} - 1)}$$

Interestingly, the budget affects the optimal target. For low budget, the agent with highest centrality b_i is selected. For large budget, such that $\mu << f(\omega)(m_{ii} - 1)$, the optimal target maximizes the index $\frac{b_i^2}{(m_{ii}-1)}$, which is close to the so called inter-centrality index $\frac{bi^2}{m_{ii}}$, which corresponds to a problem of dropping an agent out of the network (see Ballester et al. (2006)).

To illustrate, Figure 6.2 presents a network with 11 agents and three type of agents, respectively represented by agents 1, 2 and 3.

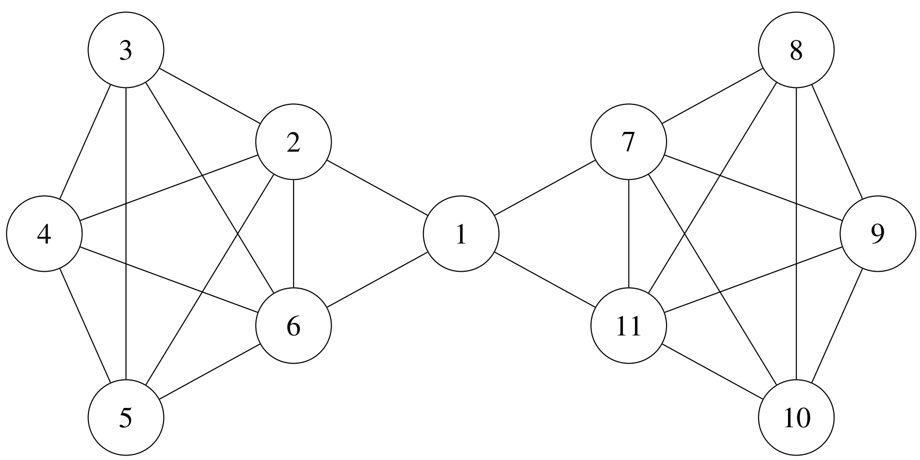

Figure 6.2 represents the variation of aggregate consumption after a rehabilitation program reducing parameter α_{cA} for one agent in the network, as a function of the budget. The blue (resp. red, green) curve represents the effect of the policy applied to agent 1 (resp. 2, 3). For each budget level, the key player is the agent with the lowest curve. This is agent 2 if the budget is low, and this is agent 1 if the budget is high.

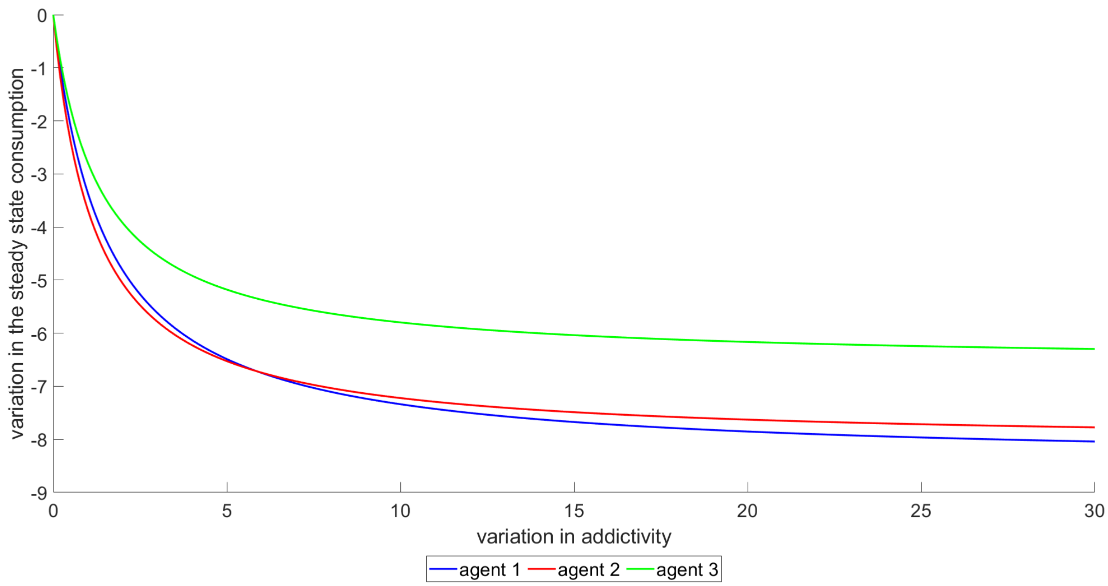

Figure 1: Variation of aggregate consumption after a rehabilitation program reducing parameter α_{cA} for one agent in the network, as a function of the budget. The blue (resp. red, green) curve represents the effects of the policy to agent 1 (resp 2, 3).

We pursue by examining the star network:

Proposition 11. *Consider the star network. The key player is the central agent when $\omega < \omega_c = f^{-1}(2 + \mu n)$ while the key player is a peripheral agent if $\omega > \omega_c$.*

The proof rests on the fact that the Bonacich centrality is favorable to the central agent while the ratio $\frac{b_i^2}{m_{ii}-1}$ is always higher for a peripheral agent (see the proof of Proposition 11). Tuning the budget from 0 to sufficiently large level, the result follows.

Remark 1. *A rehabilitation program to agent i could also alternatively induce a reduction in the private benefit α_c for agent i. Suppose that with budget ω the program leads to a private benefit α_c' such that $\kappa' = \kappa - f_i(\omega)$. Considering then a policymaker undertaking a private benefit-oriented key-player policy, with a limited budget ω, and in the aim of reducing aggregate consumption, it is readily shown that the optimal policy consists in targeting the agent i maximizing $f_i(\omega) \cdot b_i$. The optimal target maximizes Bonacich centrality when all consumers are equally sensitive to the rehabilitation program. In opposite to a program affecting parameter α_{cA}, the optimal target does not depend on the budget level.*

7 Conclusion

This paper has addressed the impact of a network of peers on the demand for addictive good. We modeled peer influence as a linear-in-sum model. The analysis shows the role of the Bonacich centrality in shaping steady state consumption levels, under both myopic, time-consistent and present-biased attitudes. A public policy intervention aiming at reducing addictive good consumption should take care of network effects, either in a context of addictive good taxation, or in a context of rehabilitation program.

This paper opens the room for further research. First, heterogeneity plays a role in predicting addictive behavior. Chaloupka (1991) suggests that some persons, particularly the young and the poor, may discount the future much more heavily than other segments of the population. Furthermore, Gruber (2000) indicates that 'black youth and those with less educated parents are much more responsive to cigarette price than are white teens and those with more educated parents, suggesting a strong correlation between price sensitivity and socioeconomic status.' The model is easily extended to heterogeneous propensities to addiction. Understanding how networked peer effects affect addiction under heterogeneity is therefore an important research agenda.[23]

Second, it would be useful to understand deeper how peer influence varies with the duration and the level of consumption in addictive good. E.g., extreme addiction might lead to social isolation. In that respect, it would be challenging exploring further the endogenous network formation of peer networks, when incentives to form links are closely related to the addiction consumption levels of involved partners.[24]

[23] Providing general insights on the role of heterogeneity is complex issue; see Appendix D in Ushchev and Zenou (2020) for an example on the impact of a mean-preserving spread on effort in the three-player star, in the context of linear-in-means models and conformism.

[24] This issue is linked to deep estimation challenges: "peer effects are notoriously difficult to estimate econometrically because in most contexts, people choose with whom they associate. Hence, while similarities in behavior among members of a group may be due to peer effects, it is difficult to rule out the possibility that group members may be similar to each other along unobserved dimensions or may have come together with the intention of achieving similar outcomes." (Kremer and Levy (2008)).

Last, on the empirical ground, Boucher et al. (2024) find that for addictive behavior like drinking, conformism is a reasonable specification, and that people tend to rely on less active agents rather than mean. It would therefore be useful to bring data to the testable implications of this model.

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A Appendix A: Proofs

A.1 Proofs under myopia

Proof of Corollary 1. By Corollary 1, the best-response consumption levels of agent i in period t and $t-1$ satisfy:

$$\begin{cases} \alpha_c - \alpha_{cc} c_{i,t} + \alpha_{cA} A_{i,t} - \alpha_p c_{i,t} + \alpha_p \bar{c}_{i,t} = 0 \\ \alpha_c - \alpha_{cc} c_{i,t-1} + \alpha_{cA} A_{i,t-1} - \alpha_p c_{i,t-1} + \alpha_p \bar{c}_{i,t-1} = 0 \end{cases}$$

Taking the first equation minus $(1-\gamma)$ times the second equation, we get:

$$\gamma\alpha_c - (\alpha_{cc} + \alpha_p)(c_{i,t} - (1-\gamma)c_{i,t-1}) + \alpha_{cA}(A_{i,t} - (1-\gamma)A_{i,t-1}) + \alpha_p(\bar{c}_{i,t} - (1-\gamma)\bar{c}_{i,t-1}) = 0$$

Observing that $A_{i,t} - (1-\gamma)A_{i,t-1} = c_{i,t-1}$ and rearranging, we find

$$c_{i,t}^{BR,M}(\bar{c}_{i,t}, c_{i,t-1}, \bar{c}_{i,t-1}) = \frac{\gamma\alpha_c}{\alpha_{cc} + \alpha_p} + \left(1 - \gamma + \frac{\alpha_{cA}}{\alpha_{cc} + \alpha_p}\right) c_{i,t-1} + \frac{\alpha_p}{\alpha_{cc} + \alpha_p}\left(\bar{c}_{i,t} - (1-\gamma)\bar{c}_{i,t-1}\right)$$

□

Proof of proposition 2. First, the consumption of addictive good converges if $\lambda(\mathbf{G}) < \frac{1}{\mu_M}$, i.e.

$$\lambda(\mathbf{G}) < \frac{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}}{\alpha_p}$$

This is guaranteed by Assumption 1. The computation of the steady state consumption is straightforward.

□

Proof of lemma 1. Let's pose $\mathbf{M} = (I - \mu\mathbf{G})^{-1}$ and $\mathbf{b} = \mathbf{M}\mathbf{1}$. Notice that $\mathbf{M}$ is the limit of a Neumann series :

$$(I - \mu\mathbf{G})^{-1} = \sum_{k=0}^{\infty} (\mu\mathbf{G})^k.$$

So,

$$\begin{aligned}\frac{\partial \mathbf{b}}{\partial \mu} &= \sum_{k=1}^{\infty} k\mu^{k-1}\mathbf{G}^k\mathbf{1} \\ &= \mathbf{G}\sum_{k=1}^{\infty} k\mu^{k-1}\mathbf{G}^{k-1}\mathbf{1} \\ &= \mathbf{G}\sum_{j=0}^{\infty} (j+1)\mu^{j}\mathbf{G}^{j}\mathbf{1}\end{aligned}$$

If we denote $\mathbf{Z} = \sum_{j=0}^{\infty} j\mu^j\mathbf{G}^j$, we have :

$$\frac{\partial \mathbf{b}}{\partial \mu} = \mathbf{G}(\mathbf{Z} + \mathrm{M})\mathbf{1}$$

We can now write $\mathbf{Z}$ as in function of M:

$$\begin{aligned}\mathbf{Z} &= \sum_{j=1}^{\infty} j\mu^j\mathbf{G}^j \\ &= \sum_{j=1}^{\infty} \mu^j\mathbf{G}^j + \mu\mathbf{G}\sum_{j=1}^{\infty} (j-1)\mu^{j-1}\mathbf{G}^{j-1} \\ &= \sum_{j=1}^{\infty} \mu^j\mathbf{G}^j + \mu\mathbf{G}\mathbf{Z} \\ &= (\mathbf{M} - \mathbf{I}) + \mu\mathbf{G}\mathbf{Z}\end{aligned}$$

Which is equivalent to :

$$\begin{aligned}\mathbf{Z} &= (\mathbf{I} - \mu\mathbf{G})^{-1}(\mathbf{M} - \mathbf{I}) \\ &= \mathrm{M}(\mathbf{M} - \mathbf{I})\end{aligned}$$

It follows that $\frac{\partial \mathbf{b}}{\partial \mu} = \mathbf{G}\mathbf{M}^2$. Yet,

$$\begin{aligned}\mathbf{GM} &= \mathbf{G}\sum_{k=0}^{\infty}(\mu G)^k \\ &= \frac{1}{\mu}\sum_{k=0}^{\infty}(\mu \mathbf{G})^{k+1} \\ &= \frac{1}{\mu}(\sum_{k=0}^{\infty}(\mu \mathbf{G})^k - I) \\ &= \frac{\mathbf{M}-\mathbf{I}}{\mu}\end{aligned}$$

So,

$$\begin{aligned}\frac{\partial \mathbf{b}}{\partial \mu} &= \frac{1}{\mu}(\mathbf{M}-\mathbf{I})\mathbf{M}\mathbf{1} \\ &= \frac{1}{\mu}(\mathbf{M}^2-\mathbf{M})\mathbf{1}\end{aligned}$$

Or, denoting $\mathbf{M}^2\mathbf{1} = \mathbf{b}_{\mathbf{b}}$:[25]

$$\frac{\partial \mathbf{b}}{\partial \mu} = \frac{1}{\mu}(\mathbf{b}_{\mathbf{b}} - \mathbf{b})$$

□

Proof of proposition 3. Derivative of the steady state consumption with respect to α_{cA}. A direct check shows $\frac{\partial \kappa_M}{\partial \alpha_{cA}} > 0, \frac{\partial \mu_M}{\partial \alpha_{cA}}$. We then deduce from (9) that $\frac{\partial \mathbf{c}^{\infty}}{\partial \alpha_{cA}} > 0$.

Derivative of the steady state consumption with respect to γ. A direct check shows $\frac{\partial \kappa_M}{\partial \gamma} < 0, \frac{\partial \mu_M}{\partial \gamma} < 0$. We then deduce from (9) that $\frac{\partial \mathbf{c}^{\infty}}{\partial \gamma} < 0$.

Derivative of the steady state consumption with respect to α_p.

[25] If $\mathbf{b}_{\mathbf{a}} = \sum_j m_{ij} a_j$ represents the Bonacich profile weighted by vector $\mathbf{a}$, $\mathbf{b}_{\mathbf{b}}$ represents the Bonacich vector weighted by the unweighted Bonacich vector.

Define $\phi = \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}$ for convenience. Direct computation entails that $\frac{\partial \mathbf{c}^{\infty}}{\partial \alpha_p} > 0$ if and only if

$$\frac{\phi}{\alpha_p + \phi} \mathbf{b_b} > \mathbf{b} \tag{16}$$

Given that

$$\frac{\phi}{\alpha_p + \phi} = \frac{\phi}{\alpha_p} \mu^M = (\frac{1}{\mu^M} - 1)\mu^M = 1 - \mu^M$$

equation (16) is also written

$$(1 - \mu^M)\mathbf{b_b} > \mathbf{b}$$

i.e., letting $\mathbf{0}$ denote the n-dimensional vector of zeros,

$$(1 - \mu^M)\mathbf{M}^2\mathbf{1} > \mathbf{M1}$$

i.e.,

$$\mathbf{M}(\mathbf{M} - \mathbf{I})\mathbf{1} - \mu^M \mathbf{M}^2\mathbf{1} > \mathbf{0}$$

i.e., given that $\mathbf{M} - \mathbf{I} = \mu^M \mathbf{GM}$,

$$\mu^M(\mathbf{G} - \mathbf{I})\mathbf{M}^2\mathbf{1} > \mathbf{0}$$

i.e., given that $\mathbf{GM} = \mathbf{MG}$ on undirected networks,

$$\mathbf{M}^2(\mathbf{G} - \mathbf{I})\mathbf{1} > \mathbf{0} \tag{17}$$

Now, letting vector $\mathbf{D} = \mathbf{G1}$ represent the profile of degrees,

$$(\mathbf{G} - \mathbf{I})\mathbf{1} = \mathbf{D} - \mathbf{1} \geq 0$$

as we assume that there is no isolated agent. As $\mathbf{M}^2 > \mathbf{0}$, it follows that inequality (17) holds.

□

A.2 Proofs under rational addiction

Proof of Proposition 4. See the proof of Proposition 7, of which it is a special case for $\beta = 1$.

□

Proof of proposition 5. We first establish the conditions of global convergence under time-consistent forward-looking behavior. Let $\mathbf{M} = (\mathbf{I}+\frac{\tau_p^+}{\tau_c^+}\mathbf{G})^{-1}$. The system of best-responses at date t is given by:

$$\mathbf{c}_{t+1} = \mathbf{k} + \mathbf{A}_0\mathbf{c}_t + \mathbf{A}_{-1}\mathbf{c}_{t-1} \tag{18}$$

with

$$\begin{cases} \mathbf{k} = -\frac{\tau_k}{\tau_c^+}\mathbf{M}\mathbf{1} \\ \mathbf{A}_0 = \mathbf{M}\Big(\frac{1}{\tau_c^+}\mathbf{I} - \frac{\tau_p}{\tau_c^+}\mathbf{G}\Big) \\ \mathbf{A}_{-1} = \mathbf{M}\Big(\frac{-\tau_c^-}{\tau_c^+}\mathbf{I} - \frac{\tau_p^-}{\tau_c^+}\mathbf{G}\Big) \end{cases}$$

The system converges if the modulus of any eigenvalue of the matrix

$$P = \begin{pmatrix} \mathbf{A}_0 & \mathbf{A}_{-1} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}$$

is strictly smaller than 1.

Since $\mathbf{G}$ is symmetric, there is an orthonormal matrix $\mathbf{U}$ such that $\mathbf{U}^T = \mathbf{U}^{-1}$ and $\mathbf{G} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^T$ with $\boldsymbol{\Lambda} = Diag(\lambda_i)_{i=1}^{i=n}$ the diagonal matrix of eigenvalues of $\mathbf{G}$. Then,

$$\mathbf{A}_0 = (\mathbf{I} + \frac{\tau_p^+}{\tau_c^+}\mathbf{G})^{-1}\Big(\frac{1}{\tau_c^+}\mathbf{I} - \frac{\tau_p}{\tau_c^+}\mathbf{G}\Big)$$

That is, developing the invert matrix as a series,

$$\mathbf{A}_0 = \mathbf{U}^T\mathbf{D}_0\mathbf{U}$$

with $\mathbf{D}_0 = Diag\Big(\xi_i^0\Big)_{i=1}^{i=n}$ and $\xi_i^0 = \frac{\frac{1}{\tau_c^+} - \frac{\tau_p}{\tau_c^+}\lambda_i}{1+\frac{\tau_p^+}{\tau_c^+}\lambda_i}$.

And similarly, $\mathbf{A}_{-1} = \mathbf{U}^T\mathbf{D}_{-1}\mathbf{U}$, with $\mathbf{D}_{-1} = Diag\Big(\xi_i^1\Big)_{i=1}^{i=n}$ and $\xi_i^1 = \frac{\frac{-\tau_c^-}{\tau_c^+} - \frac{\tau_p^-}{\tau_c^+}\lambda_i}{1+\frac{\tau_p^+}{\tau_c^+}\lambda_i}$.

Now, let $\mathbf{x} = \mathbf{U}\mathbf{c}$. Then the convergence of the dynamics of (29) is given by the system with null constant, that can be written

$$\mathbf{U}^T\mathbf{x}_{t+1} = \mathbf{A}_0\mathbf{U}^T\mathbf{x}_t + \mathbf{A}_{-1}\mathbf{U}^T\mathbf{x}_{t-1}$$

I.e.,

$$\mathbf{U}^T\mathbf{x}_{t+1} = \mathbf{U}^T\mathbf{D}_0\mathbf{x}_t + \mathbf{U}^T\mathbf{D}_0\mathbf{x}_{t-1}$$

Or,

$$\mathbf{x}_{t+1} = \mathbf{D}_0\mathbf{x}_t + \mathbf{D}_{-1}\mathbf{x}_{t-1}$$

The characteristic polynomial associated with eigenvalue λ_i is $r^2 - \xi_i^0 r - \xi_i^1 = 0$. Global convergence is therefore guaranteed by checking that all the roots of the characteristic polynomials lie strictly inside the unit circle.

Second, we compute the steady state by setting $c_{i,t}^{TC} = c_{i,t-1}^{TC} = c_{i,t+1}^{TC} = c_{i,\infty}^{TC}$ and $\bar{c}_{i,t} = \bar{c}_{i,t-1} = \bar{c}_{i,t-2} = \bar{c}_{i,\infty}^{TC}$. The individual steady state consumption of time-consistent for agent i is written:

$$c_{i,\infty}^{TC} = \kappa^{TC} + \mu^{TC}\bar{c}_{i,\infty}^{TC} \tag{19}$$

with

$$\begin{cases} \kappa^{TC} = \frac{\alpha_c}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\delta}{\gamma}\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)}} \\ \\ \mu^{TC} = \frac{\alpha_p}{\alpha_c} \cdot \kappa^{TC} \end{cases}$$

Inverting the system of n equations of individual steady states in matrix form, few computation give the vector of steady state time-consistent consumptions.

□

Proof of Proposition 7. First, we calculate instantaneous utility as a function of past and present consumption of addictive goods by replacing $A_{i,t}$ by its expression as a function of the history of consumption $c_i(t) = (c_{i,0}, .., c_{i,t})$. As a reminder we have :

$$A_{i,t} = \sum_{\tau=0}^{t-1} (1-\gamma)^{t-1-\tau} c_{i,\tau}$$

and

$$u_t(c_i(t)) = \alpha_c c_{i,t} - \frac{1}{2}\alpha_{cc} c_{i,t}^2 + \alpha_{cA} c_{i,t} A_{i,t} - \frac{1}{2}\alpha_{AA} A_{i,t}^2 + \alpha_p c_{i,t} \bar{c}_{i,t}$$

For an infinite history of consumption $c_i(\infty)$, the stream of agent i's utilities at date t is given by

$$U_t(c_i(\infty)) = u_t(c_i(t)) + \beta \textstyle\sum_{\tau=1}^{\infty} \delta^\tau u_{t+\tau}(c_i(t+\tau))$$

Then at the individual optimum at time t individual i chooses $c_{i,t}$ such that :

$$\frac{\partial u_t}{\partial c_{i,t}} = -\beta \sum_{\tau=1}^{\infty} \delta^\tau \frac{\partial u_{t+\tau}}{\partial c_{i,t}}$$

Noticing that

$$\frac{\partial u_{t+\tau}}{\partial c_{i,t}} = (1-\gamma)^{\tau-1}(\alpha_{cA} c_{i,t+\tau} - \alpha_{AA} A_{i,t+\tau})$$

we find

$$\frac{\partial u_t}{\partial c_{i,t}} = -\beta\delta \sum_{\tau=1} \delta^{\tau-1}(1-\gamma)^{\tau-1}(\alpha_{cA} c_{t+\tau} - \alpha_{AA} A_{i,t+\tau})$$

Now we compute the two following differences :

$$\delta(1-\gamma)\frac{\partial u_t}{\partial c_{i,t}} - \frac{\partial u_{t-1}}{\partial c_{t-1}} = \beta\delta(\alpha_{cA} c_{i,t} - \alpha_{AA} A_{i,t}) \tag{20}$$

$$\delta(1-\gamma)\frac{\partial u_{t+1}}{\partial c_{i,t+1}} - \frac{\partial u_t}{\partial c_{i,t}} = \beta\delta(\alpha_{cA}c_{i,t+1} - \alpha_{AA}A_{i,t+1}) \tag{21}$$

The next step is to compute the difference (21) $-(1-\gamma)$(20), in order to eliminate the discounted sum of all past consumption. We get

$$\delta(1-\gamma)\frac{\partial u_{t+1}}{\partial c_{i,t+1}} - (\delta(1-\gamma)^2+1)\frac{\partial u_t}{\partial c_{i,t}} + (1-\gamma)\frac{\partial u_{t-1}}{\partial c_{i,t-1}} =$$

$$\beta\delta\Big(\alpha_{cA}c_{i,t+1} - \alpha_{AA}A_{i,t+1} - (1-\gamma)\alpha_{cA}c_{i,t} + (1-\gamma)\alpha_{AA}A_{i,t}\Big) \tag{22}$$

with

$$\frac{\partial u_t}{\partial c_{i,t}} = \alpha_c - (\alpha_{cc}+\alpha_p)c_{i,t} + \alpha_{cA}A_{i,t} + \alpha_p\bar{c}_{i,t} \tag{23}$$

We then plug the expression of $\frac{\partial u_t}{\partial c_{i,t}}$ given in (23) into (22) to obtain:

$$\begin{aligned}
\delta(1-\gamma)\Big[\alpha_c - (\alpha_{cc}+\alpha_p)c_{i,t+1} + \alpha_p\bar{c}_{i,t+1}\Big] \\
-(1+\delta(1-\gamma)^2)\Big[\alpha_c - (\alpha_{cc}+\alpha_p)c_{i,t} + \alpha_p\bar{c}_{i,t}\Big] \\
+(1-\gamma)\Big[\alpha_c - (\alpha_{cc}+\alpha_p)c_{i,t-1} + \alpha_p\bar{c}_{i,t-1}\Big] \\
+\alpha_{cA}\Big[\delta(1-\gamma)A_{i,t+1} - (1+\delta(1-\gamma)^2)A_{i,t} + (1-\gamma)A_{i,t-1}\Big] \\
= \beta\delta\Big[\alpha_{cA}c_{i,t+1} - (1-\gamma)\alpha_{cA}c_{i,t} + \alpha_{AA}\big((1-\gamma)A_{i,t} - A_{i,t+1}\big)\Big]
\end{aligned}$$

Observing that

$$(1-\gamma)A_{i,t} - A_{i,t+1} = -c_{i,t}$$

and

$$\delta(1-\gamma)A_{i,t+1} - (1+\delta(1-\gamma)^2)A_{i,t} + (1-\gamma)A_{i,t-1} = \delta(1-\gamma)c_{i,t} - c_{i,t-1}$$

we get

$$\begin{aligned}
\delta(1-\gamma)\Big[\alpha_c - (\alpha_{cc}+\alpha_p)c_{i,t+1} + \alpha_p \bar{c}_{i,t+1}\Big] \\
-(1+\delta(1-\gamma)^2)\Big[\alpha_c - (\alpha_{cc}+\alpha_p)c_{i,t} + \alpha_p \bar{c}_{i,t}\Big] \\
+(1-\gamma)\Big[\alpha_c - (\alpha_{cc}+\alpha_p)c_{i,t-1} + \alpha_p \bar{c}_{i,t-1}\Big] \\
+\alpha_{cA}\Big[\delta(1-\gamma)c_{i,t} - c_{i,t-1}\Big] \\
= \beta\delta\Big[\alpha_{cA}c_{i,t+1} - (1-\gamma)\alpha_{cA}c_{i,t} - \alpha_{AA}c_{i,t}\Big]
\end{aligned}$$

from which the period-t best-response of naive consumer follows directly.

□

Proof of Corollary 2. Let $c_0^N(\beta)$ be the steady-state consumption of a naive isolated agent. We show that the consumption of an isolated agent is decreasing in β. Actually,

$$c_0^N(\beta) = \frac{\theta_\beta^0 + \delta\kappa^{TC}\theta_\beta^+}{1-\theta_\beta} \tag{24}$$

That is,

$$c_0^N(\beta) = \frac{a\beta + b}{c\beta + d} \tag{25}$$

with

$$\begin{cases}
a = \delta\alpha_{cA}\kappa^{TC} \\
b = \gamma\alpha_c(1-\delta(1-\gamma)) + (1-\gamma)(\alpha_{cc}+\alpha_p)\kappa^{TC} \\
c = \delta((1-\gamma)\alpha_{cA} + \alpha_{AA}) \\
d = (\gamma + \delta(1-\gamma)^2)(\alpha_{cc}+\alpha_p) - (1-\delta(1-\gamma))\alpha_{cA}
\end{cases}$$

Then, $\frac{\partial c_0^N(\beta)}{\partial \beta} > 0$ iff $ad > bc$. We will see that this cannot happen. Let $\phi = \alpha_{cc} + \alpha_p$ for convenience. Then, $ad > bc$ means

$$\kappa^{TC}\alpha_{cA}((\gamma + \delta(1-\gamma)^2)\phi - (1-\delta(1-\gamma))\alpha_{cA})$$

$$> ((1-\gamma)\alpha_{cA} + \alpha_{AA})((1-\delta(1-\gamma))\gamma\alpha_c + (1-\gamma)\phi\delta\kappa^{TC})$$

That is, developing κ^{TC},

$$\psi_1 + \psi_2 > 0$$

with

$$\begin{cases} \psi_1 = \alpha_c\alpha_{cA}(\gamma\phi - \alpha_{cA}) - \gamma\alpha_c((1-\gamma)\alpha_{cA} + \alpha_{AA})(\phi - \frac{\alpha_c}{\gamma}) \\ \psi_2 = \delta\left[\alpha_c\alpha_{cA}f_1 - ((1-\gamma)\alpha_{cA} + \alpha_{AA})\Big(\gamma\alpha_c f_2 + (\phi - \frac{\alpha_{cA}}{\gamma})f_3 + \delta f_2 f_3\Big)\right] \\ f_1 = (1-\gamma)((1-\gamma)\phi + \alpha_{cA}) \\ f_2 = \frac{1}{\gamma}\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)} \\ f_3 = (1-\gamma)(\phi\kappa^{TC} - \gamma\alpha_c) \end{cases}$$

We show that $\psi_1 < 0$. Indeed,

$$\psi_1 = \alpha_c(\gamma\phi - \alpha_{cA})\Big(\alpha_{cA} - ((1-\gamma)\alpha_{cA} + \alpha_{AA})\Big)$$

I.e.,

$$\psi_1 = \alpha_c(\phi - \frac{\alpha_{cA}}{\gamma})(\gamma\alpha_{cA} - \alpha_{AA})$$

And since $\gamma\alpha_{cA} < \alpha_{AA}$, we deduce that $\psi_1 < 0$.

We show that $\psi_2 \leq 0$. Supposing $\psi_2 > 0$ means

$$\alpha_{cA} \cdot \alpha_c(1-\gamma)((1-\gamma)\phi + \alpha_{cA}) > ((1-\gamma)\alpha_{cA} + \alpha_{AA}) \cdot Q$$

with

$$Q = \alpha_c\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)} + (\phi - \frac{\alpha_{cA}}{\gamma})(1-\gamma)(\phi\kappa^{TC} - \gamma\alpha_c) + \delta\frac{\alpha_{AA} - \gamma\alpha_{cA}}{\gamma(1-\delta(1-\gamma))}(1-\gamma)(\phi\kappa^{TC} - \gamma\alpha_c)$$

Now, because $\gamma\alpha_{cA} < \alpha_{AA}$, we observe that

$$\alpha_{cA} \leq (1-\gamma)\alpha_{cA} + \alpha_{AA}$$

it is therefore sufficient, to show the contradiction, that

$$\alpha_c(1-\gamma)((1-\gamma)\phi+\alpha_{cA})\leq Q$$

That is,

$$(1-\gamma)\phi+\alpha_{cA}\leq$$

$$\frac{1}{1-\gamma}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{\gamma(1-\delta(1-\gamma))}+\alpha_{cA}-\delta\frac{\alpha_{AA}-\gamma\alpha_{cA}}{\gamma(1-\delta(1-\gamma))}+\phi\left(\phi\frac{\kappa^{TC}}{\alpha_c}-\gamma-\alpha_{cA}\frac{\kappa^{TC}}{\gamma\alpha_c}+\delta\frac{\kappa^{TC}}{\gamma\alpha_c}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}\right)$$

That is,

$$(1-\gamma)\phi\leq\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\gamma}+\phi\left(\frac{\phi\kappa^{TC}}{\alpha_c}-\gamma-\alpha_{cA}\frac{\kappa^{TC}}{\gamma\alpha_c}+\delta\frac{\kappa^{TC}}{\gamma\alpha_c}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}\right)$$

I.e.,

$$1\leq(\phi-\frac{\alpha_{cA}}{\gamma})\frac{\kappa^{TC}}{\alpha_c}+\frac{\delta}{\gamma}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}\frac{\kappa^{TC}}{\alpha_c}+\frac{1}{\phi}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\gamma}$$

I.e.,

$$1\leq\frac{\kappa^{TC}}{\alpha_c}\left(\phi-\frac{\alpha_{cA}}{\gamma}+\frac{\delta}{\gamma}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}\right)+\frac{1}{\phi}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\gamma}$$

That is, given that the member in the bracket is the inverse of $\frac{\kappa^{TC}}{\alpha_c}$,

$$1\leq 1+\frac{1}{\phi}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\gamma}$$

and we are done.

□

A.3 Proofs of public policy

Proof of Proposition 10. Let consumer i face a reduction of its addiction parameter α_{cA} following the rehabilitation program. First note that $\lim_{\omega\to\infty}\Delta_i(\omega)\leq\alpha_{cA}$ guarantees a positive steady state consumption after the rehabilitation program.

Then, line i in the linear system of interaction defining steady state consumption is modified as follows: for agent i (only), μ is modified by $\mu' = \mu - f_i(\omega)$ and κ is modified by $\kappa' = \kappa - h_i(\omega)$, with

$$f_i(\omega) = \frac{\alpha_p \Delta(\omega)}{\gamma} \cdot \frac{1}{(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma})(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\Delta_i(\omega)}{\gamma})}$$

and $h_i(\omega) = \frac{\alpha_c}{\alpha_p} f_i(\omega)$. Then, denoting the initial consumption $\mathbf{c} = \kappa \mathbf{M1}$, and the modified inverse linear matrix of the modified system $\mathbf{M}' = \mathbf{M} - f_i(\omega)\mathbf{W}$, and the modified consumption profile $\mathbf{c}' = \mathbf{M}'(\kappa \mathbf{1} - h_i(\omega)\mathbf{1}_i)$, we find that

$$\mathbf{c}' = (\mathbf{M} - f_i(\omega)\mathbf{W})\Big(\kappa \mathbf{1} - \frac{\alpha_c}{\alpha_p} f_i(\omega)\mathbf{1}_i\Big)$$

so that

$$\mathbf{1}^T\mathbf{c}' - \mathbf{1}^T\mathbf{c} = -\kappa f_i(\omega)\mathbf{1}^T\mathbf{W1} - \frac{\alpha_c}{\alpha_p} f_i(\omega)\mathbf{1}^T\mathbf{M1}_i + \frac{\alpha_c}{\alpha_p} f_i(\omega)^2\mathbf{1}^T\mathbf{W1}_i \tag{26}$$

We need to identify matrix $\mathbf{W}$. To proceed, we use the Sherman-Morrison formulae, that states the following property: Suppose $\mathbf{Q}$ is an invertible n-square matrix with real entries and $\mathbf{r}, \mathbf{s} \in \mathbb{R}^n$ are column vectors. Then $\mathbf{Q} + \mathbf{rs}^T$ is invertible if and only if $1 + \mathbf{s}^T\mathbf{Q}^{-1}\mathbf{r} \neq 0$. If $\mathbf{Q} + \mathbf{rs}^T$ is invertible, its inverse is given by

$$(\mathbf{Q} + \mathbf{rs}^T)^{-1} = \mathbf{Q}^{-1} - \frac{\mathbf{Q}^{-1}\mathbf{rs}^T\mathbf{Q}^{-1}}{1 + \mathbf{s}^T\mathbf{Q}^{-1}\mathbf{r}}$$

Let vector $\mathbf{g}_i = (g_{i1}, \cdots, g_{in})^T$ be the i's column of matrix $\mathbf{G}$; let matrix $\mathbf{G}_i$ be the n-square matrix with row i equal to row i in matrix $\mathbf{G}$ and all other rows with zero entries. We apply this formula with $\mathbf{Q} = \mathbf{I} - \mu\mathbf{G}$, $\mathbf{r} = f_i(\omega)\mathbf{1}_i$ and $\mathbf{s} = \mathbf{g}_i$. We then find $\mathbf{M}' = \mathbf{M} - f_i(\omega)\mathbf{W}$ with

$$\mathbf{W} = \frac{\mathbf{MG}_i\mathbf{M}}{1 + f_i(\omega)\sum_k g_{ik}m_{ki}}$$

That is, recalling that $\mu \mathbf{G}\mathbf{M} = \mathbf{M} - \mathbf{I}$,

$$\mathbf{M}' = \mathbf{M} - f_i(\omega)\mu \frac{\mathbf{M}\mathbf{G}_i\mathbf{M}}{\mu + f_i(\omega)(m_{ii} - 1)}$$

and after simplification of the numerator, we get

$$\mathbf{m}'_{kl} = \mathbf{m}_{kl} - f_i(\omega)\frac{m_{ki}(m_{il} - 1_{l=i})}{\mu + f_i(\omega)(m_{ii} - 1)}$$

where $1_{l=i}$ means 1 if $l = i$, 0 otherwise. Plugging that expression into the aggregate steady state consumption, we find

$$\mathbf{1}^T\mathbf{W}\mathbf{1} = \frac{b_i(b_i - 1)}{\mu + f_i(\omega)(m_{ii} - 1)} \tag{27}$$

Also,

$$\mathbf{1}^T\mathbf{W}\mathbf{1}_i = \frac{b_i(m_{ii} - 1)}{\mu + f_i(\omega)(m_{ii} - 1)} \tag{28}$$

Noticing that $\mathbf{1}^T\mathbf{M}\mathbf{1}_i = b_i$ and plugging (27) and (28) into (26), and recalling that $\frac{\alpha_c}{\alpha_p}\mu = \kappa$, we find after rearrangement

$$\mathbf{1}^T\mathbf{c}' - \mathbf{1}^T\mathbf{c} = -\kappa f_i(\omega) \cdot \frac{b_i^2}{\mu + f_i(\omega)(m_{ii} - 1)}$$

□

Proof of Proposition 11. Let $\mathbf{G}_s$ represent the adjacency matrix of the star network. Let subscript c stand for central agent, p for peripheral agent. We first show that $\frac{b_c^2}{m_{cc}-1} < \frac{b_p^2}{m_{pp}-1}$ for all $\mu < \bar{\mu} = \frac{1}{\sqrt{n-1}}$ (recalling that the maximal eigenvalue of $\mathbf{G}_s$ is equal to $\sqrt{n-1}$), then we prove the proposition.

Let $\mathbf{M} = (\mathbf{I} - \mu\mathbf{G}_s)^{-1}$. A few computations implies

$$\begin{cases} b_c = \frac{1+(n-1)\mu}{1-(n-1)\mu^2} \\ m_{cc} = \frac{1}{1-(n-1)\mu^2} \\ b_p = \frac{1+\mu}{1-(n-1)\mu^2} \\ m_{pp} = \frac{1-(n-2)\mu^2}{1-(n-1)\mu^2} \end{cases}$$

Then, $\frac{b_c^2}{m_{cc}-1} < \frac{b_p^2}{m_{pp}-1}$ whenever

$$\frac{(1+(n-1)\mu)^2}{(1-(n-1)\mu^2)(n-1)\mu^2} < \frac{(1+\mu)^2}{(1-(n-1)\mu^2)\mu^2}$$

That is, after simplification,

$$(n-2)((n-1)\mu^2-1) < 0$$

or,

$$\delta < \bar{\mu}$$

Hence, for all $\mu < \bar{\mu}$, $\frac{b_c^2}{m_{cc}-1} < \frac{b_p^2}{m_{pp}-1}$.

Let us now prove that $\frac{b_c^2}{\mu+f(\omega)(m_{cc}-1)} < \frac{b_p^2}{\mu+f(\omega)(m_{pp}-1)}$ whenever $f(\omega) > 2+\mu n$. Indeed,

$$\frac{b_c^2}{\mu+f(\omega)(m_{cc}-1)} < \frac{b_p^2}{\mu+f(\omega)(m_{pp}-1)}$$

means

$$\frac{(1+(n-1)\mu)^2}{\mu+f(\omega)\frac{(n-1)\mu^2}{1-(n-1)\mu^2}} < \frac{(1+\mu)^2}{\mu+f(\omega)\frac{\mu^2}{1-(n-1)\mu^2}}$$

Or, after simplification,

$$(n-2)\mu\Big((2+n\mu)(1-(n-1)\mu^2)+f(\omega)((n-1)\mu^2-1)\Big) < 0$$

That is,

$$(1-(n-1)\mu^2)(2+n\mu-f(\omega)) < 0$$

And given that $1-(n-1)\mu^2 = det(\mathbf{I}-\mu\mathbf{G}_s)^{-1} > 0$, we get

$$f(\omega) > 2+n\delta$$

□

B Appendix B: Global convergence under time-consistent forward-looking behavior

Let $\mathbf{M} = (\mathbf{I} + \frac{\tau_p^+}{\tau_c^+}\mathbf{G})^{-1}$. The system of best-responses at date t is given by:

$$\mathbf{c}_{t+1} = \mathbf{k} + \mathbf{A}_0\mathbf{c}_t + \mathbf{A}_{-1}\mathbf{c}_{t-1} \tag{29}$$

with

$$\begin{cases} \mathbf{k} = -\frac{\tau_k}{\tau_c^+}\mathbf{M1} \\ \mathbf{A}_0 = \mathbf{M}\Big(\frac{1}{\tau_c^+}\mathbf{I} - \frac{\tau_p}{\tau_c^+}\mathbf{G}\Big) \\ \mathbf{A}_{-1} = \mathbf{M}\Big(\frac{-\tau_c^-}{\tau_c^+}\mathbf{I} - \frac{\tau_p^-}{\tau_c^+}\mathbf{G}\Big) \end{cases}$$

The system converges if the modulus of any eigenvalue of the matrix

$$P = \begin{pmatrix} \mathbf{A}_0 & \mathbf{A}_{-1} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}$$

is strictly smaller than 1.

Since $\mathbf{G}$ is symmetric, there is an orthonormal matrix $\mathbf{U}$ such that $\mathbf{U}^T = \mathbf{U}^{-1}$ and $\mathbf{G} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$ with $\mathbf{\Lambda} = Diag(\lambda_i)_{i=1}^{i=n}$ the diagonal matrix of eigenvalues of $\mathbf{G}$. Then,

$$\mathbf{A}_0 = (\mathbf{I} + \frac{\tau_p^+}{\tau_c^+}\mathbf{G})^{-1}\Big(\frac{1}{\tau_c^+}\mathbf{I} - \frac{\tau_p}{\tau_c^+}\mathbf{G}\Big)$$

That is, developing the invert matrix as a series,

$$\mathbf{A}_0 = \mathbf{U}^T\mathbf{D}_0\mathbf{U}$$

with $\mathbf{D}_0 = Diag\Big(\xi_i^0\Big)_{i=1}^{i=n}$ and $\xi_i^0 = \frac{\frac{1}{\tau_c^+} - \frac{\tau_p}{\tau_c^+}\lambda_i}{1+\frac{\tau_p^+}{\tau_c^+}\lambda_i}$.

And similarly, $\mathbf{A}_{-1} = \mathbf{U}^T\mathbf{D}_{-1}\mathbf{U}$, with $\mathbf{D}_{-1} = Diag\Big(\xi_i^1\Big)_{i=1}^{i=n}$ and $\xi_i^1 = \frac{\frac{-\tau_c^-}{\tau_c^+} - \frac{\tau_p^-}{\tau_c^+}\lambda_i}{1+\frac{\tau_p^+}{\tau_c^+}\lambda_i}$.

Now, let $\mathbf{x} = \mathbf{U}\mathbf{c}$. Then the convergence of the dynamics of (29) is given by the system with null constant, that can be written

$$\mathbf{U}^T\mathbf{x}_{t+1} = \mathbf{A}_0\mathbf{U}^T\mathbf{x}_t + \mathbf{A}_{-1}\mathbf{U}^T\mathbf{x}_{t-1}$$

I.e.,

$$\mathbf{U}^T\mathbf{x}_{t+1} = \mathbf{U}^T\mathbf{D}_0\mathbf{x}_t + \mathbf{U}^T\mathbf{D}_0\mathbf{x}_{t-1}$$

Or,

$$\mathbf{x}_{t+1} = \mathbf{D}_0\mathbf{x}_t + \mathbf{D}_{-1}\mathbf{x}_{t-1}$$

The characteristic polynomial associated with eigenvalue λ_i is $r^2 - \xi_i^0 r - \xi_i^1 = 0$. Global convergence is therefore guaranteed by checking that all the roots of the characteristic polynomials lie strictly inside the unit circle.

C Appendix C: Comparing behaviors

The following tables illustrate the impact of peers on individual steady state consumption on various network structures and parameters, by reporting the ratio of consumption over the consumption of an isolated consumer. We consider a regular network (where consumers have the same number of neighbors given by parameter k below), the star network. The parameters used are the following :

$\alpha_c = 0.5, \alpha_{cc} = 1.5, \alpha_{AA} = 0.8, \alpha_{cA} = 0.2, \alpha_p = 0.3, \gamma = 0.5, \delta = 0.1, \beta = 0.5, n = 11, k = 4$

Network	c^M	$c^N(\beta = 0.1)$	$c^N(\beta = 0.5)$	c^{TC}
Regular	1.20	1.18	1.17	1.19
Star (central)	1.86	1.79	1.75	1.05
Star (périph.)	1.15	1.14	1.13	1.14

Table 1: $\delta = \frac{1}{40}, \alpha_p = 0.1$. Ratio of consumption over consumption of isolated consumer, as a function of network position.

Network	c^M	$c^N(\beta = 0.1)$	$c^N(\beta = 0.5)$	c^{TC}
Regular	2.14	2.01	1.94	2.09
Star (central)	9.44	7.61	6.78	8.58
Star (périph.)	3.51	2.91	2.64	3.23

Table 2: $\delta = \frac{1}{40}, \alpha_p = 0.4$. Ratio of consumption over consumption of isolated consumer, as a function of network position.

Network	c^M	$c^N(\beta = 0.1)$	$c^N(\beta = 0.5)$	c^{TC}
Regular	1.20	1.12	1.10	1.15
Star (central)	1.86	1.53	1.45	1.65
Star (périph.)	1.15	1.08	1.06	1.10

Table 3: $\delta = \frac{5}{40}, \alpha_p = 0.1$. Ratio of consumption over consumption of isolated consumer, as a function of network position.

Network	c^M	$c^N(\beta = 0.1)$	$c^N(\beta = 0.5)$	c^{TC}
Regular	2.14	1.60	1.49	1.77
Star (central)	9.44	4.01	3.27	8.58
Star (périph.)	3.52	1.76	1.53	2.18

Table 4: $\delta = \frac{5}{40}, \alpha_p = 0.4$. Ratio of consumption over consumption of isolated consumer, as a function of network position.

The next table presents the absolute consumption levels used to establish the ratios in the above tables.

Parameters	**Network**	c^M	c^{TC}	$c^N(\beta = 0.1)$	$c^N(\beta = 0.5)$
$\delta = \frac{1}{40}, \alpha_p = 0.1$	Empty	0.416667	0.371094	0.406570	0.389782
	Regular	0.500000	0.435780	0.485336	0.461631
	Star (central)	0.777778	0.651259	0.747782	0.700970
	Star (périph.)	0.481481	0.419429	0.467216	0.444307
$\delta = \frac{1}{40}, \alpha_p = 0.4$	Empty	0.333333	0.303514	0.326933	0.315975
	Regular	0.714286	0.590062	0.683817	0.637813
	Star (central)	3.148148	2.059693	2.807131	2.407167
	Star (périph.)	1.172840	0.803632	1.057876	0.922220
$\delta = \frac{5}{40}, \alpha_p = 0.1$	Empty	0.416667	0.234375	0.316865	0.272529
	Regular	0.500000	0.258621	0.364420	0.306261
	Star (central)	0.777778	0.339975	0.523035	0.419047
	Star (périph.)	0.481481	0.250311	0.351193	0.295657
$\delta = \frac{5}{40}, \alpha_p = 0.4$	Empty	0.333333	0.205479	0.265273	0.233962
	Regular	0.714286	0.306122	0.471665	0.376380
	Star (central)	3.148148	0.673196	1.417677	0.938092
	Star (périph.)	1.172840	0.316142	0.580073	0.412418

Table 5: Absolute consumption levels.

The following table shows the steady state consumptions on a regular network with various degrees k. Simulations are preformed with the following parameters (ensuring convergence).

$$\alpha_c = 0.5, \alpha_{cc} = 3.5, \alpha_{AA} = 0.8, \alpha_{cA} = 0.1, \alpha_p = 0.3, \gamma = 0.8, \delta = 0.1, \beta = 0.5$$

k	Type M	Type TC	Type N
2	1.1951	1.1895	1.1922
4	1.4848	1.4675	1.4758
6	1.9600	1.9152	1.9364
8	2.8824	2.7559	2.8149
10	5.4444	4.9122	5.1520

Table 6: Ratio $c_{regular}/c_{empty}$ as a function of the degree k