# The Dark Side of Peers: Demotivation through Social Comparison in Networks*

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**Abstract**

This paper incorporates demotivation into a model of social comparison on networks, where status is determined by relative performance. Demotivated agents experience both a reduced marginal return to effort and lower

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overall effort. In the absence of demotivating status concerns, social comparison increases effort but diminishes welfare. However, introducing demotivation generates a game of strategic substitutes. While demotivation lowers the well-being of affected individuals, it can generate welfare benefits by alleviating social pressure to perform and generating positive status spillovers.

**Keywords.** Social Comparison; Demotivation; Networks; Strategic Substitutes, Equilibrium Welfare.

**JEL Classification.** C72; D85; D91

# 1 Introduction

Comparing oneself with others is a core aspect of human experience. For instance, social comparison is prevalent for people at school, at work, and in fact in nearly all areas of social life, and across sport and culture. While upward comparison can motivate individuals to increase their effort, it can also produce the opposite effect when the performance gap appears too large or improvement seems out of reach. In such cases, individuals may become demotivated and reduce the importance they attach to social comparison.[1] For instance, Goulas and Megalokonomou (2021) show that students in cohorts that did observe their peers' scores and who learned they were low achieving compared to their peers performed 0.3 standard deviations worse the next time they took the exam, compared to cohorts that did not learn their relative position.[2]

[1]To some extent, this idea echoes Festinger's seminal theory of social comparison (although Festinger's work did not examined status-based incentives), which posits that the propensity to compare oneself to others declines as the perceived gap in ability or opinion increases: *"The tendency to compare oneself with some other specific person decreases as the difference between his opinion or ability and one's own increases." (Festinger (1954), p. 120).*

[2]Bénabou and Tirole (2003) show how external social comparison can undermine intrinsic motivation and reduce individuals' effort and performance. Lazear and Rosen (1981) examine how individuals' motivation to exert effort depends on their relative rank in a competitive environment. Similarly, Murphy and Weinhardt (2020) and Dening et al. (2023) establish that an individual's ordinal position within a group significantly impacts later objective outcomes. Such evidence strongly suggests a causal link between negative social comparison —such as being ranked low— and a decline in motivation and effort. The experimental literature on worker performance in organizations has also stressed how payments schemes can lead to diminished returns to effort linked to demotivation under peer pressure. See for instance Eriksson et al. (2009), or Bellemare et al. (2010). See also Golman et al. (2017), p.114-115, that surveys studies documenting the role

Such evidence suggests that when individuals suffer from large unfavorable comparisons, they may reduce the weight they place on social status. In environments where status is a key motivator of effort, this psychological response may in turn reduce the perceived return to exerting effort. Moreover, demotivation can entail significant psychological costs, such as diminished self-esteem or a weakened sense of self-worth.[3] By significantly lowering the returns to effort and inducing psychological distress, demotivation can have far-reaching consequences for economic behavior and outcomes.

In general, people tend to compare themselves to the people they interact with most frequently, i.e. social comparisons are localized to close social contacts in a social network. The structure of the social network can play a crucial role in the emergence of demotivation. An individual's position within the network, as well as the social comparisons facilitated by their connections, can significantly influence their perception of effort and success, potentially leading to demotivation. On the other hand, when someone experiences demotivation, it can affect not only their own social status but also the status of others within the network. By improving the social status of immediate peers, demotivation can create indirect effects that ripple through the network, influencing the motivation of more distant individuals. In this way, social status and motivation form a dynamic feedback loop, where both local comparisons and network structure interact. Understanding how demotivation emerges in the network is therefore a complex issue.

This paper incorporates demotivation into a simple model of social compar-

of information into motivation maintenance.

[3]The literature in social psychology stresses that upward comparison can undermine self-esteem. See for instance Tesser et al. (1988), Collins (1996), Lockwood and Kunda (2003) and Rogers and Feller (2016).

ison in networks. In this work, agents exert costly effort and derive utility not only from their own effort but also from their relative standing within their social network. To capture social comparison, we introduce in the utility function a status component which is an increasing function of the difference between own effort and neighbors' average. We formally introduce demotivation by assuming that demotivation induces a lower return to effort. We model this variation of the return to effort through a kink in the social status function: when own effort is sufficiently far below local peers' effort, the marginal return to effort drops. This kink introduces a convexity in the loss domain. Convexity in the loss domain is also assumed by Kahneman and Tbersky (1979) (see Figure 3, pp. 279 there-in). However, they were mainly interested in the concavity induced by loss aversion, that generates steeper slopes the smaller the losses, while we rather focus on the slope far below in the loss domain. We assume that is that region, the slopes can be lower than the slopes in the region of gain. Our goal is to understand how the network structure influences the emergence of demotivation, and their further consequences on economic outcomes like effort and social welfare.

In the benchmark case with social comparison but without demotivation (that we call the standard status concerns scenario), this model induces a unique equilibrium, in which all agents exert a high effort level. This leads to lower welfare as compared to the case in which there is no social comparison at all (that we call the no-status concerns scenario). This is the standard arm race result known in the literature on conspicuous goods. Introducing demotivating social concerns, the best-response of an agent to local peers' effort consists in choosing a high effort level (and be motivated) when others exert a low effort level, and choosing a low effort level (and be demotivated) when others exert a high effort level. That is, an agent is demotivated when neighbors' average effort exceeds a critical value. Characterizing equilibria, we obtain that an agent is demotivated when the proportion of motivated neighbors exceeds a threshold, that depends on the severity of the kink, but not on the network structure. This equilibrium characterization corresponds to *a network game of strategic substitutes with binary effort choice.* This sharply contrasts with the complementarity-driven incentives commonly found in the status games studied in the literature. In our model, the emergence of strategic substitutes is a direct consequence of the demotivation-related kink in the status function.

We identify a potential function ensuring equilibrium existence.[4] The network structure matters in shaping equilibria in many respects. In this world of strategic substitutes, equilibrium multiplicity can be huge under complex network structures, which raises the issue of finding all equilibria. We show that *the set of equilibria is in general a NP-complete problem*, by establishing a correspondence with the so-called MaxCut problem in the simplest version of the game. Moreover, the impact of network on demotivation can be very strong: in certain networks and for certain values of percentage of others effort below which an agent becomes demotivated, an agent may be demotivated (or motivated) across all equilibria due to their sole position, meaning that the network then fully predicts demotivation. This sharp theoretical prediction echoes stylized facts suggesting that individuals' location in a social network can create persistent patterns of low effort or disengagement.[5]

[4]The formal structure of equilibria, as well as the existence of a potential function, echoes the literature on anti-coordination games. E.g., Blume (1993), Young (1998), or more recently Bramoullé (2007).

[5]For instance, in the field of education, see Blansky et al. (2013), Gerharda et al. (2018),

We show that no equilibrium Pareto-dominates another, reinforcing the non-trivial welfare implications induced by demotivation. We then consider a utilitarian welfare approach. Contrary to the conventional view that status concerns drive excessive effort and reduce welfare, the introduction of demotivation can alter these conclusions. Indeed, the presence of demotivated agents contribute to reduce the social pressure on effort and to improve the social status of their neighbors.

We then study equilibrium welfare, by comparing the welfare of an equilibrium with demotivating concerns scenario to respectively the standard status concerns scenario and the no-status concerns scenario. Addressing comparison with the standard status concerns scenario, the reduction of social pressure always benefits motivated agents through improved social status, and this can even benefit demotivated agents through both reduced effort and and the attenuation of utility loss for demotivated agents that is due to the convexity of the status function. In total, for any network, the welfare of any equilibrium is higher than the equilibrium welfare in the standard status concerns scenario. When demotivation leads to a sufficiently large drop in the return to effort, equilibrium welfare can even exceed that of the no-status concerns, reversing the predictions known in the literature regarding the welfare effects of status effects. Again, the threshold is equilibrium-dependent (and thus network-dependent). These messages, highlighting how demotivating social concerns can generate welfare gains, should be taken with caution. First, the behavioral literature has also pointed out the existence of loss aversion near the reference point (here, neighbors' average effort), which introduces an additional source of utility loss not captured in the current

Kaufman et al. (2020), Korpershoek et al. (2020) and references there-in.

model. Second, demotivation often entails psychological costs revealing significant diminishing well-being, which may offset potential gains and that we study in a separate section.

Introducing psychological costs to demotivation has two main consequences. First, such cost affect the anti-coordination threshold - high costs constitute a threat that makes demotivation less likely; and second, high psychological costs have direct negative consequences for equilibrium welfare. Interestingly, we identify a countervailing effect of psychological costs on equilibrium welfare. Increasing these costs can raise the welfare of the second-best equilibrium — i.e., the equilibrium that yields the highest welfare. The intuition is that higher psychological costs strengthen incentives to remain motivated, potentially leading to a reconfiguration of the set of motivated agents that reduces the aggregate negative externalities that motivated agents exert on others, thus enhancing welfare.

Finally, we extend our model in several directions. We introduce local synergies in the network beyond status effects, heterogeneous agents' characteristics, and more general utility functions. Across these extensions, the emergence of strategic substitutes remains a robust outcome.

*Relationship to the literature.* The literature on status goods[6] has a long tradition in economics. Relative concerns has been modeled through the "keeping up with the Joneses" formulation, in which agents derive utility from consumption relative to an aggregate or group benchmark rather than in absolute terms. Seminal contributions include Veblen (1899), Duesenberry (1949), Frank (1985), Abel

[6]This literature can itself be considered as being inserted in the more general models of aspiration (for a recent survey, see Genicot and Ray (2020)). Our model of demotivating status concerns echoes, so some extent, the aspiration-frustration model of Genicot and Ray (2017), whose intertemporal mechanism is however based on a very different principle.

(1990), Gali (1994), Clark and Oswald (1996), Hopkins and Kornienko (2004), Luttmer (2005), Frank (2005). In these models, social comparisons generate a consumption externality: higher consumption by others reduces the marginal utility of one's own consumption, thereby creating incentives to increase consumption just to maintain relative status. The resulting strategic complementarities amplify aggregate fluctuations, alter effective risk aversion, and often lead to over–consumption in equilibrium.

Our paper inserts more specifically in the literature on status games played on networks. Ghiglino and Goyal (2010) introduce a networked positional good, by which agents derive utility from consumption relative to that of their neighbors, and optimal consumption decisions are strategic complements with networked peers. Their specification leads to predict the consumption levels of the positional good through the so-called Bonacich centrality. Langtry (2023) assumes that agents form a social reference point based on the (weighted) sum of their neighbors consumption (with no loss aversion). They show that an increase in the strength of social comparisons, even by only a few agents, increases consumption and decreases welfare for everyone; and that a higher marginal cost of consumption can increase welfare. Further papers incorporated behavioral economics considerations, especially the loss-aversion phenomenon as introduced by the prospect theory of Kahneman and Tbersky (1979), which introduces an asymmetry around the reference point of status concern. Bramoullé and Ghiglino (2024) incorporate loss aversion into the framework of Ghiglino and Goyal (2010) and find a continuum of equilibria in which all consumers consume the same quantity of the status good on the network when agents' incomes are sufficiently close to each other, while there is a single equilibrium when income dispersion is sufficiently large. Immorlica et al. (2017) examines status concern, a situation in which agents only care about neighbors with higher actions. In their model, the disutility of falling behind others drives effort, which can be interpreted as a form of loss aversion. The main finding is that the cohesion of player sets determines the extent of status-seeking activity. At equilibrium, players stratify into social classes, with each class's action level pinned down by its cohesion. With respect to the literature on status concerns, and in particular to games played on networks, our paper aims at capturing the empirically plausible complementary behavioral phenomenon that falling far behind common standards often reduces the perceived benefits of additional effort (demotivation). Our paper contributes by showing that demotivating social concerns induces a game of strategic substitutes, where the general view in the literature status effect, with or without loss-aversion ingredient, is the presence of strategic complementarity in effort.

In a recent work, Lopez-Pintado and Melendez-Jimenez (2021) present a model in which agents exert a costly effort to obtain a utility gain in case of positive comparison with their reference group; there is no status loss possibility (in case of unfavorable social comparison). Agents repeatedly draw partners from a random process with a fixed degree distribution, and optimal efforts are substitutable. Their context is very different from ours and they do not examine welfare considerations. Yet the mechanism generating effort substitutability echoes ours to a certain extent. In their context agents renounce to incur a high cost to be above others, while in our setting, in which status effect can either reward or penalize according to the relative position with respect to the reference group, agents renounce to maintaining their status, trading lower effort cost with status loss.

Our equilibria with binary substitutes echo the literature on anti-coordination games played on networks. In particular, Bramoullé (2007) examines a binary anti-coordination game played on a fixed network. Anti-coordination also arise in congestion games (Rosenthal (2017)), or for instance in fashion games (Cao et al. (2013)). Our model can be seen as a providing a possible micro-economic foundations to anti-coordination. Moreover, in the literature on network games with linear interaction and strategic substitutes, some equilibria can involve binary actions (Bramoullé and Kranton (2007), Bramoullé et al. (2014)).

Our paper is also related to the literature modeling discouragement of workers in organizations. Gil and Prowse (2012) structurally estimate a model of disappointment aversion in a two-agent real-effort tournament, where only the winner receives a prize. Modeling disappointment-aversion through choice-acclimating reference point, they find that effort can be strategic substitutes (as an agent may reduce effort following an increase of the other agent when their chances of winning are low), which is interpreted as a discouragement effect. Our paper complements these findings by proposing a different source of discouragement, stemming from unfavorable social comparison. In addition, the tractability of our model enables to undertake a network analysis.

There is also a literature on labor supply providing mixed evidence that disadvantageous pay inequality has a negative impact on employee's effort. For studies reporting evidence of such impact, see Gächter and Thöni (2010), Noscenzo (2013), Bracha et al. (2015) for laboratory experiments; see Hennig-Schmidt et al. (2010), Sseruyange and Bulte (2020) for field experiments.

Social psychology has also studied the factors contributing to motivation. For instance, Deci and Ryan (2000) self-determination theory highlights that motivation is shaped by the social environment through the satisfaction of three basic psychological needs—autonomy, competence, and relatedness, and that when these needs are supported, individuals display stronger, more autonomous, and more lasting motivation. Bandura (1977) self-efficacy theory Bandura's framework emphasizes that motivation depends crucially on individuals' beliefs in their own ability to succeed. When people perceive themselves as capable, they invest more effort, persist longer, and show greater resilience in the face of setbacks.

The paper is organized as follows. The networked game of social comparison is presented in Section 2. Section 3 studies equilibria of the game, and Section 4 analyzes the welfare properties of these equilibria. Section 5 introduces psychological cost affecting demotivated agents, while section 6 introduces local synergies. Section 7 concludes. All proofs are relegated in Appendix A. Appendix B extends the model to agents' heterogeneity and to more general utilities.

# 2 Model

Let $\mathcal{N} = \{1, 2, \cdots, n\}$ be a finite set of agents organized in a network of social contacts $\mathbf{G} = (g_{ij})_{(i,j)\in\mathcal{N}^2}$, with $g_{ij} \in \{0, 1\}$ for all $i, j$, $g_{ii} = 0$ by convention, and $\mathbf{G}^T = \mathbf{G}$; the network is therefore binary and undirected. When $g_{ij} = 1$, agents $i$ and $j$ are called neighbors. Let $\mathbf{1}$ represent the n-dimensional vector of ones, let $\mathbf{d} = (d_i)_{i\in\mathcal{N}} = \mathbf{G1}$ be the profile of degrees in network $\mathbf{G}$. To avoid trivialities, we assume that no agent is isolated, so that $\mathbf{d} \geq \mathbf{1}$. Let $\tilde{\mathbf{G}} = (\tilde{g}_{ij})$, with $\tilde{g}_{ij} = \frac{g_{ij}}{d_i}$ be the normalized adjacency matrix in which all entries are divided by agent's degree. Let $x_i \in \mathbb{R}^+$ represent agent $i$'s effort level, and $\mathbf{x} = (x_i)_{i\in\mathcal{N}}$ a profile of effort; let $\overline{x}_i = \sum_{j\in\mathcal{N}} \tilde{g}_{ij} x_j$ the average effort level of agent $i$'s social contacts.

We specify the following utility function for an agent $i$:

$$u_i(x_i, x_{-i}) = ax_i - \frac{1}{2}x_i^2 + \underbrace{S(x_i - \overline{x}_i)}_{\text{status function}} \tag{1}$$

where parameter $a$ represents agent $i$'s private return of effort (See Section 6 for heterogeneous private returns and for more general utilities). Agent $i$'s utility is separable in a private returns to costly effort and a status effect reflecting the utility of social comparison. The status effect is a function of the difference between own effort and average neighbors' effort. Importantly, status can be positive or negative, depending on whether agent's effort is above or below peers' effort.[7]

We now describe the status function, that incorporates demotivation features in line with the stylized facts exhibited in the introduction. The status function is parameterized by three parameters. Let $\gamma_H, \gamma_L, \beta$ be three real numbers such that $0 \leq \gamma_L \leq \gamma_H$, $0 < \beta \leq 1$. We consider a stylized piecewise-linear status function $S(.)$ given by

$$\begin{cases} S(x_i - \overline{x}_i) = \gamma_H(x_i - \overline{x}_i) & \text{if} \quad x_i \geq \beta\overline{x}_i \\ S(x_i - \overline{x}_i) = \gamma_L(x_i - \overline{x}_i) - (1-\beta)(\gamma_H - \gamma_L)\overline{x}_i & \text{if} \quad x_i < \beta\overline{x}_i \end{cases}$$

Figure 5 illustrates the status function, which has several features. An agent suffers a utility loss when effort is below the average of neighbors' effort and experiences a utility gain when effort is higher. Second, the status function incorporates demotivation through the following additional features. One the one hand, above a percentage $\beta$ of neighbors' effort, the marginal return of effort is equal to $\gamma_H$,

[7] Immorlica et al. (2017) use same utility specification, but focus on a very different status function, in which agents are exclusively (negatively) impacted by those neighbors whose effort is larger than theirs. Their game can be viewed as a game of loss aversion, of which it shares the strategic complementarities of agents' actions.

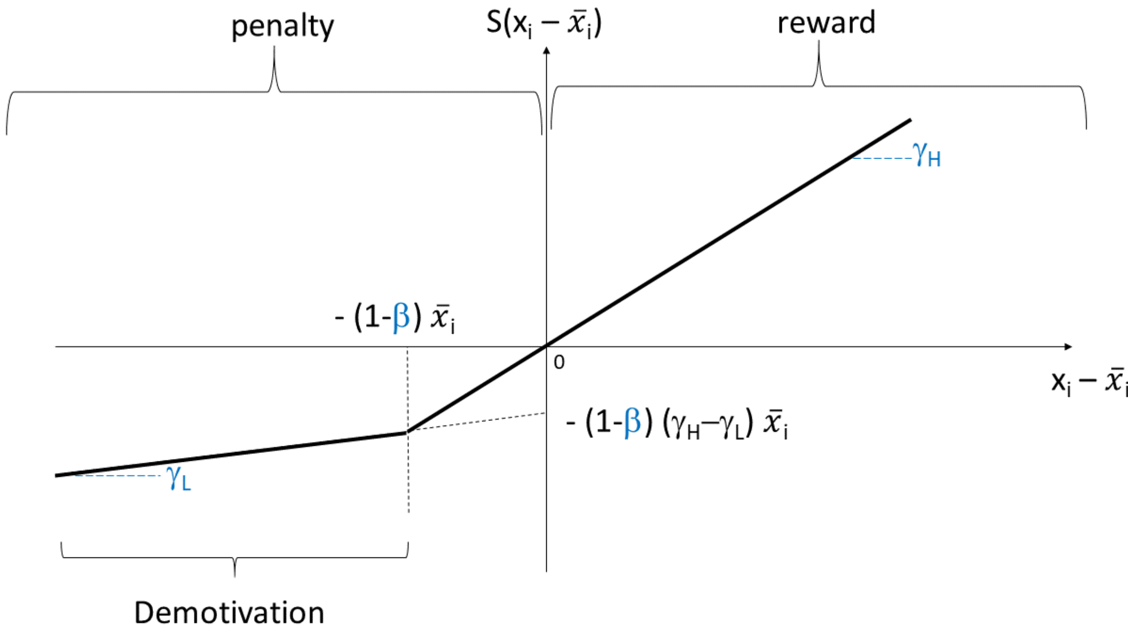



Figure 1: The status function $S$ with demotivation effect.

while below that percentage, the return to effort is lowered to $\gamma_L$ as a result of demotivation.

To summarize, in this model a demotivated agent puts less focus on social comparison (Section 5 studies the case in which a demotivated agent also suffers a psychological cost).

It can be helpful to note the modelling distinctions to Bramoullé and Ghiglino (2024), although their paper shares with our model a kink generating a piecewise-linear status function. First, in their paper, the kink goes the other way, entailing a concavity in the status function, and the kink is at the reference point (the average neighbors' effort), in order to capture loss aversion effect. We rather consider a convex kink potentially far below the average neighbors' effort, with the aim of capturing demotivating status concerns. Second, another salient difference is the utility function per se. They focus on Cobb-Douglass specification and linear interaction, that generates strategic complementarity in effort as a result of social comparison. This implies that the slopes of best-responses to neighbors' effort are always positive, even in the absence of loss aversion. By contrast, with our linear quadratic specification of utilities, status seeking does not generate complementarities in actions in absence of demotivation-related kink, which allows us to focus on the demotivation aspect of status concerns. This is why our best-responses are step functions (as we will see thereafter).

It will be useful to define two benchmark cases. We call the situation where $\gamma_L = \gamma_H = 0$ the no-status concerns scenario, and the situation where $\gamma_L = \gamma_H > 0$ the standard status concerns scenario (of course, parameter $\beta$ is not relevant in those two scenarios). Our main case study, $\gamma_L < \gamma_H$, is called demotivating status concerns scenario.

Throughout the paper, it will be useful to introduce the following notation. For a given effort profile $\mathbf{x}$, we define $\mathbf{e}(\mathbf{x}) \in \{0,1\}^n$, such that $e_i = 1 \Leftrightarrow x_i \geq \beta \overline{x}_i$. Hence, an agent $i$ such that $e_i = 1$ is said *motivated*, and an agent $i$ such that $e_i = 0$ is said *demotivated*. Given profile $\mathbf{e}$, let $d_i^1(\mathbf{e}) = [\mathbf{G}\mathbf{e}]_i$ represent the number of agent $i$'s motivated neighbors, and $d_i^0(\mathbf{e}) = [\mathbf{G}(\mathbf{1} - \mathbf{e})]_i$ represent the number of agent $i$'s demotivated neighbors. Define the index $\rho_i^1(\mathbf{e}) = \frac{d_i^1(\mathbf{e})}{d_i}$ $\big($resp. $\rho_i^0(\mathbf{e}) = \frac{d_i^0(\mathbf{e})}{d_i}\big)$, the proportion of agent $i$'s motivated (resp. demotivated) neighbors.

# 3 Demotivation brings anti-coordination

In this section, we analyze the equilibria of the game. The main insight of this section is that incorporating demotivation into social comparison generates a binary network game of strategic substitutes sort between motivated versus demotivated agents. This message is robust to several generalizations presented later on (see Section 5, Section 6, and Appendix B).

## 3.1 A main result

We analyze the best-responses of the game, and then we characterize equilibria.

*Best-responses.* This model generates simple best-responses.

**Proposition 1** *Let $\varphi = \frac{1}{\beta}\left(a + \frac{\gamma_H+\gamma_L}{2}\right)$. Agent $i$'s best-response to $\overline{x}_i$ is given by:*

$$\begin{cases} x_i^{BR}(\overline{x}_i) = a + \gamma_H & \textit{if} \quad \overline{x}_i \leq \varphi \\ x_i^{BR}(\overline{x}_i) = a + \gamma_L & \textit{if} \quad \overline{x}_i \geq \varphi \end{cases}$$

Figure 2 illustrates the shape of best-responses, which rests on the tradeoff between effort cost and utility gain on status. Motivated agents have to exert a high effort level to gain status, but when the average of neighbors' effort is too high, the reward in utility in terms of status the agent is low, meaning that the agent is better off reducing effort (thus effort cost) at the expense of a loss in status.[8] Importantly,

[8]In this model, the best-response effort of a demotivated agent is larger than the effort exerted under no status effect. In some real-life situations, the psychological impact of demotivation can even lead to annihilate effort incentives. This can be done in the present model without difficulty, by incorporating an impact of demotivation on private return to effort or an impact on effort cost. The analysis is easily extended to such setting, where a demotivated agent can exert a very low effort level, possibly below the effort they would exert in absence of status effect. However, it

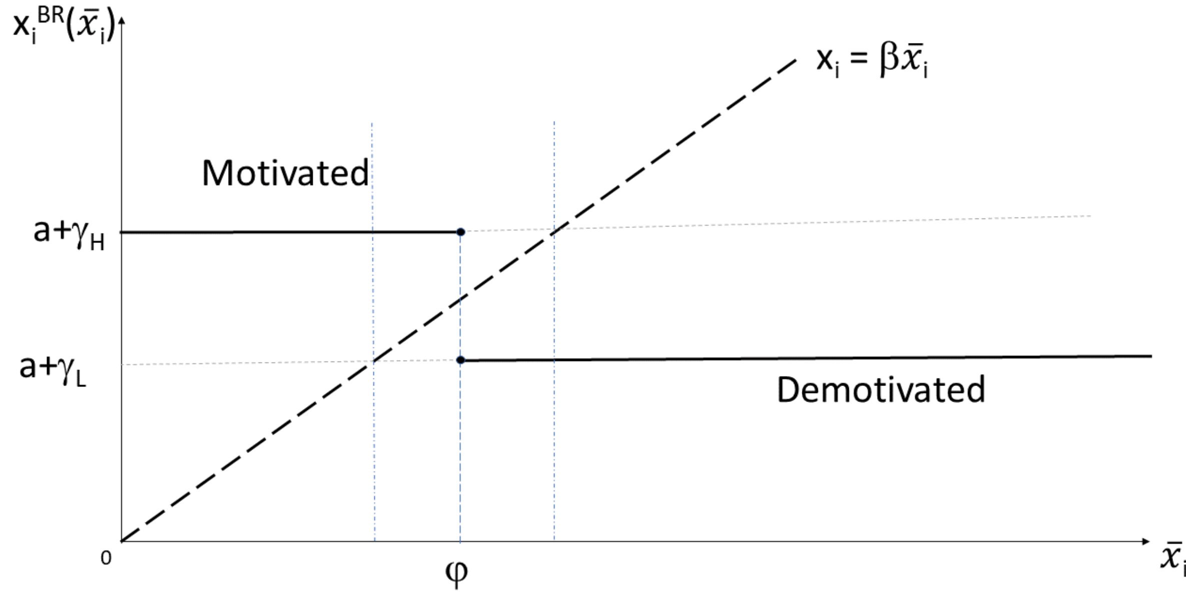



Figure 2: The best-response of an agent $i$ under demotivating status concerns.

the best-response play of a motivated (resp. demotivated) agent is always strictly greater (resp. strictly lower) than average of neighbors' effort. When $\overline{x}_i = \varphi$, the agent has two best-responses, as playing either motivated or demotivated generate same payoff.

*Equilibria.* As a preliminary remark, we observe that, in the standard status concerns scenario, i.e. for $\gamma_L = \gamma_H$, there is a unique equilibrium in which there is no strategic interaction in decisions. Agent $i$ exerts effort $x_i^* = a + \gamma_H$, and reaches an equilibrium utility $u_i^* = \frac{a^2-\gamma_H^2}{2}$. Indeed, the search for status pushes

should be noted that, without introducing a kink in the status function, there is no equilibrium with demotivated agent.

agents to increase effort, and since all efforts are identical at equilibrium, there is no gain in status for each agent. This generates a clear-cut message about the impact of status effect on equilibria in the absence of demotivation:

**Observation 1** *When $\gamma_L = \gamma_H$, there is a unique equilibrium, in which $x_i^* = a + \gamma_H$ for all $i$. At equilibrium, effort is increasing in $\gamma_H$, and all individual utilities are decreasing in $\gamma_H$.*

An immediate implication of Observation 1 is that standard status concerns (i.e., absent any demotivation consideration), status effect entails higher effort and decreased utilities for all with respect to the no-status concerns scenario.

We turn now to demotivating status concerns, i.e. $0 \leq \gamma_L < \gamma_H$ and $0 < \beta \leq 1$. A key aspect is that individuals can decide of being in either of two states: motivated or demotivated.[9] At any equilibrium $\mathbf{x}^*$, not only vector $\mathbf{e}^*(\mathbf{x}^*)$ keeps track of the status of agents in terms of demotivation, but also it fully reveals the (only) two equilibrium effort levels $x_i^* = a + \gamma_L + (\gamma_H - \gamma_L)e_i^*$. For notational convenience, we shall write equilibrium $\mathbf{e}^*$ and omit the reference to $\mathbf{x}^*$. We define, for $\gamma_L < \gamma_H$,

$$\kappa(a, \gamma_L, \gamma_H, \beta) = \frac{2(1-\beta)a + \gamma_H + (1-2\beta)\gamma_L}{2\beta(\gamma_H - \gamma_L)} \tag{2}$$

For convenience, we omit reference to the parameters in what follows and speak about threshold $\kappa$. We observe that $\kappa$ is increasing in $a$, decreasing in $\beta$, and $\kappa \geq \frac{1}{2}$ for all parameters values (that minimal bound is attained when $\beta = 1$). The kink induced by demotivation in the status function brings strategic interaction.

[9] Referring to demotivation as a ‘decision’ is merely a notational convenience to describe optimal actions in the game. It does not necessarily imply that individuals consciously choose to be demotivated in real-life settings.

In particular, an equilibrium $\mathbf{e}^*$ satisfies the following first-order conditions (as shown in the proof of Theorem 1 thereafter):

$$e_i^* = 1 \;\; \Rightarrow \;\; d_i^1(\mathbf{e}^*) \leq \kappa d_i$$
$$e_i^* = 0 \;\; \Rightarrow \;\; d_i^0(\mathbf{e}^*) \leq (1-\kappa) d_i$$

An agent plays motivated if the proportion of motivated neighbors is less than $\kappa$ (which implies reward in status), and similarly an agent plays demotivated if the proportion of demotivated neighbors is less than $1-\kappa$ (which implies loss in status). Thus agents anti-coordinate with those neighbors of same category, motivated and demotivated, and that two-group partition is endogenous at equilibrium.[10]

Actually, equilibria are solutions to a maximization problem with concave objective function

$$P(\mathbf{e}) = (\kappa \mathbf{1} - \frac{1}{2}\mathbf{e})^T \mathbf{G}\mathbf{e} \tag{3}$$

This function is called a potential of the game[11] since $\mathbf{G}^T = \mathbf{G}$. Indeed, setting $\mathbf{e}_{-i} = (e_j)_{j \neq i}$,

$$P(1, \mathbf{e}_{-i}) - P(0, \mathbf{e}_{-i}) = \kappa d_i - d_i^1(\mathbf{e})$$

meaning that, when agent $i$ becomes motivated, this improves the potential function whenever the first-order conditions of the game hold. The potential function guarantees equilibrium existence, and we obtain:

[10] In graph theory, a $k$-dependent set is a subset of vertices such that no vertex in the subset is adjacent to more than $k$ vertices of the subset. $f$-dependent sets generalize $k$-dependent sets to heterogeneous thresholds. Hence, the set of equilibria is an $f$-dependent set with heterogeneous thresholds, where, for vertex $v_i$, the threshold $f(v_i) = \kappa d_i$ (see Diks et al. (1994)).

[11] See Monderer and Shapley (1996) or Voorneveld (2000).

**Theorem 1** *Let the status function be such that $0 \leq \gamma_L < \gamma_H, \beta \leq 1$. There is always an equilibrium. Agents play a binary network game of strategic substitutes. A profile $\mathbf{e}^*$ is a Nash equilibrium if and only if*

$$e_i = 1 \quad \Rightarrow \quad \rho_i^1(\mathbf{e}) \leq \kappa$$
$$e_i = 0 \quad \Rightarrow \quad \rho_i^0(\mathbf{e}) \leq 1 - \kappa$$

*with $\kappa$ defined as in equation (2).*

Theorem 1 gives a powerful message about existence. Due to the potential function, there exists a Nash equilibrium on any network and for any parameter values. From the shape of the potential, and since the support of actions is compact, a maximum of the potential exists and is then a Nash equilibrium. Theorem 1 also provides a powerful characterization of Nash equilibria, expressing that agents are demotivated as soon as the share of motivated neighbors exceeds $\kappa$. Nash stability thus boils down to a simple graph-related criterion.[12]

Equilibria have the following general property. *Consider a configuration $\mathbf{e}$ and an equilibrium $\mathbf{e}^* \neq \mathbf{e}$. If either $\mathbf{e}^* \leq \mathbf{e}$ or $\mathbf{e}^* \geq \mathbf{e}$, then $\mathbf{e}$ is not an equilibrium.*[13] That two equilibria are not nested implies a clear distinction between the groups of demotivated agents.

We give now general insights about how networks affect equilibrium characterization. It is important to stress that demotivation can emerge in regular net-

[12]The characterization given in Theorem 1 is formally equivalent to Bramoullé (2007), Proposition 1. However, in our model with continuous actions, effort selection is binary only at equilibrium; Furthermore, Theorem 1 relates the anti-coordination threshold $\kappa$ to the primitives of our model of status effect with demotivation.

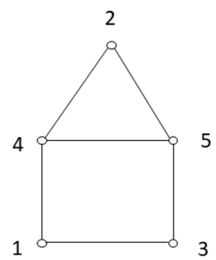
[13]Consider otherwise two distinct and nested equilibria $\mathbf{e}^* \leq \mathbf{e}^{*\prime}$. Then, there is an agent $i$ such that $e_i^* = 0$ and $e_i^{*\prime} = 1$. That is, exploiting equilibrium conditions for agent $i$, $d_i^1(\mathbf{e}^{*\prime}) \leq \kappa d_i < d_i^1(\mathbf{e}^*)$. But since $\mathbf{e}^* \leq \mathbf{e}^{*\prime}$, $\mathbf{G}\mathbf{e}^* \leq \mathbf{G}\mathbf{e}^{*\prime}$, which contradicts that $d_i^1(\mathbf{e}^{*\prime}) < d_i^1(\mathbf{e}^*)$.

works. For instance, for the pair network with two agents, an equilibrium necessarily contains a single demotivated agent: the status effect demotivates agents when the other agent is motivated, whose status effect is enhanced by the demotivation of the other agent. On the complete network of even size, there is unique equilibrium in which the society is shared between two groups of equal size; with an odd number of agents, multiplicity emerges without further refinement on stability solution.[14] There are two equilibria in complete bipartite networks. This is because agents on the same side have the same neighborhood. For general network structures, there can be a high number of Nash equilibria in this game.

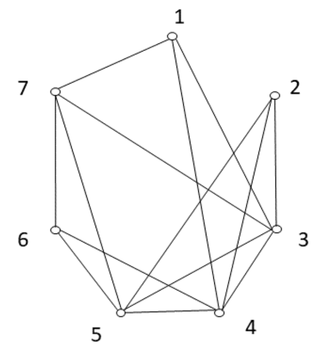
By equation (2), following a decrease in parameter $\beta$, agents anti-coordinate with those of same category, but motivated agents tolerate more motivated neighbors, while demotivated neighbors tolerate less demotivated neighbors. This asymmetry increases with parameter $\beta$. In particular, for $\beta$ sufficiently low, $\kappa$ becomes larger than unity, inducing a single equilibrium with no demotivation; This is in sharp contrast with the case $\beta = 1$, which exhibits a huge multiplicity in general.

An interesting property of equilibria is that, *by her position on the network, an agent can be demotivated in all equilibria*. As well, an agent can be motivated in all equilibria. To illustrate, the Left-panel of Figure 3 depicts a five-agent network, with $a = 1, \gamma_H = 1, \gamma_L = 0, \beta = 0.92$ (which induces $\kappa = 0.63$). There are 3 equilibria, and agent 2 is never demotivated. The Right-panel of Figure 3 depicts a seven-agent network with same parameters. There are 6 equilibria, and agent 2 is

[14]For instance, we might consider a slight refinement to Nash equilibrium by imposing that, in case of indifference, an agent always plays in the motivated region; This could be rationalized through the introduction of a small cost to choosing demotivation. Under that refinement, there is always a unique equilibrium in the complete network.




| Agent | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Eq 1 | 1 | 1 | 1 | 0 | 0 |
| Eq 2 | 1 | 1 | 0 | 0 | 1 |
| Eq 3 | 0 | 1 | 1 | 1 | 0 |




| Agent | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Eq 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| Eq 2 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| Eq 3 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| Eq 4 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| Eq 5 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| Eq 6 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |

Figure 3: Two networks containing locked agents for $a = 1, \gamma_H = 1, \gamma_L = 0, \beta = 0.92$. The state of agents in Nash equilibria are given in respective tables below; 1 means motivated, 0 means demotivated.

always demotivated.[15] Interestingly, demotivation traps are sensitive to parameter $\kappa$, suggesting that targeted public interventions —through modifications of the underlying parameters shaping $\kappa$— could help agents escape demotivation traps.

The model identifies network structures in which an individual is demotivated in all equilibria, generating a demotivation trap driven purely by network position. This theoretical prediction echoes stylized facts documented in education, organi-

[15]Finding a general network property ensuring that at least one agent is demotivated in all equilibria as a function of $\kappa$ is an open issue.

zations, and labor networks, where individuals located in peripheral or disadvantaged network positions consistently exert lower effort or disengage, even when their underlying ability remains comparable. Such persistent under-performance is often attributed to structural disadvantage, relative performance gaps, and status externalities within social networks, and a number of empirical studies document patterns consistent with the demotivation traps highlighted by the model. In educational settings, students' network position is strongly associated with their level of engagement and academic performance: pupils who are socially isolated or located at the periphery of peer networks exhibit significantly lower motivation and achievement, even after controlling for ability (e.g., Kaufman et al. (2020); Korpershoek et al. (2020) and references there-in). Similarly, the structure of classroom help networks predicts academic outcomes, with centrally positioned students performing better and peripheral ones persistently under-performing (Gerharda et al. (2018)). And relative performance within one's friendship network shapes subsequent academic trajectories (Blansky et al. (2013)). Taken together, these stylized facts suggest that individuals' location in a social network can create persistent patterns of low effort or disengagement, providing empirical resonance to the model's prediction that some agents may be trapped into demotivation due to their network position.

To finish, we present additional properties in the limit case $\beta = 1$. Even if this may not be the most likely situation in which social concerns is demotivating, as we expect this to occur when agents' effort is relatively far below from neighbors' effort, the case is instructive to fulfill the analysis. In this situation, demotivation occurs as soon as effort is below neighbors' effort. From equation (2), we deduce $\kappa = \frac{1}{2}$, and thus, by Theorem 1, a configuration $\mathbf{e}^*$ is a Nash equilibrium if and only if

$$e_i = 1 \Rightarrow \rho_i^1(\mathbf{e}) \leq \frac{1}{2}$$
$$e_i = 0 \Rightarrow \rho_i^0(\mathbf{e}) \leq \frac{1}{2}$$

Note that the set of equilibria does not depend on the slopes $\gamma_L, \gamma_H$ of the piecewise-linear status function.

We can state the following property: *When* $\beta = 1$*, for any equilibrium* $\mathbf{e}^*$*, the profile* $\mathbf{1} - \mathbf{e}^*$ *is also an equilibrium.* This follows directly from the fact that both motivated and demotivated agents play an anti-coordination game with the same threshold at equilibrium. Interestingly, this property rules out the possibility of an agent being locked into demotivation (or motivation) solely based on their position in the network (as we have seen before, that possibility emerges when $\beta < 1$).

Furthermore, when $\beta = 1$, finding the set of equilibria is a complex problem. Indeed, as $\kappa = \frac{1}{2}$, the potential function given in equation (3) counts the number of cross links between motivated and demotivated agents. There is therefore a direct correspondence with the MaxCut problem. Indeed, any solution to the MaxCut is, among all two-group partitions, a partition maximizing the number of links between the two groups.[16] Therefore, any two-group partition of the society that is a solution to the MaxCut problem is a maximizer of the potential function, inducing two possible equilibria in which the set of motivated agents coincides with one of the two groups. The MaxCut problem being NP-complete, we deduce:

[16]This is a classical problem of combinatorial optimization, see e.g. Garey and Johnson (1979), Goemans and Williamson (1995).

**Proposition 2** *Assume $\beta = 1$, and $0 \leq \gamma_L < \gamma_H$. The problem of finding the set of equilibria is NP-complete.*

# 4 Demotivation benefits equilibrium welfare

This section examines equilibrium utilities and the welfare implications of equilibria, highlighting how demotivating social concerns can generate welfare gains through three distinct channels: positive externalities on the status of connected individuals, the relaxation of social pressure that reduces costly effort, and the attenuation of utility loss for demotivated agents—reflected in the convexity induced by the kink in the status function. While these mechanisms can lead to overall welfare improvements, which next results will confirm, it is important to stress that two potential caveats. First, demotivation often entails psychological costs that may significantly diminish well-being and offset potential gains; this aspect is further explored in Appendix 5. Second, empirical evidence suggests the possible presence of loss aversion near the reference point (here, neighbors' average effort), which introduces an additional source of utility loss not captured in the current model.

Since parameter $\beta$ plays no pivotal role on result, we assume $\beta = 1$ for simplicity throughout the section. We consider a standard utilitarian approach, in which the social welfare for a profile of effort $\mathbf{x}$ is

$$W(\mathbf{x}; \mathbf{G}) = \sum_{i \in \mathcal{N}} u_i(\mathbf{x}; \mathbf{G})$$

We start here by analyzing the standard status concerns scenario, for which $\gamma_L = \gamma_H > 0$. In that situation, there is a single Nash equilibrium $\mathbf{x}^*$ in which every agent exerts same effort level $x_i^* = a + \gamma_H$. This implies, for any network $\mathbf{G}$,

$$W(\mathbf{x}^*; \mathbf{G}) = \frac{n}{2}(a^2 - \gamma_H^2) \tag{4}$$

Equilibrium welfare does not depend on the network when there is no kink. Furthermore, equilibrium welfare is lowered compared to the no-status scenario. This is because higher effort means higher effort cost but supplementary effort with respect to the no-status scenario generates no status-related utility gain as everyone does same effort. These results are in line with the conclusions of the economic literature on the impact of status effect on effort and welfare. From observation 1, we deduce:

*For any network $\mathbf{G}$, the equilibrium welfare in the standard status concerns scenario (for which $0 < \gamma_L = \gamma_H$) is lower than the equilibrium welfare in the no-status concerns scenario (for which $\gamma_L = \gamma_H = 0$).*

We examine now the impact of demotivating status concerns on any equilibrium. Network effects induce heterogeneous externalities on social status along three dimensions. To see this, consider agent $i$ switching from demotivation to motivation, thus generating negative status-related externalities to others. First, there is a composition effect across agent $i$'s neighborhood as the negative impact of the switch is larger on the utility of a motivated neighbor. Moreover, the impact of agent $i$'s switch on a neighbor $j$ is larger when agent $j$'s degree is lower, and the impact is larger when agent $i$'s degree is higher.

At equilibrium $\mathbf{e}^*$, agent $i$'s effort is given by $x_i^* = a + \gamma_L + (\gamma_H - \gamma_L)e_i^*$. Equilibrium utility is then given by

$$u_i(\mathbf{x}^*) = \frac{a^2 - \gamma_i^2}{2} + S(x_i^* - \overline{x}_i^*)$$

The above utility clarifies the utilities of motivated and demotivated agents. Motivated agents exert a higher effort than demotivated agent, which is detrimental to their private part of utility. However, motivated agents have a utility gain from status where demotivated agents suffer a utility loss from status.

Define $h_i^1(\mathbf{e}) = \sum_{j \in N} \frac{g_{ij}}{dj} e_j$ (this index differs from $\rho_i^1(\mathbf{e}) = \frac{1}{d_i} \sum_{j \in N} g_{ij} e_j$), and $h_i^0(\mathbf{e}) = \sum_{j \in N} \frac{g_{ij}}{dj}(1 - e_j)$. Define $e^* = \mathbf{1}^T \mathbf{e}^*$ for convenience. A few computations provides a characterization of equilibrium welfare:

$$
\begin{aligned}
W(\mathbf{e}^*; \mathbf{G}) = \frac{n(a^2 - \gamma_L^2)}{2} &+ \left( \frac{(\gamma_H - \gamma_L)^2}{2} \right) e^* \\
&- (\gamma_H - \gamma_L) \left( \gamma_L \sum_i e_i^* h_i^0(\mathbf{e}^*) + \gamma_H \sum_i e_i^* h_i^1(\mathbf{e}^*) \right)
\end{aligned} \tag{5}
$$

The welfare of an equilibrium depends three factors: it is increasing in the number of motivated agents $e^*$, it is decreasing in $\sum_i e_i^* h_i^0$, that captures the aggregate status-related externality generated by motivated agents on demotivated neighbors, and it is decreasing in $\sum_i e_i^* h_i^1$, that captures the aggregate status-related externality generated by motivated agents on motivated neighbors. The impact on motivated neighbors is larger than the impact on demotivated neighbors. Given the conflicting forces shaping equilibrium welfare, the second-best (i.e. the equilibrium with highest welfare) may not contain the largest set of motivated agents.

## 4.1 Comparing equilibrium welfare of demotivating status concerns with no-status concerns scenarios

*Comparison with no-status concerns scenario.* Demotivation entails both positive and negative effect. One the one hand, demotivation is good for motivated agents's utility: because demotivated agents reduce their effort level, this enhances their status gain. On the other hand, demotivation has mixed effect on demotivated agents: by reducing their effort level, they trade effort cost against status.

**Proposition 3** *Consider equilibrium* $\mathbf{e}^*$ *on any network* $\mathbf{G}$*. Both motivated and demotivated agents are better off compared to the no-status concerns scenario, meaning that the equilibrium welfare is higher compared to the no-status concerns scenario.*

*Comparison with no-status scenario.* Standard status effect (in absence of demotivating status concerns), is detrimental to all agents. Demotivating status concerns modifies the pictures. It can be good for motivated agents with respect to no-status scenario because, by reducing the social pressure on effort of demotivated agents, it improves the status of motivated agents. However, it is always bad for demotivated agents, who suffer both penalty on status.

Recall that $\rho_i^1(\mathbf{e}^*)$ denotes the share of agent $i$'s neighbors who are motivated in equilibrium $\mathbf{e}^*$. Comparing to the no-status scenario, demotivated agents always experience unfavorable social comparisons, incurring both higher effort costs and a status penalty. In opposite, motivated agents can benefit from status effect when the proportion of motivated neighbors is sufficiently low (ensuring high status reward), in spite incurring high effort cost:

**Result 1** *A demotivated agent is strictly worse off compared to the no-status concerns scenario. A motivated agent is strictly better off compared to the no-status concerns scenario if and only if*

$$\rho_i^1(\mathbf{e}^*) \leq \frac{1}{2} - \frac{\gamma_L}{2(\gamma_H - \gamma_L)}$$

As intuition suggests, the condition under which motivated agents are better off than in the no-status concerns scenario is less demanding when the gap $\gamma_H - \gamma_L$

is larger. Note in particular that, for $\gamma_L = 0$, a motivated agent always benefits from status effect, because the inequality $\rho_i^1(\mathbf{e}^*) \leq \frac{1}{2}$ holds at equilibrium. And in opposite, a motivated agent is always penalized by status effect when $\gamma_L$ is sufficiently close from $\gamma_H$, in particular when $\gamma_H < 2\gamma_L$ (as this means $\frac{1}{2} - \frac{\gamma_L}{2(\gamma_H - \gamma_L)} < 0$).

We can then compare equilibrium welfare to the no-status concerns scenario. The key driver is the ratio $\frac{\gamma_L}{\gamma_H}$. For convenience, we denote $\phi^* = \sum_i e_i^*(1 - 2\rho_i^1(\mathbf{e}^*)) \geq 0$ (as, in any equilibrium, for all $i : e_i^* = 1$, $\rho_i^1(\mathbf{e}^*) \leq \frac{1}{2}$) and $\phi^{**} = \sum_i \rho_i^1(\mathbf{e}^*)$. Recall that in any equilibrium, for all $i : e_i^* = 0$, $\rho_i^1(\mathbf{e}^*) \geq \frac{1}{2}$. We obtain:

**Proposition 4** *Assume $\beta = 1$, and $0 \leq \gamma_L < \gamma_H$. For any network $\mathbf{G}$, the welfare at equilibrium $\mathbf{e}^*$ is larger than the equilibrium welfare in the no-status concerns scenario (i.e., $\gamma_L = \gamma_H = 0$) if and only if the ratio $\frac{\gamma_L}{\gamma_H}$ is lower than the following threshold $\tau_c(\mathbf{e}^*)$:*

*If $\forall i : e_i^* = 0$, $\rho_i^1(\mathbf{e}^*) = \frac{1}{2}$, then $\tau_c(\mathbf{e}^*) = \frac{\phi^*}{2(\phi^* + \phi^{**})}$. Otherwise,*

$$\tau_c(\mathbf{e}^*) = \frac{\phi^* + \phi^{**} - \sqrt{(\phi^{**})^2 + n\phi^*}}{\phi^* - n + 2\phi^{**}}$$

Note that many equilibria are such that, for all demotivated agents, the proportion of motivated neighbors is equal to $\frac{1}{2}$ (so that the relevant condition in the above proposition is the first one).[17] By Proposition 4, the presence of demotivated agents induces a complete reversal in the qualitative effect of status on welfare. While standard status concerns reduce equilibrium welfare, a sufficiently

[17]For instance, consider a three-agent complete network, with two demotivated agents, or consider a eight-agent circle, in which agents $1, 2, 5, 6$ are motivated, and agents $3, 4, 7, 8$ are demotivated.

pronounced demotivating kink can lead to welfare gains, as demotivated agents strongly enhance the status, and thus the utility, of motivated agents. Again, the threshold ratio for which welfare with status effect dominates welfare without status effect is equilibrium-dependent (and thus also network-dependent).

In the extreme case where $\gamma_L = 0$, demotivated agents are not affected by status effect, meaning that, for them, both effort and utilities are equal to those in the absence of status. Then, only motivated agents are affected by status. By the presence of demotivated agent, motivated agents get status-related reward, and that reward dominates the extra effort cost necessary to be motivated by construction of the equilibrium. Therefore:[18]

**Corollary 1** *Assume $\beta = 1, \gamma_L = 0$ and $\gamma_H > 0$. The welfare of an equilibrium $\mathbf{e}^*$ on a given network $\mathbf{G}$ is given by*

$$W(\mathbf{e}^*; \mathbf{G}) = \frac{na^2}{2} + \frac{\gamma_H^2}{2} \sum_i e_i^*(1 - 2\rho_i^1(\mathbf{e}^*))$$

*Hence, for all networks, the welfare of any equilibrium is larger than the welfare of the equilibrium in the no-status concerns scenario.*

## 4.2 Statics on the kink

We examine how the strength of social comparison in the status function affects equilibrium welfare through separate comparative statics with respect to parameters $\gamma_L, \gamma_H$. As observed earlier, the equilibria do not depend on $\gamma_L, \gamma_H$. For simplicity we undertake these two statics assuming that the set of equilibria is unaffected by a change in these parameters; this means that we disregard parameters

[18]The proof of Corollary 1 is immediate, recalling that $\rho_i^1(\mathbf{e}^*) \leq \frac{1}{2}$ for all $i : e_i^* = 1$ on any equilibrium.

for which there is a knife-edge agent.[19] Interestingly, the two statics do not have a symmetric effect on equilibrium welfare.[20]

*Statics on $\gamma_L$.* To start with, we examine how a marginal decrease in parameter $\gamma_L$ affects agents' utilities:

**Proposition 5** *Assume $\beta = 1$ and $0 \leq \gamma_L < \gamma_H$. Let $\mathbf{G}$ be any network, and $\mathbf{e}^*$ any corresponding equilibrium robust to a marginal decrease of $\gamma_L$. When $\gamma_L$ decreases marginally, this improves the equilibrium utilities of both motivated and demotivated agents.*

By Proposition 5, sharpening the kink through a reduction of $\gamma_L$ induces a reduction of the pressure of social comparison, which benefits both demotivated and motivated agents. Hence, decreased $\gamma_L$ improves equilibrium welfare.

*Statics on $\gamma_H$.* We examine how a marginal increase in parameter $\gamma_H$ affects agents' utilities:

**Result 2** *Assume $\beta = 1$ and $0 \leq \gamma_L < \gamma_H$. Let $\mathbf{G}$ be any network, and $\mathbf{e}^*$ any equilibrium robust to a marginal increase of $\gamma_H$. When $\gamma_H$ increases marginally, this reduces the equilibrium utilities of demotivated agents, and this increases the equilibrium utility of a motivated agent $i$ if and only if*

$$\rho_i^1(\mathbf{e}^*) < \frac{1}{2 + \frac{\gamma_L}{\gamma_H - \gamma_L}}$$

By Result 2, sharpening the kink through an increase of $\gamma_H$ induces more pressure from social comparison, which is detrimental to demotivated agents but leads motivated agents to be better off through a gain in status when the proportion of their

[19] That is, for all $i : e_i = 1$, $\rho_i^1(\mathbf{e}^*) < \kappa$ and for all $i : e_i = 0$, $\rho_i^1(\mathbf{e}^*) > \kappa$.

[20] Hence, a rotation of the kink preserving its angle is not neutral for welfare.

motivated neighbors is low enough (which ensures a high status reward). Overall, the qualitative impact on equilibrium welfare depends on the network:

**Proposition 6** *Assume $\beta = 1$, and $0 \leq \gamma_L < \gamma_H$. For any network $\mathbf{G}$, any equilibrium $\mathbf{e}^*$, the equilibrium welfare is increasing in parameter $\gamma_H$ if and only if*

$$\sum_i e_i^*(1 - 2\rho_i^1(\mathbf{e}^*)) > \frac{\gamma_L}{\gamma_H - \gamma_L} \sum_i \rho_i^1(\mathbf{e}^*) \tag{6}$$

By Proposition 6, the impact of increased $\gamma_H$, through a higher social pressure, is equilibrium-dependent. The condition given in equation (6) is more favorable to welfare improvement for lower values of $\gamma_L$ – It is met for $\gamma_L = 0$ and it fails for $\gamma_L$ sufficiently close to $\gamma_H$.

## 4.3 Pareto-dominance

We also investigate whether some equilibria Pareto-dominate others. Take the 4-star network, which has two equilibria. It is easily shown that no equilibrium Pareto-dominates the other. That message is confirmed more generally:

**Proposition 7** *Assume $\beta \leq 1$, and $0 \leq \gamma_L < \gamma_H$. For any network $\mathbf{G}$, no equilibrium Pareto-dominates another equilibrium.*

The absence of Pareto-dominance among equilibria stems from the fact that it is always better to be motivated in a given equilibrium than to be demotivated in any equilibrium. Then the point follows from the non-nestedness property of equilibria that stems from the nature of the game (as presented before).

# 5 Demotivation-related psychological cost

Demotivating social concerns can entail significant psychological costs, such as diminished self-esteem or a weakened sense of self-worth. We thus assume now that demotivated agents suffer a loss in utilities that captures some psychological costs $\psi > 0$.[21]

*Equilibria.* We incorporate psychological cost $\psi > 0$ related to demotivation into the social status function. The status function is now parameterized by four parameters. Let $\gamma_H, \gamma_L, \beta, \psi$ be four real numbers such that $0 \leq \gamma_L < \gamma_H$, $0 < \beta \leq 1$, and $\psi > 0$. We consider a stylized piecewise-linear status function $S(.)$ given by

$$\begin{cases} S(x_i - \overline{x}_i) = \gamma_H(x_i - \overline{x}_i) & \text{if} \quad x_i \geq \beta\overline{x}_i \\ S(x_i - \overline{x}_i) = \gamma_L(x_i - \overline{x}_i) - (1-\beta)(\gamma_H - \gamma_L)\overline{x}_i - \psi & \text{if} \quad x_i < \beta\overline{x}_i \end{cases}$$

Figure 5 illustrates the status function. Demotivation induces a utility loss $\psi$ at the kink, aimed to capture possible psychological costs, like for instance lowered self-esteem, self-worth.[22]

[21] The literature in social psychology stresses that upward comparison can undermine self-esteem. For instance, Tesser et al. (1988) provide empirical evidence that when another outperforms the self on a task high in relevance to the self, the closer the other the greater the threat to self-evaluation. Wheeler and Miyake (1992) find that students reported feeling depressed and discouraged when they compared themselves with superior people. Exploring the impact of upward social comparison on self-evaluations, Collins (1996) underlines that 'expecting to be different from an upward target should lead to a contrast effect, feelings of inferiority, and more negative self-appraisals'. Lockwood and Kunda (2003) and Rogers and Feller (2016) show that exposure to exemplary peer performances can undermine motivation and success by causing people to perceive that they cannot attain their peers' high levels of performance.

[22] Modeling psychological costs with a discontinuity in the utility enables to maintain the property that return to effort is increasing with effort, which is crucial in the context of demotivation.

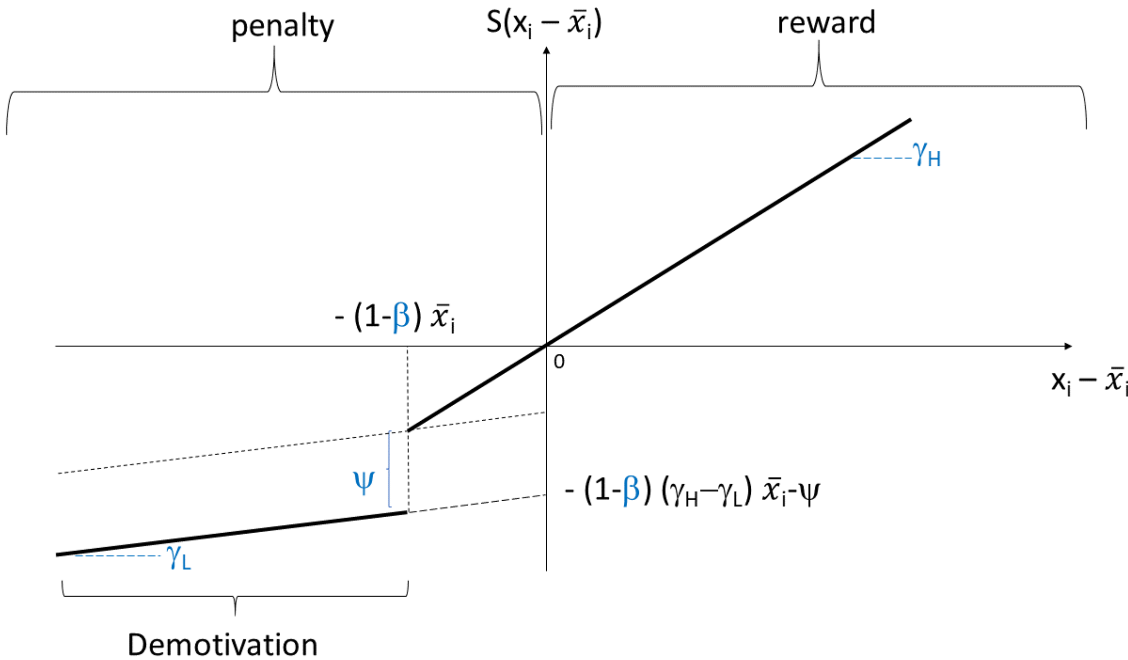



Figure 4: The status function $S$ with demotivation effect.

We define, for $\gamma_L < \gamma_H$ (abusing the notation by relating $\kappa$ to the sole primitive $\psi$ for convenience),

$$\kappa(\psi) = \frac{2(1-\beta)a + \gamma_H + (1-2\beta)\gamma_L}{2\beta(\gamma_H - \gamma_L)} + \frac{\psi}{\beta(\gamma_H - \gamma_L)^2} \tag{7}$$

Then we get the same shape in best-responses as in Proposition 1, modulo a modification of parameter $\varphi$. Letting $\varphi(\psi) = \frac{1}{\beta}\left(a + \frac{\gamma_H+\gamma_L}{2} + \frac{\psi}{\gamma_H-\gamma_L}\right)$, agent $i$'s best-response to $\overline{x}_i$ is given by:

$$\begin{cases} x_i^{BR}(\overline{x}_i) = a + \gamma_H & \text{if} \quad \overline{x}_i \leq \varphi(\psi) \\ x_i^{BR}(\overline{x}_i) = a + \gamma_L & \text{if} \quad \overline{x}_i \geq \varphi(\psi) \end{cases}$$

Then, Theorem 1 straightforwardly extends to that setting by replacing $\kappa$ by $\kappa(\psi)$:

**Theorem 2** *Let the status function be such that* $0 \leq \gamma_L < \gamma_H, \beta \leq 1, 0 \leq \psi$. *There is always an equilibrium. Agents play a binary network game of strategic substitutes. A profile* $\mathbf{e}^*$ *is a Nash equilibrium if and only if*

$$e_i = 1 \quad \Rightarrow \quad \rho_i^1(\mathbf{e}) \leq \kappa(\psi)$$
$$e_i = 0 \quad \Rightarrow \quad \rho_i^0(\mathbf{e}) \leq 1 - \kappa(\psi)$$

*with* $\kappa(\psi)$ *defined as in equation (7).*

To understand better the specific role of utility loss at the kink, we neutralize parameter $\beta$, assuming $\psi > 0, \beta = 1$. This entails a change of slope and a discontinuity at 0.From equation (7), we find

$$\kappa = \frac{1}{2} + \frac{\psi}{(\gamma_H - \gamma_L)^2}$$

Hence, incorporating a utility loss affects equilibria. The threshold number of motivated neighbors above which an agent becomes demotivated is higher, reflecting that the agent prefers to incur a higher cost to exerting high effort and avoiding the utility loss arising under demotivation. Motivation is thus enhanced by the utility loss. Note that the threshold is decreasing in the gap $\gamma_H - \gamma_L$ whereas it does not depend on the gap when $\psi = 0$. One consequence of increased threshold is that, as for $\beta < 1$, for some values of parameter $\psi$, agents can be locked to demotivation (or motivation) in all equilibria.

*Impact of psychological cost on welfare.* At equilibrium $\mathbf{e}^*$, agent $i$'s effort is given by $x_i^* = a + \gamma_L + (\gamma_H - \gamma_L)e_i^*$. Equilibrium utility is then given by

$$u_i(\mathbf{x}^*) = \frac{a^2 - \gamma_i^2}{2} + S(x_i^* - \overline{x}_i^*) - (1 - e_i^*)\psi$$

Motivated agents exert a higher effort than demotivated agent, which is detrimental to their private part of utility. However, motivated agents have a utility gain from status where demotivated agents suffer a utility loss from status plus a psychological cost.

Recall that $h_i^1(\mathbf{e}) = \sum_{j \in N} \frac{g_{ij}}{dj} e_j$, and $h_i^0(\mathbf{e}) = \sum_{j \in N} \frac{g_{ij}}{dj}(1 - e_j)$, $e^* = \mathbf{1}^T \mathbf{e}^*$. Equilibrium welfare is written:

$$W(\mathbf{e}^*; \mathbf{G}) = \frac{n(a^2 - \gamma_L^2)}{2} + \left(\frac{(\gamma_H - \gamma_L)^2}{2} + \psi\right) e^* - n\psi \\ - (\gamma_H - \gamma_L)\left(\gamma_L \sum_i e_i^* h_i^0(\mathbf{e}^*) + \gamma_H \sum_i e_i^* h_i^1(\mathbf{e}^*)\right) \tag{8}$$

Next Proposition summarizes the implications in terms of welfare.

**Proposition 8** *Consider equilibrium* $\mathbf{e}^*$ *on any network* $\mathbf{G}$*. The equilibrium welfare is higher compared to the no-demotivation scenario if and only if*

$$\psi \leq \frac{\gamma_H - \gamma_L}{2(n - e^*)}\left(n(\gamma_L + \gamma_H) + (\gamma_H - \gamma_L)e^* - 2\left(\gamma_H \sum_i e_i^* h_i^1(\mathbf{e}^*) + \gamma_L \sum_i e_i^* h_i^0(\mathbf{e}^*)\right)\right)$$

By Proposition 3, the impact of demotivation is positive to all agents through the relaxation of social pressure on demotivated agents. Indeed, motivated agents benefit from status reward, while demotivated agents advantageously exert less effort at the expense of status. In opposite, significant psychological costs have heavy consequence on equilibrium welfare. Even if a demotivated agent is better off than being motivated, the presence of demotivated agents as a whole induces substantial aggregate utility losses. Note that the threshold on $\psi$ above which the introduction of demotivation is detrimental to equilibrium welfare is equilibrium-dependent (and thus network-dependent).

We can then compare equilibrium welfare to the no-status scenario. Overall, the impact on equilibrium welfare depends on both the severity of the kink and the magnitude of psychological cost:

**Proposition 9** *Consider equilibrium* $\mathbf{e}^*$ *on any network* $\mathbf{G}$*. The equilibrium welfare is higher compared to the no-status scenario if and only if*

$$\psi \leq \frac{(\gamma_H - \gamma_L)^2 e^* - (\gamma_H - \gamma_L)\Big(\gamma_H \sum_i e_i^* h_i^1(\mathbf{e}^*) + \gamma_L \sum_i e_i^* h_i^0(\mathbf{e}^*)\Big) - n\gamma_L^2}{2(n - e^*)}$$

By Proposition 9, demotivation can be welfare-improving with respect to the no-status scenario when demotivated agents suffer sufficiently low psychological costs, and when the kink is sufficiently pronounced. The critical bound on psychological cost is equilibrium-dependent.

*Impact of psychological cost on second-best equilibrium welfare.* The impact of the psychological costs of demotivated agents on the welfare of equilibria is subtle. To see this, we consider an increase of $\psi$. When the increase does not affect the set of equilibria, a higher utility cost $\psi$ lowers the welfare of all equilibria containing demotivated agents. However, increasing $\psi$ can affect the set of equilibria by increasing incentives to be motivated, and this can result in higher equilibrium welfare.

We illustrate how this countervailing effect operates by focusing on the second-best equilibrium, assuming $\gamma_L = 0$ for simplicity. Few computation gives the welfare of an equilibrium:

$$W(\mathbf{e}^*) = \frac{na^2}{2} + \frac{\gamma_H^2}{2} e^* - \gamma_H^2 \sum_i e_i^* \rho_i^1(\mathbf{e}^*) - (n - e^*)\psi$$

Hence, the equilibrium welfare takes into account the aggregate utility loss $(n - e^*)\psi$ generated by demotivated agents, and it also takes into account the negative aggregate impact of status among motivated agents, as measured by the sum over all motivated agents of the shares of their motivated neighbors. In the example given by the 11-agent network depicted in Figure 5, increased $\psi$ enhances the welfare at the second-best equilibrium for the following parameter values. We fix $a = 2, \gamma_H = 1, \gamma_L = 0, \beta = 1$, and we consider $\psi = 0.16$ and $\psi = 0.17$. Setting $\psi = 0.16$, we get $\kappa = 0.66$. With these parameter values, the network depicted in the figure has ten equilibria. The second-best equilibrium, presented in the Left-panel, reaches a welfare of $22.26$. We note that the sum of the shares of motivated neighbors over all motivated agents is equal to $2.6$. For $\psi = 0.17$, we obtain $\kappa =$

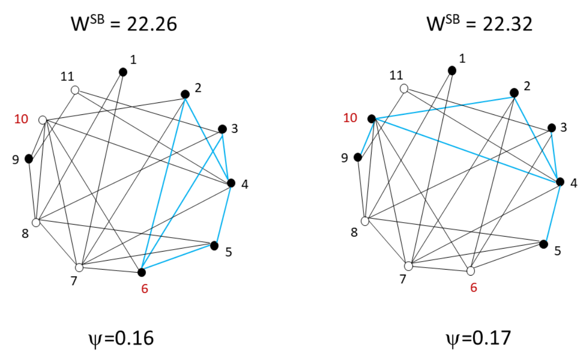



Figure 5: Increasing the psychological cost $\psi$ improves the welfare of the second-best equilibrium; $n = 11$, $a = 2, \gamma_H = 1, \gamma_L = 0, \beta = 1$. Black nodes (resp. white nodes) are motivated (resp. demotivated). The links among motivated agents are in blue.

$0.67$. Again, there are ten equilibria, but the second-best equilibrium is modified. The second-best equilibrium for $\psi = 0.17$ is depicted in the Right-panel. This slight increase in the psychological cots incurred by demotivated agents modifies the incentives of agents 10 and 6: Agent 10 becomes motivated by the increase of $\kappa$, but that change makes agent 6 become demotivated. The consequence on the welfare at second-best is as follows: given that the number of motivated agents is unchanged between the two second-bests, the increase in $\psi$ is neutral with respect to the aggregate utility losses. The only difference is the sum of the shares of motivated neighbors over motivated agents, that is now equal to $2.5$ (note that the share of motivated neighbors is modified for many agents). This reduction of the aggregate (negative) impact of motivated agents on others is good for the welfare.

# 6 Local synergies

We incorporate local synergies in sum in the model. For instance, this can fit with applications related to education, or workers.[23] Like status effects, synergies tend to higher effort as a source of strategic complementarities. The analysis mainly suggests that even in presence of synergies the strategic substitute nature of interactions is a robust mechanism. However the analysis is challenging, and identifying who is demotivated is more complex than examining the sole neighbors' behavior.

We assume the following specification:

$$u_i = x_i - \frac{1}{2}x_i^2 + \delta x_i d_i \overline{x}_i + S(x_i - \overline{x}_i)$$

with parameter $\delta \geq 0$ representing the intensity of synergies among neighbors. For simplicity, we assume $\beta = 1$ in the status function (the proof of Theorem 3 below is presented for $\beta \leq 1$). The network intervenes twice, shaping local synergies and social comparison. Local synergies are the sum of neighbors' bilateral synergies, and agents compare their effort to the average of their peers (the benchmark model studied in the paper corresponds to assuming $\delta = 0$). The equilibrium

[23]See Calvó-Armengol et al. (2009) for empirical evidence of synergies in school context, or Cornelissen et al. (2017) at the workplace.

conditions are as follows (see the proof of Theorem 3 thereafter for more details):

$$[(\mathbf{M}-\mathbf{I})\mathbf{e}]_i \leq \mathbf{k}(\mathbf{G},\delta)$$

where vector $k(\mathbf{G},\delta) = (\kappa_i(\mathbf{G},\delta))_{i\in\mathcal{N}}$ is such that

$$\kappa_i(\mathbf{G},\delta) = \frac{\delta d_i(\gamma_H-\gamma_L)+2(a+\gamma_L)}{2(\gamma_H-\gamma_L)(1-\delta d_i)} - \frac{a+\gamma_L}{\gamma_H-\gamma_L} b_i \qquad (9)$$

Shortly speaking, the first-order conditions indicate that an agent is motivated when her connection to other motivated agents is sufficiently low, where 'connection' is no longer restricted to direct neighbors, but is extended to account for paths of any length with decay. One interest with that specification is that, like the no-synergy case, under symmetry of matrix $\mathbf{G}$, the game still admits a potential function:[24]

$$P(\mathbf{e}) = \mathbf{k}(\mathbf{G},\delta)^T\mathbf{e} - \frac{1}{2}\mathbf{e}^T(\mathbf{M}-\mathbf{I})\mathbf{e}$$

This guarantees equilibrium existence. The general picture is then as follows. For $\gamma_L = \gamma_H$ and $\delta > 0$, the model is the game of local synergies. There is a single equilibrium given by standard Bonacich centralities. For $\delta = 0$, we get the anti-coordination game, exhibiting multiple equilibria. In-between, i.e. for $\gamma_L < \gamma_H$ and $\delta > 0$, the game incorporates both local synergies and anti-coordination. Uniqueness should therefore be confined to a region of the parameter space such that the anti-coordination effect is dominated by the local synergy effect. Let matrix $\mathbf{M} = (\mathbf{I}-\delta\mathbf{G})^{-1}$, vector $\mathbf{b} = \mathbf{M}\mathbf{1}$. We obtain:

**Theorem 3** *The game with status effect including demotivation and local synergies in sum is a potential game. Let* $\mathbf{e}^*$ *be an equilibrium. Then*

[24] In opposite, there is no potential with linear-in-means synergies, because in that case the interaction stays asymmetric even if $\mathbf{G}$ is symmetric.

- *if* $d_i \geq \frac{1}{\delta}$, $e_i^* = 1$.

- *if* $d_i < \frac{1}{\delta}$, $e_i^* = 1$ *if and only if*

$$[(\mathbf{M} - \mathbf{I})\mathbf{e}^*]_i \leq \kappa_i(\mathbf{G}, \delta) \tag{10}$$

*with* $\kappa_i$ *defined as in (9). The equilibrium effort profile is given by*

$$\mathbf{x}^* = (a + \gamma_L)\mathbf{b} + (\gamma_H - \gamma_L)\mathbf{M}\mathbf{e}^*$$

Conform to intuition, more demotivation, through lower $\gamma_L$, leads to more anti-coordination and thus favors multiplicity. However, the role of synergies is ambiguous with respect to demotivation because increased effort can induce mixed effect on social status. There are at least two consequences. First, more synergies can increase the number of equilibria. Second, more synergies can reduce the number of motivated agents (simple examples illustrate both claims).

The case of extreme synergies calls for interest. Recall that the intensity of synergies is not larger than $\bar{\delta} = \frac{1}{\mu(\mathbf{G})}$:

**Corollary 2** *When* $\delta$ *is sufficiently close to* $\bar{\delta}$*, there is a single equilibrium. In that equilibrium, agent* $i$ *is demotivated if and only if* $d_i \leq \mu(\boldsymbol{G})$.

Here the synergies strongly dominate anti-coordination, so that the uniqueness property that holds under synergies and no anti-coordination extends straightforwardly.

# 7 Conclusion

This paper analyzes demotivation stemming from unfavorable social comparison with local peers in networks. We modeled effort decisions with a status-dependent utility function, and introduced demotivation through a lower return to effort when effort below a fixed percentage of neighbors' effort. The introduction of demotivation entails a binary potential game of strategic substitutes played on networks, generating multiple Nash equilibria in general. Our findings also highlight that demotivated agents can become locked into low-effort outcomes due to their placement in the network, and that demotivation can have a positive impact on equilibrium welfare, by improving the social status of peers. These insights provide a theoretical foundation for understanding better how network structures influence demotivation and performance in social and economic settings.

This theoretical work opens up several avenues for future research. First, the model might generate testable predictions linking network position, relative performance, and individual effort levels — in particular, it predicts that the proportion of demotivated neighbors is lower for demotivated agents than for motivated agents. Second, social comparison —and its link to demotivation— likely plays a key role in shaping social networks. Introducing endogenous network formation in a setting where effort incentives depend on relative comparisons could shed light on how individuals strategically choose their social ties in order to avoid being demotivated.[25] Third, given empirical evidence in behavioral economics that incentives are strongest when individuals perform close to their peers, it would be worthwhile to consider a social-comparison model that incorporates both loss aversion— which raises effort for agents who are just behind their peers— and demotivation, which discourages effort when peers are far ahead.

[25] See Akerlof (2017) for a model with two agents linking socialization, value formation, and status concerns.

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# A Appendix A: Proofs

[Proof of Proposition 1] To define the best-response to others' play, consider the restriction of utilities in each domain (no-demotivation and demotivation).

For convenience, write $x_i^{\gamma_H}$ (resp. $x_i^{\gamma_L}$) the best-response of agent $i$ under no-demotivation (resp. under demotivation). Without demotivation,

$$x_i^{\gamma_H} = a + \gamma_H$$

which holds for $x_i^{\gamma_H} \geq \beta \overline{x}_i$. Feasibility then implies $\overline{x}_i \leq \overline{x}^0 = \frac{a+\gamma_H}{\beta}$. With demotivation,

$$x_i^{\gamma_L} = a + \gamma_L$$

which holds for $x_i^{\gamma_L} \leq \beta \overline{x}_i$. Feasibility then implies $\overline{x}_i \geq \overline{x}^1 = \frac{a+\gamma_L}{\beta}$. Clearly $\overline{x}^1 < \overline{x}^0$ since $\gamma_L < \gamma_H$. We then need to compare utilities for $\overline{x}_i \in [\overline{x}^1, \overline{x}^0]$. We find $U_i(x_i^{\gamma_H}) \geq U_i(x_i^{\gamma_L})$ whenever

$$\frac{1}{2}(a_i + \gamma_H)^2 - \gamma_H \overline{x}_i \geq \frac{1}{2}(a_i + \gamma_L)^2 - ((1-\beta)\gamma_H + \beta\gamma_L)\overline{x}_i - \psi$$

That is:

$$\overline{x}_i \leq \varphi = \frac{1}{\beta}\left(a + \frac{\gamma_H + \gamma_L}{2} + \frac{\psi}{\gamma_H - \gamma_L}\right)$$

[Proof of Theorem 1] We start with the characterization, and then we turn to existence.

*Characterization.* Let $\mathbf{e} \in \{0,1\}^n$ be a binary vector such that $e_i = 1$ if agent $i$'s plays is not in the demotivation region, and $e_i = 0$ if agent $i$'s plays is in the demotivation region. Inverting the system of best-responses, an equilibrium can be written

$$\mathbf{x}^* = (a + \gamma_L)\mathbf{1} + (\gamma_H - \gamma_L)\mathbf{e}^*$$

Noticing that $\tilde{\mathbf{G}}\mathbf{1} = \mathbf{1}$, we get

$$\overline{\mathbf{x}}^* = (a + \gamma_L)\mathbf{1} + (\gamma_H - \gamma_L)\tilde{\mathbf{G}}\mathbf{e}^*$$

Then agent $i$ plays in the motivated region whenever $\overline{x}_i^* \leq \varphi$, i.e.,

$$a + \gamma_L + (\gamma_H - \gamma_L)\rho_i^1(\mathbf{e}) \leq \frac{1}{\beta}\left(a + \frac{\gamma_H + \gamma_L}{2} + \frac{\psi}{\gamma_H - \gamma_L}\right)$$

That is,

$$\rho_i^1(\mathbf{e}) \leq \kappa = \frac{2\beta a + \gamma_H + (1-2\beta)\gamma_L}{2\beta(\gamma_H - \gamma_L)} + \frac{\psi}{\beta(\gamma_H - \gamma_L)^2}$$

*Existence.* Let $d_i^1 = [\mathbf{GE}]_i$ represent the number of agent $i$'s neighbors being in state $1$. We have

$$e_i^* = 1 \Rightarrow d_i^1(\mathbf{e}) \leq \kappa d_i$$
$$e_i^* = 0 \Rightarrow d_i^1(\mathbf{e}) \geq \kappa d_i$$

Consider the function

$$P(\mathbf{e}) = \kappa \mathbf{1}^T\mathbf{G}\mathbf{e} - \frac{1}{2}\mathbf{e}^T\mathbf{G}\mathbf{e}$$

with binary matrix $\mathbf{G}$ representing the network. This function is a potential of the demotivation game when $\mathbf{G}^T = \mathbf{G}$. Indeed, the impact of a switch to motivation is as follows. Let $\mathbf{e}$ and $i : e_i = 0$, and let $\mathbf{1}_i = (0, 0, \cdots, 1, 0, \cdots, 0)^T$ the vector of zeros but a one at entry $i$. Since $\mathbf{G}^T = \mathbf{G}$,

$$P(\mathbf{e} + \mathbf{1}_i) - P(\mathbf{e}) = \kappa d_i - d_i^1(\mathbf{e})$$

Hence, function $P$ increases with the switch whenever $\frac{d_i^1(\mathbf{e})}{d_i} \leq \kappa$, which is equivalent to playing $e_i = 1$ as a best-response. That potential has a maximum (the strategy space is finite). Hence any maximum of the potential over the strategy space is a Nash equilibrium.

[Proof of Result 1] In the no-status scenario ($\gamma_L = \gamma_H = 0$), there is a single equilibrium in which every agent's effort is $a$, and equilibrium utility is $\frac{a^2}{2}$.

A demotivated agent $i$ is worse off:

$$u_i^* = \frac{(a + \gamma_L)^2}{2} - \gamma_l \overline{x}_i^* \leq \frac{(a + \gamma_L)^2}{2} - \gamma_l(a + \gamma_L) = \frac{a^2 - \gamma_L^2}{2} < \frac{a^2}{2}$$

Consider now a motivated agent $i$, whose equilibrium utility can be written

$$u_i^* = \frac{(a + \gamma_H)^2}{2} - \gamma_H\big(a + \gamma_L + (\gamma_H - \gamma_L)\rho_i^1(\mathbf{e}^*)\big)$$

Then, $u_i^* \geq \frac{a^2}{2}$ whenever

$$\rho_i^1(\mathbf{e}^*) \leq \frac{1}{2} - \frac{\gamma_L}{2(\gamma_H - \gamma_L)}$$

[Proof of Proposition 4] At equilibrium $\mathbf{e}^*$, the first-order conditions defining effort are given by $x_i^* = a + \gamma_L + (\gamma_H - \gamma_L)e_i^*$ for agent $i$. We then get

$$W(\mathbf{e}^*) = \frac{1}{2}\sum_{i \in N}(x_i^*)^2 - \sum_{i \in N}\big(\gamma_L + (\gamma_H - \gamma_L)e_i^*\big)\overline{x}_i^*$$

That is, plugging effort into equilibrium welfare,

$$W(\mathbf{e}^*) = \frac{1}{2}\sum_i(a+\gamma_L+(\gamma_H-\gamma_L)e_i^*)^2 - \sum_i(\gamma_L+(\gamma_H-\gamma_L)e_i^*)(a+\gamma_L+(\gamma_H-\gamma_L)\rho_i^1(\mathbf{e}^*))$$

That is,

$$W(\mathbf{e}^*) = \frac{na^2}{2} - \frac{n}{2}\gamma_L^2 - (\gamma_H - \gamma_L)\Big(\sum_i \rho_i^1(\mathbf{e}^*)\Big)\gamma_L + \frac{(\gamma_H-\gamma_L)^2}{2}\Big(\sum_i e_i^*(1-2\rho_i^1(\mathbf{e}^*))\Big)$$

That is, denoting $\phi^* = \sum_i e_i^*(1-2\rho_i^1(\mathbf{e}^*)) \geq 0$ and $\phi^{**} = \sum_i \rho_i^1(\mathbf{e}^*)$ for convenience,

$$W(\mathbf{e}^*) = \frac{na^2}{2} + P_2(\gamma_L)$$

with

$$P_2(\gamma_L) = \underbrace{\left(\frac{\phi^* - n + 2\phi^{**}}{2}\right)}_{a_2}\gamma_L^2 + \underbrace{\gamma_H\Big(-\phi^* - \phi^{**}\Big)}_{a_1}\gamma_L + \underbrace{\frac{1}{2}\gamma_H^2\phi^*}_{a_0}$$

Clearly $a_0 > 0, a_1 < 0$. Also, $a_2 \geq 0$: indeed

$$a_2 = \sum_i (1-e_i^*)(2\rho_i^1(\mathbf{e}^*) - 1)$$

and, for all $e_i^* = 0$, and all $\beta \leq 1$, $\rho_i^1(\mathbf{e}^*) \geq \frac{1}{2}$. Then two cases can arise.

Case 1: $a_2 = 0$; Then $\frac{\partial W(\mathbf{e}^*)}{\partial \gamma_L} < 0$, and $W(\mathbf{e}^*) \geq \frac{na^2}{2}$ whenever

$$\gamma_L \leq \gamma_H \cdot \frac{\phi^*}{2(\phi^* + \phi^{**})}$$

Case 2: $a_2 > 0$; Then $P_2(\gamma_L)$ has two positive roots $\gamma_l', \gamma_L''$. Since $P_2(\gamma_H) < 0 < P_2(0)$, it follows that $0 < \gamma_L' < \gamma_H < \gamma_L''$. Therefore, $W(\mathbf{e}^*) \geq \frac{na^2}{2}$ whenever

$$\gamma_L \leq \gamma_H \cdot \frac{\phi^* + \phi^{**} - \sqrt{(\phi^{**})^2 + n\phi^*}}{\phi^* - n + 2\phi^{**}}$$

[Proof of Proposition 5] Assume $0 \leq \gamma_L < \gamma_H$ and consider an equilibrium $\mathbf{e}^*$. For a motivated agent $i$, equilibrium effort is $x_i^* = a + \gamma_H$, and equilibrium utility $u_i^* = \frac{(a+\gamma_H)^2}{2} - \gamma_H \overline{x}_i^*$. For a demotivated agent $i$, equilibrium effort is $x_i^* = a + \gamma_L$, and equilibrium utility $u_i^* = \frac{(a+\gamma_L)^2}{2} - \gamma_L \overline{x}_i^*$.

Then, for a motivated agent $i$: $\frac{\partial u_i^*}{\partial \gamma_L} = -\gamma_H(1-\rho_i^1(\mathbf{e}^*))$. For a demotivated agent $i$, $\frac{\partial u_i^*}{\partial \gamma_L} = -(\gamma_H - \gamma_L)\rho_i^1(\mathbf{e}^*) - \gamma_L(1-\rho_i^1(\mathbf{e}^*))$. The proposition follows directly.

[Proof of Result 2] Assume $0 \leq \gamma_L < \gamma_H$ and consider an equilibrium $\mathbf{e}^*$. For a motivated agent $i$, equilibrium effort is $x_i^* = a + \gamma_H$, and equilibrium utility $u_i^* = \frac{(a+\gamma_H)^2}{2} - \gamma_H \overline{x}_i^*$. For a demotivated agent $i$, equilibrium effort is $x_i^* = a+\gamma_L$, and equilibrium utility $u_i^* = \frac{(a+\gamma_L)^2}{2} - \gamma_L \overline{x}_i^*$.

Then, for a motivated agent $i$: $\frac{\partial u_i^*}{\partial \gamma_H} = (\gamma_H - \gamma_L)(1-\rho_i^1(\mathbf{e}^*)) - \gamma_H \rho_i^1(\mathbf{e}^*)$. For a demotivated agent $i$, $\frac{\partial u_i^*}{\partial \gamma_H} = -\gamma_L \rho_i^1(\mathbf{e}^*)$. The proposition follows directly.

[Proof of Proposition 6] Deriving (8) with respect to $\gamma_H$, we find

$$\frac{\partial W(\mathbf{e}^*;\mathbf{G})}{\partial \gamma_H} = (\gamma_H - \gamma_L)e^* - \gamma_L \sum_i e_i^* h_i^0(\mathbf{e}^*) - (2\gamma_H - \gamma_L)\sum_i e_i^* h_i^1(\mathbf{e}^*)$$

Or equivalently, recalling $h_i^0(\mathbf{e}^*) + h_i^1(\mathbf{e}^*) = h_i(\mathbf{e}^*)$,

$$\frac{\partial W(\mathbf{e}^*;\mathbf{G})}{\partial \gamma_H} = (\gamma_H - \gamma_L)\sum_i e_i^*(1 - 2h_i^1(\mathbf{e}^*)) - \gamma_L \sum_i e_i^* h_i(\mathbf{e}^*)$$

meaning

$$\frac{\partial W(\mathbf{e}^*;\mathbf{G})}{\partial \gamma_H} > 0$$

if and only if

$$(\gamma_H - \gamma_L)\sum_i e_i^*(1 - 2h_i^1(\mathbf{e}^*)) > \gamma_L \sum_i e_i^* h_i(\mathbf{e}^*)$$

I.e., noticing that $\sum_i e_i^* h_i^1(\mathbf{e}^*) = \sum_i e_i^* \rho_i^1(\mathbf{e}^*)$ and $\sum_i e_i^* h_i(\mathbf{e}^*) = \sum_i \rho_i^1(\mathbf{e}^*)$,

$$\sum_i e_i^*(1 - 2\rho_i^1(\mathbf{e}^*)) > \frac{\gamma_L}{\gamma_H - \gamma_L}\sum_i \rho_i^1(\mathbf{e}^*)$$

[Proof of Proposition 7] The proposition is shown for any $\beta \leq 1$. By the property that two equilibria cannot be nested, there is no two distinct equilibria being nested. That none of the two equilibria can Pareto-dominate the other is then a direct implication of the following lemma:

**Lemma 1** *Assume $a_i = a$ for all $i$. For any two distinct equilibria $\boldsymbol{e}^*, \boldsymbol{e}^{'*}$, and any pair of agents $(i, j)$ such that $e_i^* = 1, e_j^{'*} = 0$, $u_i(\boldsymbol{e}^*) > u_j(\boldsymbol{e}^{'*})$.*

*Proof of Lemma 1.* Consider any equilibrium with agent $i$ being motivated, and any equilibrium with agent $j$ being demotivated. The respective equilibrium utilities are written after few computation as

$$\begin{cases} u_i(\mathbf{x}^*) = \frac{(a+\gamma_H)^2}{2} - \gamma_H \overline{x}_i \\ u_j(\mathbf{x}^{'*}) = \frac{(a+\gamma_L)^2}{2} - ((1-\beta)\gamma_H + \beta\gamma_L)\overline{x}_j \end{cases}$$

Hence, $u_i(\mathbf{e}^*) > u_j(\mathbf{e}^{'*})$ whenever

$$(\gamma_H - \gamma_L)\Big(a + \frac{\gamma_H + \gamma_L}{2}\Big) > \gamma_H \overline{x}_i - ((1-\beta)\gamma_H + \beta\gamma_L)\overline{x}_j \tag{11}$$

By the property of equilibrium, we have

$$\overline{x}_i \leq \varphi < \overline{x}_j$$

A sufficient condition to get (11) is when $\overline{x}_i = \overline{x}_j = \varphi$ (since these conditions minimize the RHS of the inequality). This means

$$(\gamma_H - \gamma_L)\Big(a + \frac{\gamma_H + \gamma_L}{2}\Big) > \beta(\gamma_H - \gamma_L)\varphi$$

i.e.,

$$\varphi < \frac{a + \frac{\gamma_H + \gamma_L}{2}}{\beta} = \varphi$$

a contradiction.

[Proof of Proposition 8] The first sentence follows from direct comparison between equation (8) and equation (4). The proof of the second sentence is a direct consequence of Proposition 5.

[Proof of Theorem 3] This proof is presented for $\beta \leq 1$, including the case $\beta = 1$. This game admits the following best-responses:

$$\begin{cases} x_i^{BR}(\overline{x}_i) = a + \gamma_H + \delta d_i \overline{x}_i \text{ if } x_i \geq \beta \overline{x}_i \\ x_i^{BR}(\overline{x}_i) = a + \gamma_L + \delta d_i \overline{x}_i \text{ if } x_i \leq \beta \overline{x}_i \end{cases}$$

Suppose first that $\beta \leq \delta d_i$. Then $a + \gamma_H + \delta d_i \overline{x}_i \geq (1 - \beta)\overline{x}_i$ whenever

$$-(a + \gamma_H) \leq (\delta d_i - 1 + \beta)\overline{x}_i$$

which is true. Hence, agent $i$'s best-response does not contain the demotivation region.

Second, Suppose that $1 - \beta > \delta d_i$. Then $a + \gamma_H + \delta d_i \overline{x}_i \geq \beta \overline{x}_i$ whenever $\overline{x}_i \leq y^1 = \frac{a+\gamma_H}{\beta - \delta d_i}$. Similarly, $a + \gamma_L + \delta d_i \overline{x}_i \leq \beta \overline{x}_i$ whenever $\overline{x}_i \geq y^0 = \frac{a+\gamma_L}{\beta - \delta d_i}$. Since $y^0 < y^1$, we have to study the best-play in the interval $\overline{x}_i \in (y^0, y^1)$. Let us define

$$\varphi_i = \frac{a + \frac{\gamma_H + \gamma_L}{2}}{\beta - \delta d_i} \tag{12}$$

where the value $\varphi_i$ stems from equating utilities at best-responses in both regions of non-demotivation and demotivation. Actually: $U_i(x_i^{\gamma_H}) \geq U_i(x_i^{\gamma_L})$ whenever

$$\frac{1}{2}(a + \gamma_H + \delta d_i \overline{x}_i)^2 - \gamma_H \overline{x}_i \geq \frac{1}{2}(a + \gamma_L + \delta d_i \overline{x}_i)^2 - ((1 - \beta)\gamma_H + \beta \gamma_L)\overline{x}_i$$

That is, $U_i(x_i^{\gamma_H}) > U_i(x_i^{\gamma_L})$ if and only if $\overline{x}_i < \varphi_i$.

Agent $i$'s best-response is then written as

$$x_i^{BR}(\overline{x}_i) = a + \gamma_L + (\gamma_H - \gamma_L)e_i + \delta d_i \overline{x}_i$$

with $e_i = 1$ if and only if $\overline{x}_i \leq \varphi_i$. Denoting $\mathbf{M} = (\mathbf{I} - \delta \mathbf{G})^{-1}$, we have

$$\mathbf{x} = (a + \gamma_L)\mathbf{M1} + (\gamma_H - \gamma_L)\mathbf{Me}$$

from which we deduce, denoting $\tilde{\mathbf{G}} = (\frac{g_{ij}}{d_i})$,

$$\overline{\mathbf{x}} = (a + \gamma_L)\tilde{\mathbf{G}}\mathbf{M}\mathbf{1} + (\gamma_H - \gamma_L)\tilde{\mathbf{G}}\mathbf{M}\mathbf{e}$$

That is, for agent $i$,

$$\overline{x}_i = (a + \gamma_L)\sum_j \frac{g_{ij}b_j}{d_i} + (\gamma_H - \gamma_L)\sum_j \frac{g_{ij}b_{\mathbf{e},j}}{d_i}$$

Now, $e_i = 1 \Leftrightarrow \overline{x}_i \leq \varphi_i$, that is, multiplying by $\delta d_i$ both RHS and LHS,

$$(a + \gamma_L)\delta \sum_j g_{ij}b_j + (\gamma_H - \gamma_L)\delta \sum_j g_{ij}b_{\mathbf{e},j} \leq \delta d_i \varphi_i$$

Recalling that $\delta\mathbf{G}\mathbf{M} = \mathbf{M} - \mathbf{I}$, and thus $\delta\mathbf{G}\mathbf{M}\mathbf{1} = \mathbf{b} - \mathbf{1}$ and $\delta\mathbf{G}\mathbf{M}\mathbf{e} = (\mathbf{M} - \mathbf{I})\mathbf{e}$, we find $e_i = 1$ if and only if

$$(a + \gamma_L)(b_i - 1) + (\gamma_H - \gamma_L)[(\mathbf{M} - \mathbf{I})\mathbf{e}]_i \leq \delta d_i \varphi_i$$

or $e_i = 1$ if and only if

$$[(\mathbf{M} - \mathbf{I})\mathbf{e}]_i \leq \kappa_i \tag{13}$$

with

$$\kappa_i = \frac{\delta d_i \varphi_i - (a + \gamma_L)(b_i - 1)}{\gamma_H - \gamma_L}$$

Plugging $\varphi_i$ from (12), we get

$$\kappa_i = \frac{\delta d_i(\gamma_H - \gamma_L) + 2(a + \gamma_L)\beta}{2(\gamma_H - \gamma_L)(\beta - \delta d_i)} - \frac{a + \gamma_L}{\gamma_H - \gamma_L} b_i \tag{14}$$

Thus, $\kappa_i$ is increasing in $d_i$, decreasing in $b_i$.

Denote $\mathbf{k} = (\kappa_i)_i$. Exploiting (13), the potential function is then:

$$P(\mathbf{e}) = \mathbf{k}^T\mathbf{e} - \frac{1}{2}\mathbf{e}^T(\mathbf{M} - \mathbf{I})\mathbf{e}$$

which guarantees existence.

# B Appendix B: Extensions

We present two extensions, introducing heterogeneous private returns and more general utility functions. The formulations preserve the strategic substitute nature of incentives, and a potential function exists in these extensions.

*Heterogeneous private returns.* We assume now that utilities are as follows:

$$u_i(x_i, x_{-i}) = a_i x_i - \frac{1}{2}x_i^2 + S(x_i - \overline{x}_i)$$

with $a_i > 0$ the private return of agent $i$. In that extended setting, best-responses are given as follows. Denote by $\overline{a}_i = \frac{1}{d_i}\sum_j g_{ij}a_j$ agent $i$'s average neighbors' private returns. Let $\varphi_i = \frac{a_i + \frac{\gamma_H+\gamma_L}{2}}{\beta}$. Agent $i$'s best-response to $\overline{x}_i$ is given by:

$$\begin{cases} x_i^{BR}(\overline{x}_i) = a_i + \gamma_H & \text{if} \quad \overline{x}_i \leq \varphi_i \\ x_i^{BR}(\overline{x}_i) = a_i + \gamma_L & \text{if} \quad \overline{x}_i > \varphi_i \end{cases}$$

Then, we define

$$\kappa_i = \frac{2(a_i - \beta\overline{a}_i) + \gamma_H + (1-2\beta)\gamma_L}{2\beta(\gamma_H - \gamma_L)}$$

Parameter $\kappa_i$ is increasing in agent $i$'s private return $a_i$, meaning that agents with higher private returns are less likely to become demotivated.

**Proposition 10** *Under heterogeneous private returns, agents play a potential game of anti-coordination, hence there is always an equilibrium. A configuration* $\mathbf{e}^*$ *is a Nash equilibrium if and only if*

$$e_i = 1 \quad \Rightarrow \quad \rho_i^1(\mathbf{e}) \leq \kappa_i$$

$$e_i = 0 \quad \Rightarrow \quad \rho_i^0(\mathbf{e}) \leq 1 - \kappa_i$$

When $\mathbf{G}^T = \mathbf{G}$, the game with $\beta \leq 1$ admits a potential function

$$P(\mathbf{e}) = \sum_{i \in \mathcal{N}} \kappa_i d_i e_i - \frac{1}{2}\mathbf{e}^T\mathbf{G}\mathbf{e}$$

which ensures equilibrium existence.

*Generalizing on utility function.* The model generates strategic substitutes under more general utility functions. We consider agent $i$'s utility:

$$u_i(x_i, \overline{x}_i) = v_i(x_i) + S(x_i - \overline{x}_i)$$

where function $v_i$ is a concave (and single-peaked function), with $v_i(0) = 0$. For simplicity, we assume $\beta = 1$ in the status function. Agent $i$'s best-response is thus

$$\begin{cases} x_i^{\gamma_H}(\overline{x}_i) = v_i'^{-1}(-\gamma_H) & \text{if} \quad x_i - \overline{x}_i \geq 0 \\ x_i^{\gamma_L}(\overline{x}_i) = v_i'^{-1}(-\gamma_L) & \text{if} \quad x_i - \overline{x}_i < 0 \end{cases}$$

The structure of best-responses is the same as the linear quadratic case. That is, agent $i$'s best-response is a step function (replacing $a_i + \gamma_H$ by $x_i^{\gamma_H}$ and $a_i + \gamma_L$ by $x_i^{\gamma_L}$). By concavity of $v_i$, function $v_i'^{-1}$ is increasing, so that $x_i^{\gamma_H} > x_i^{\gamma_L}$. Letting $v_i^{\gamma_L} = v_i(v_i'^{-1}(-\gamma_L)), v_i^{\gamma_H} = v_i(v_i'^{-1}(-\gamma_H))$, the threshold $\varphi_i$ below which agents play demotivated satisfies $u_i(x_i^{\gamma_H}, \overline{x}_i) = u_i(x_i^{\gamma_L}, \overline{x}_i)$, that is,

$$\varphi_i = \frac{v_i^{\gamma_H} - v_i^{\gamma_L} + \gamma_H(x_i^{\gamma_H} - x_i^{\gamma_L})}{\gamma_H - \gamma_L}$$

Let binary profile $\mathbf{e}$ describe the status of agents. At equilibrium $\mathbf{e}^*$,

$$x_i^* = v_i^{\gamma_H} + (v_i^{\gamma_H} - v_i^{\gamma_L})e_i^*$$

Let $\mathbf{H}$ be such that $h_{ij} = g_{ij}(v_j^{\gamma_H} - v_j^{\gamma_L})$ and $\kappa_i = \varphi_i - \frac{1}{d_i}\sum_j g_{ij} v_j^{\gamma_L}$ for convenience. We obtain $\overline{x}_i \leq \varphi_i$ if and only if

$$\sum_j h_{ij} e_j^* \leq \kappa_i d_i$$

When function $v_i = v$ for all $i$, matrix $\mathbf{H}$ is symmetric. Hence, as with linear quadratic function:

**Proposition 11** *When function $v_i = v$ for all $i$, the game is a potential game with strategic substitutes.*