Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach

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This paper examines the robustness of explanatory variables in cross-country economic growth regressions. It introduces and employs a novel approach, Bayesian Averaging of Classical Estimates (BACE), which constructs estimates by averaging OLS coefficients across models. The weights given to individual regressions have a Bayesian justification similar to the Schwarz model selection criterion. Of 67 explanatory variables we find 18 to be significantly and robustly partially correlated with long-term growth and another three variables to be marginally related. The strongest evidence is for the relative price of investment, primary school enrollment, and the initial level of real GDP per capita. (*JEL* O51, O52, O53)

Following the seminal work of Roger C. Kormendi and Philip Meguire (1985), Kevin B. Grier and Gordon Tullock (1989), and Robert J. Barro (1991), the recent empirical literature on economic growth has identified a substantial number of variables that are partially correlated with the rate of economic growth. The basic methodology consists of running cross-country or panel regressions of the form

$$\gamma = \alpha + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \cdots + \beta_n \cdot x_n + \varepsilon \tag{1}$$

where γ is the vector of rates of economic growth, α is a constant, and $x_1, \ldots, x_n$ are vectors of explanatory variables which vary across researchers and across papers. Each paper typically reports a (possibly nonrandom) sample of the regressions actually run by the researcher. Variables like the initial level of income, the investment rate, various measures of education, some policy indicators, and many other variables have been found to be significantly correlated with growth in regressions like (1).

The problem faced by empirical economists is that growth theories are not explicit enough about what variables x_j belong in the "true" regression. Hence, creative theorizing will generate models that "predict" that things like market distortions, distortionary taxes, maintenance of property rights, degree of monopoly, weather, attitudes toward work, et cetera, should be included in the growth regression. Note that these potential theories are not mutually exclusive (for example, education could very well be an important determinant of long-term growth, and this does not imply that financial development is unimportant).

The multiplicity of possible regressors is one of the major difficulties faced by researchers trying to make sense of the empirical evidence on economic growth. However, the problem is hardly unique to the growth literature: "artistic" economic theory is often capable of suggesting an enormous number of potential explanatory variables in any economic field. In principle,

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this is strictly a small-sample problem since, as the number of observations becomes large, all of the variables which do not belong in the regression will have coefficients that converge to zero. Classical statistics offers us little help since it suggests that we include all of the regressors and "let the data sort through them." In many applications, however, we do not have the luxury of having a large enough sample size to allow us to draw conclusions on the importance of potential regressors. Cross-country regressions provide perhaps the most extreme example: the number of proposed regressors exceeds the number of countries in the world, rendering the all-inclusive regression computationally impossible. Some empirical economists have therefore resorted to simply "trying" combinations of variables which could be potentially important determinants of growth and report the results of their preferred specification. Such "data-mining" could lead to spurious inference. So how should we proceed?

A natural way to think about model uncertainty is to admit that we do not know which model is "true" and, instead, attach probabilities to different possible models. While intuitively appealing, this requires a departure from the classical framework in which conditioning on a model is essential. This approach has recently come to be known as *Bayesian Model Averaging*. The procedure does not differ from the most basic Bayesian reasoning: the idea dates at least to Harold Jeffreys (1961), although fleshed out by Edward E. Leamer (1978). In this paper, we show that this approach can be used in a way that is well grounded in statistical theory, intuitively appealing, easy to understand, and easy to implement.

The fully Bayesian approach is entirely feasible and has been applied to various problems by a number of authors. Examples include Jeremy C. York et al. (1995) and Adrian E. Raftery et al. (1997); a summary of much of the recent work can be found in Jennifer A. Hoeting et al. (1999). In the growth context, Carmen Fernandez et al. (2001) apply techniques from the Bayesian statistics literature to the data set of Sala-i-Martin (1997a). A pure Bayesian approach requires specification of the prior distributions of all of the relevant parameters *conditional on each possible model.* Under ideal conditions, elicitation of prior parameters is difficult and is indeed one of the major reasons for Bayesian approaches remaining relatively unpopular. When the number of possible regressors is K, the number of possible linear models is 2^K, so with K large, fully specifying priors is infeasible. Thus, authors implementing the fully Bayesian approach have used priors which are essentially arbitrary. This makes the ultimate estimates dependent on arbitrarily chosen prior parameters in a manner which is extremely difficult to interpret. In existing applications of this approach, the impact of these prior parameters has been neither examined nor explained.

To address the issue of fragility of econometric inference with respect to modeling choices, Leamer (1983, 1985) proposes a sensitivity analysis of the results with respect to changes in the prior distribution of parameters. In particular, Leamer proposes an *extreme bounds analysis* to identify "robust" empirical relations. For a variable of interest, z, the *extreme bounds* of the distribution of the associated coefficient estimate β_z are calculated as the smallest and largest value that the coefficient can take on when combinations of additional regressors x_j enter the regression model (1). When the two extreme bounds are of opposite sign, then variable z is labeled "fragile." Ross E. Levine and David Renelt (1992) employ a version of this test to cross-country data and find that, according to extreme bounds analysis, very few (or no) variables are robustly related with growth. Another interpretation, however, is that the test is too strong for any variable to pass: any one regression model (no matter how well or poorly fitting) carries a veto. This problem is well recognized and solutions have been proposed to restrict attention to better fitting models, such as the *reasonable extreme bounds* proposed by Clive W. J. Granger and Harald F. Uhlig (1990) or to introduce a lower bound on the prior variance matrix as suggested by Leamer (1985).

In this paper we use the Bayesian approach to averaging across models, while following the classical spirit of most of the empirical growth literature.[1] We propose a model-averaging tech-

[1] We build on the work by Sala-i-Martin (1997a, b) who proposed to average estimates of mean and standard deviations for variables across regressions, using weights proportional to the likelihoods of each of the models. Sala-i-Martin calculates a likelihood-weighted sum of normal

nique which we call *Bayesian Averaging of Classical Estimates* or *BACE,* to determine the significance of variables in cross-country growth regressions. We show that the weighting method can be derived as a limiting case of a standard Bayesian analysis as the prior information becomes "dominated" by the data. BACE combines the averaging of estimates across models, which is a Bayesian concept, with classical ordinary least-squares (OLS) estimation which comes from the assumption of diffuse priors. This name is chosen to highlight the fact that while averaging across models is an inherently Bayesian idea, BACE limits the effect of prior information and uses an approach otherwise familiar to classical econometricians.

Our BACE approach has several important advantages over previously used model-averaging and robustness-checking methods: firstly, in contrast to a standard Bayesian approach that requires the specification of a prior distribution for all parameters, BACE requires the specification of only one prior hyper-parameter: the expected model size $\bar{k}$. This parameter is easy to interpret, easy to specify, and easy to check for robustness.[2] Secondly, the interpretation of the estimates is straightforward for economists not trained in Bayesian inference: the weights applied to different models are proportional to the logarithm of the likelihood function corrected for degrees of freedom (analogous to the Schwarz model selection criterion). Thirdly, our estimates can be calculated using only repeated applications of OLS. Fourthly, in contrast to Levine and Renelt and Sala-i-Martin, we consider models of all sizes and no variables are held "fixed" and therefore "untested." Fifth, we calculate the entire distribution of coefficients across models and do not focus solely on the bounds of the distribution.

When we examine the cross-country data usually used by growth empiricists using BACE, we find striking and surprisingly clear conclusions. The data identify a set of 18 variables that are significantly related to economic growth. They have a great deal of explanatory power and are very precisely estimated. Another three variables are marginal: they would be reasonable regressors if a researcher had a strong prior belief in their relevance. The remaining 46 variables have weak explanatory power and are imprecisely estimated.

The rest of the paper is organized as follows. Section I outlines the statistical theory, Section II describes the data set used, and Section III presents the main empirical results of the paper. The final section concludes.

I. Statistical Theory

A. *Statistical Basics*

In classical statistics a parameter has a true, though unknown, value, so it cannot have a density because it is not random. In the Bayesian framework parameters are considered to be uncertain. Following is a quick exposition of the basic reasoning and the language needed for understanding our approach. A more detailed presentation of these ideas can be found in Dale J. Poirier (1995). We begin with Bayes' rule in densities:

$$g(\beta|y) = \frac{f(y|\beta)g(\beta)}{f(y)}. \tag{2}$$

This is true for any random variables y and β. In equation (2), $g(\beta)$ is the *prior density* of a parameter vector β, interpreted as the researcher's information about β *prior* to seeing the data. The likelihood function $f(\beta|y)$ summarizes the information about β contained in the data. The vector y is the observed data with prior density $f(y)$, reflecting our prior opinions about the data. $g(\beta|y)$, is the density of β conditional on the data and is called the *posterior density.*

"Model averaging" is a special case of Bayes' rule. Suppose we divide the parameter space into two regions and label them M_0 and M_1. These regions could be what we would usually call hypotheses (e.g., $\beta > 0$ versus $\beta \leq 0$) or something we would usually call models (e.g.,

cumulative distribution functions as the measure of significance of a variable and finds that a number of variables are significantly correlated with growth, therefore contesting the pessimistic conclusions reached by Levine and Renelt (1992).

[2] In the standard Bayesian sense, we can calculate estimates for a range of different values of $\bar{k}$. Thus we can make statements of the form, "whether you think a good model size is three regressors or 12 regressors, this one particular variable is important."

$\beta_1 = 0, \beta_2 \neq 0$ versus $\beta_1 \neq 0, \beta_2 = 0$). Each of these has a prior probability specified by the researcher as $P(M_i)$. These prior probabilities summarize the researcher's beliefs concerning the relative likelihood of the two regions (models). Given the two regions, Bayes' rule implies $g(\beta|y) = P(M_0)[f(y|\beta)g(\beta|M_0)/f(y)] + P(M_1)[f(y|\beta)g(\beta|M_1)/f(y)]$. Rewriting this in terms of the posterior probabilities conditional on the two regions (models) we get:

$$(3) \quad g(\beta|y) = P(M_0|y)\frac{f(y|\beta)g(\beta|M_0)}{f(y|M_0)} + P(M_1|y)\frac{f(y|\beta)g(\beta|M_1)}{f(y|M_1)}$$

where $P(M_i|y)$ is the posterior probability of the ith region, the probability of that region conditional on the data. Equation (3) says that the posterior distribution of the parameters is the weighted average of the two possible conditional posterior densities with the weights given by the posterior probabilities of the two regions. In this paper we will be considering linear regression models for which each model is a list of included variables, with the slope coefficients for all of the other possible regressors set equal to zero.

B. *Diffuse Priors*

As we explained above, fully specifying priors is infeasible when the set of possible regressors is large. In applications of Bayesian theory, if a researcher is incapable or unwilling to specify prior beliefs, a standard remedy is to apply *diffuse priors*. If the parameter space is bounded, then a diffuse prior is a uniform distribution. When the parameter space is unbounded, as in the usual multiple linear regression model, a uniform distribution cannot be directly imposed and instead we must take a limit as the prior distribution becomes flat. In many contexts, imposing diffuse priors generates classical results: in the linear regression model standard diffuse priors and Bayesian regression yields posterior distributions identical to the classical sampling distribution of OLS.

We would like to work with diffuse priors but they create a problem when different regression models contain different sets of variables. As noted above, when the parameter space is unbounded, we must get results for diffuse priors by taking a limit of informative priors. The informative prior must specify prior information concerning both β, the vector of slope coefficients, and σ, the error standard deviation. There are no difficulties taking the limit as our prior information concerning σ becomes uninformative so the equations below all reflect a diffuse prior with respect to σ. Equation (4) below gives the ratio of the posterior probabilities of two regression models (called the posterior odds ratio) with different sets of included variables, X for M_0 and Z for M_1

$$(4) \quad \frac{P(M_0|y)}{P(M_1|y)} = \frac{P(M_0)}{P(M_1)} \times \left(\frac{|A|/|A + X'X|}{|B|/|B + Z'Z|}\right)^{1/2}\left(\frac{SSE_0 + Q_0}{SSE_1 + Q_1}\right)^{-T/2}$$

where $P(M_i)$ is the prior probability of model i as specified by the researcher. This expression assumes that the marginal prior density for β is multivariate normal with variance-covariance matrices given by A^{-1} under M_0, and by B^{-1} under M_1. SSE_i is the OLS sum of squared errors under model i, T is the sample size, and Q_i is a quadratic form in the OLS estimated and prior parameters that need not concern us here. This is a textbook expression (see, for example, Arnold Zellner, 1971). Making the priors diffuse requires taking the limit of (4) as A and B approach zero so that the variance of our prior density goes to infinity. The mathematical difficulty with this is the factor in (4) with the ratio of the determinants of A and B. Both determinants approach zero as the variance goes to infinity, so their ratio depends on the *rate* at which each goes to zero. Depending on precisely how one parameterizes the matrices one gets different answers when evaluating this limit.[3] One limit is the likelihood-weighting method of Sala-i-Martin (1997a). If we specify the prior precision matrices as $A = gX'X$ and

[3] Leamer (1978) provides some intuition for why such problems occur, but argues, in Bayesian spirit, that one should not be interested in diffuse priors.

$B = gZ'Z$ (the g-priors suggested by Zellner, 1986) and take the limit of (4) as g goes to zero, we get:

$$\frac{P(M_0|y)}{P(M_1|y)} = \frac{P(M_0)}{P(M_1)} \left(\frac{SSE_0}{SSE_1}\right)^{-T/2}. \tag{5}$$

The second factor on the right-hand side is equal to the likelihood ratio of the two models. This weighting is troublesome because models with more variables have lower SSE's; the posterior mean model size (average of the different model sizes weighted by their posterior probabilities) will always be bigger than the prior model size, irrespective of the data. Thus it is not sensible to use this approach when considering models of different sizes.

The indeterminacy of the limit in (4) suggests that, for fairly diffuse priors, the exact specification of the prior precision matrix, which will in practice be arbitrary, may generate large differences in the results. There is, however, another limit one can take: the limit of (4) as the information in the data, $X'X$ and $Z'Z$, become large. Instead of taking the limit as the prior becomes flat we are taking the limit as the data becomes very informative relative to the prior information (that is, as the prior becomes "dominated" by the data). If we assume that the variance-covariance matrix of the X's exists and take the limit of (4) as $X'X$ (and $Z'Z$, respectively) goes to infinity we get:[4]

$$\frac{P(M_0|y)}{P(M_1|y)} = \frac{P(M_0)}{P(M_1)} T^{(k_1 - k_0)/2} \left(\frac{SSE_0}{SSE_1}\right)^{-T/2} \tag{6}$$

where k_i is the number of included regressors in model M_i. This provides an approximation to the odds ratios generated by a wide range of reasonably diffuse prior distributions. The degrees-of-freedom correction should be familiar, since it is the ratio of the Schwarz model selection criteria for the two models, exponentiated. The similarity to the Schwarz criterion is not coincidental: Gideon Schwarz (1978) used the same approximation to the odds ratio to justify the criterion. In our empirical work we will use the approximation in equation (6).

In order to get weights for different models we need the posterior probabilities of each model, not the odds ratio. We thus normalize the weight of a given model by the sum of the weights of all possible models, i.e., with K possible regressors:

$$P(M_j|y) = \frac{P(M_j)T^{-k_j/2}SSE_j^{-T/2}}{\sum_{i=1}^{2^K} P(M_i)T^{-k_i/2}SSE_i^{-T/2}}. \tag{7}$$

Once the model weights have been calculated, Bayes' rule says that the posterior density of a parameter is the average of the posterior densities conditional on the models as shown in (3) for two models. A posterior mean is defined to be the expectation of a posterior distribution. Taking expectations with respect to β across (3) (with 2^K terms instead of only two) gives:

$$E(\beta|y) = \sum_{j=1}^{2^K} P(M_j|y)\hat{\beta}_j \tag{8}$$

where $\hat{\beta}_j = E(\beta|y, M_j)$ is the OLS estimate for β with the regressor set that defines model j. In Bayesian terms, $\hat{\beta}_j$ is the posterior mean conditional on model j.[5] Note that any variable excluded from a particular model has a slope coefficient with degenerate posterior distribution at zero. The posterior variance of β is given by:

$$\mathrm{Var}(\beta|y) = \sum_{j=1}^{2^K} P(M_j|y)\mathrm{Var}(\beta|y, M_j) + \sum_{j=1}^{2^K} P(M_j|y)(\hat{\beta}_j - E(\beta|y))^2. \tag{9}$$

Leamer (1978) provides a simple derivation for

[4] See Leamer (1978, p. 112) equation (4.16) for a discussion of the limiting argument leading to this expression. This precise expression arises only if we take the limit using g-priors. For other sorts of priors it is an approximation.

[5] The difficulty with making the prior diffuse applies only to the comparison, or averaging, of *different* models. Conditional on one particular set of included variables the mean of the Bayesian regression posterior is simply the OLS estimate.

(9). Inspection of (9) demonstrates that the posterior variance incorporates both the estimated variances in individual models as well as the variance in estimates of the β's across different models.

We can also estimate the posterior probability that a particular variable is in the regression (i.e., has a nonzero coefficient). We will call this the *posterior inclusion probability* for the variable and it is calculated as the sum of the posterior model probabilities for all of the models including that variable. We will also report the posterior mean and variance conditional on the inclusion of the variable.

C. *Model Size*

We have not yet discussed the specification of the $P(M_i)$'s, the prior probabilities attached to the different models. One common approach to this problem in the statistical literature has been to assign equal prior probability to each possible model. While this is sensible for some applications, for linear regression with a large number of potential regressors it has odd and troubling implications. In particular it implies a very strong prior belief that the number of included variables should be large. It also implies that the expected model size, which is equal to $K/2$, increases with the number of explanatory variables available to researchers. We will instead specify our model prior probabilities by choosing a prior mean model size, $\bar{k}$, with each variable having a prior probability $\bar{k}/K$ of being included, independent of the inclusion of any other variables, where K is the total number of potential regressors.[6] Equal probability for each possible model is the special case in which $\bar{k} = K/2$. In our empirical work we focus on a relatively small $\bar{k}$ on the grounds that most researchers prefer relatively modest parameterizations. We examine the robustness of our conclusions with respect to this hyperparameter in Section III, subsection B.

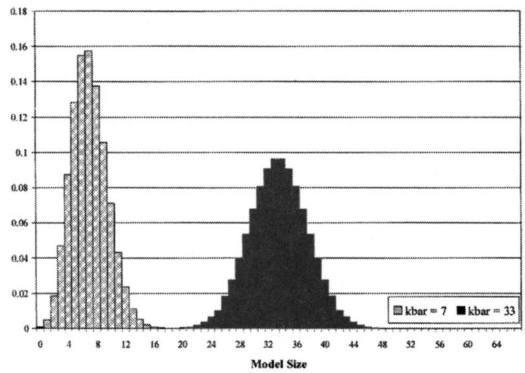

FIGURE 1. PRIOR PROBABILITIES BY MODEL SIZE

Note: Benchmark with prior model size $\bar{k} = 7$ and equal model probability with $\bar{k} = 33$.

In order to illustrate further this issue, Figure 1 plots the prior probability distributions by model size for our baseline model with $\bar{k} = 7$ and with equal probabilities for all models, $\bar{k} = 33$, given the 67 potential regressors we consider in our empirical work. Note that in the latter case, the great majority of the prior probability is focused on models with many included variables: more than 99 percent of the prior probability is located in models with 25 or more included variables. It is our opinion that few researchers actually have such prior beliefs. Thus, while we will calculate results for large models ($\bar{k} = 22$ and 28) below, we do not choose to focus attention on these cases. Alternatively, Figure 2 illustrates the weights that equation (7) gives to models of different sizes: starting with prior model size $\bar{k} = 7$, a researcher would need to observe an adjusted R^2 as shown in the figure to be indifferent between models of different sizes.

D. *Sampling*

Equations (7), (8), and (9) all face the problem that they include sums running over 2^K terms: for many problems for which model averaging is attractive this is an infeasibly large number even though each term only requires the computation of an OLS regression. For our baseline estimation, with $K = 67$, this would mean estimating 1.48×10^{20} regressions, which

[6] In most applications the prior probability of including a particular variable is not, for most researchers, independent of the probability of including any other variable. For example, in a growth regression if a variable proxying political instability is included, such as a count of revolutions, many researchers would think it less likely that another measure, such as the number of assassinations, be included as well. While this sort of interdependence can be readily incorporated into our framework, we do not presently pursue this avenue.

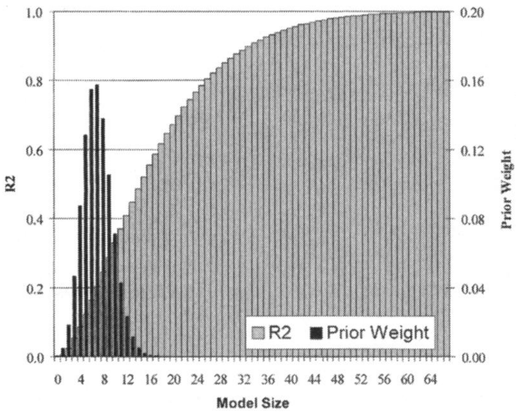

FIGURE 2. PRIOR PROBABILITIES $P(M_j)$ AND CORRESPONDING ADJUSTED R^2 NEEDED TO MAKE A RESEARCHER INDIFFERENT BETWEEN MODELS OF DIFFERENT SIZES k_j

Note: Calculated from equation (7) as $SSE_j = [P(M_j|y)T^{-k_j/2}/P(M_j)]^{-2/T}$ and $R^2 = [(SSE_0 - SSE_j)/SSE_0]$, where $P(M_j)$ are the prior probabilities for the benchmark case ($\bar{k} = 7$) and $P(M_j|y)$ is the flat posterior probability equal to $1/68 = 0.015$.

is computationally not feasible. As a result, only a relatively small subset of the total number of possible regressions can be run.

Several stochastic algorithms have been proposed for dealing with this issue, including the Markov-Chain Monte-Carlo Model Composition (MC^3) technique (David Madigan and Jeremy C. York, 1995), stochastic search variable selection (SSVS) (Edward I. George and Robert E. McCulloch, 1993), and the Gibb's sampler-based method of John F. Geweke (1994). We will take a simpler approach that matches the form of the prior distribution. We select models by randomly including each variable with independent sampling probability $P_s(\beta_i)$. So long as the sampling probabilities are strictly greater than zero and strictly less than one, any values will work in the sense that, as the number of random draws grows, the sampled versions of (7), (8), and (9) will approach their true values. Merlise Clyde et al. (1996) have shown that this procedure, when implemented with $P_s(\beta_i)$ equal to the prior inclusion probability, (called by the authors "random sampling") has computational efficiency not importantly lower than that of the MC^3 and SVSS algorithms (for at least one particular data set). For the present application, we found that sampling models using their prior probabilities produced unacceptably slow convergence. Instead, we sampled one set of regressions using the prior probability sampling weights and then used the approximate posterior inclusion probabilities calculated from those regressions for the subsequent sampling probabilities. This "stratified sampling" accelerates convergence. The Technical Appendix (available on the AER Web site at http://www.aeaweb.org/aer/contents/) discusses computational and convergence issues in detail and may be of interest to researchers looking to apply these techniques.

As an alternative to model averaging, Leamer (1978) suggests orthogonalizing[7] the explanatory variables and estimating the posterior means of the effects of the $K + 1$ principle components. An advantage of this approach is the large reduction in the computational burden, which is especially relevant for prediction. The problem, however, is that with the transformation of the data the economic interpretation of the coefficients associated with the original variables is lost. As we are particularly interested in the interpretation of variables as determinants of economic growth, we do not follow this approach.

II. Data

Of the many variables that have been found to be significantly correlated with growth in the literature, we selected 67 variables using the following criteria. First, we kept the variables that represent *"state variables"* of a dynamic optimization problem. Hence, we choose variables measured as closely as possible to the beginning of the sample period (which is 1960) and eliminate all those variables that were computed for the later years only. This leads to the exclusion of some widely used political variables that were published for the late 1980's (in this category, for example, we neglect the widely used bureaucracy and corruption variables, which were computed for 1985 only). This is partly done to deal with the endogeneity problem.

The second selection criterion derives from our need for a "balanced" data set. By balanced,

[7] An orthogonalization of the explanatory variables would make BACE results invariant with respect to linear transformations of the data (see also the discussion in Leamer, 1985).

TABLE 1—DATA DESCRIPTION AND SOURCES

Rank	Variable	Description and source	Mean	Standard deviation
	Average growth rate of GDP per capita 1960–1996	Growth of GDP per capita at purchasing power parities between 1960 and 1996. From Alan Heston et al. (2001).	0.0182	0.019
1	East Asian dummy	Dummy for East Asian countries.	0.1136	0.3192
2	Primary schooling in 1960	Enrollment rate in primary education in 1960. Barro and Jong-Wha Lee (1993).	0.7261	0.2932
3	Investment price	Average investment price level between 1960 and 1964 on purchasing power parity basis. From Heston et al. (2001).	92.4659	53.6778
4	GDP in 1960 (log)	Logarithm of GDP per capita in 1960. From Heston et al. (2001).	7.3549	0.9011
5	Fraction of tropical area	Proportion of country's land area within geographical tropics. From John L. Gallup et al. (2001).	0.5702	0.4716
6	Population density coastal in 1960's	Coastal (within 100 km of coastline) population per coastal area in 1965. From Gallup et al. (2001).	146.8717	509.8276
7	Malaria prevalence in 1960's	Index of malaria prevalence in 1966. From Gallup et al. (2001).	0.3394	0.4309
8	Life expectancy in 1960	Life expectancy in 1960. Barro and Lee (1993).	53.7159	12.0616
9	Fraction Confucian	Fraction of population Confucian. Barro (1999).	0.0156	0.0793
10	African dummy	Dummy for Sub-Saharan African countries.	0.3068	0.4638
11	Latin American dummy	Dummy for Latin American countries.	0.2273	0.4215
12	Fraction GDP in mining	Fraction of GDP in mining. From Robert E. Hall and Charles I. Jones (1999).	0.0507	0.0769
13	Spanish colony	Dummy variable for former Spanish colonies. Barro (1999).	0.1705	0.3782
14	Years open 1950–1994	Number of years economy has been open between 1950 and 1994. From Jeffrey D. Sachs and Andrew M. Warner (1995).	0.3555	0.3444
15	Fraction Muslim	Fraction of population Muslim in 1960. Barro (1999).	0.1494	0.2962
16	Fraction Buddhist	Fraction of population Buddhist in 1960. Barro (1999).	0.0466	0.1676
17	Ethnolinguistic fractionalization	Average of five different indices of ethnolinguistic fractionalization which is the probability of two random people in a country not speaking the same language. From William Easterly and Ross Levine (1997).	0.3476	0.3016
18	Government consumption share 1960's	Share of expenditures on government consumption to GDP in 1961. Barro and Lee (1993).	0.1161	0.0745
19	Population density 1960	Population per area in 1960. Barro and Lee (1993).	108.0735	201.4449
20	Real exchange rate distortions	Real exchange rate distortions. Levine and Renelt (1992).	125.0341	41.7063
21	Fraction speaking foreign language	Fraction of population speaking foreign language. Hall and Jones (1999).	0.3209	0.4136
22	Openness measure 1965–1974	Ratio of exports plus imports to GDP, averaged over 1965 to 1974. This variable was provided by Robert Barro.	0.5231	0.3359
23	Political rights	Political rights index. From Barro (1999).	3.8225	1.9966
24	Government share of GDP in 1960's	Average share government spending to GDP between 1960–1964. From Heston et al. (2001).	0.1664	0.0712
25	Higher education in 1960	Enrollment rates in higher education. Barro and Lee (1993).	0.0376	0.0501
26	Fraction population in tropics	Proportion of country's population living in geographical tropics. From Gallup et al. (2001).	0.3	0.3731
27	Primary exports 1970	Fraction of primary exports in total exports in 1970. From Sachs and Warner (1997).	0.7199	0.2827
28	Public investment share	Average share of expenditures on public investment as fraction of GDP between 1960 and 1965. Barro and Lee (1993).	0.0522	0.0388
29	Fraction Protestant	Fraction of population Protestant in 1960. Barro (1999).	0.1354	0.2851
30	Fraction Hindu	Fraction of the population Hindu in 1960. Barro (1999).	0.0279	0.1246
31	Fraction population less than 15	Fraction of population younger than 15 years in 1960. Barro and Lee (1993).	0.3925	0.0749
32	Air distance to big cities	Logarithm of minimal distance (in km) from New York, Rotterdam, or Tokyo. From Gallup et al. (2001).	4,324.1705	2,613.7627
33	Nominal government GDP share 1960's	Average share of nominal government spending to nominal GDP between 1960 and 1964. Calculated from Heston et al. (2001).	0.149	0.0584
34	Absolute latitude	Absolute latitude. Barro (1999).	23.2106	16.8426
35	Fraction Catholic	Fraction of population Catholic in 1960. Barro (1999).	0.3283	0.4146
36	Fertility in 1960's	Fertility in 1960's. Barro and Sala-i-Martin (1995).	1.562	0.4193
37	European dummy	Dummy for European economies.	0.2159	0.4138
38	Outward orientation	Measure of outward orientation. Levine and Renelt (1992).	0.3977	0.4922
39	Colony dummy	Dummy for former colony. Barro (1999).	0.75	0.4355
40	Civil liberties	Index of civil liberties index in 1972. Barro (1999).	0.5095	0.3259
41	Revolutions and coups	Number of revolutions and military coups. Barro and Lee (1993).	0.1849	0.2322

TABLE 1—*Continued.*

Rank	Variable	Description and source	Mean	Standard deviation
42	British colony	Dummy for former British colony after 1776. Barro (1999).	0.3182	0.4684
43	Hydrocarbon deposits in 1993	Log of hydrocarbon deposits in 1993. From Gallup et al. (2001).	0.4212	4.3512
44	Fraction population over 65	Fraction of population older than 65 years in 1960. Barro and Lee (1993)	0.0488	0.029
45	Defense spending share	Average share public expenditures on defense as fraction of GDP between 1960 and 1965. Barro and Lee (1993).	0.0259	0.0246
46	Population in 1960	Population in 1960. Barro (1999).	20,308.08	52,538.3866
47	Terms of trade growth in 1960's	Growth of terms of trade in the 1960's. Barro and Lee (1993).	−0.0021	0.0345
48	Public education spending share in GDP in 1960's	Average share public expenditures on education as fraction of GDP between 1960 and 1965. Barro and Lee (1993).	0.0244	0.0096
49	Landlocked country dummy	Dummy for landlocked countries.	0.1705	0.3782
50	Religious intensity	Religion measure. Barro (1999).	0.7803	0.1932
51	Size of economy	Logarithm of aggregate GDP in 1960.	16.1505	1.8202
52	Socialist dummy	Dummy for countries under Socialist rule for considerable time during 1950 to 1995. From Gallup et al. (2001).	0.0682	0.2535
53	English-speaking population	Fraction of population speaking English. From Hall and Jones (1999).	0.084	0.2522
54	Average inflation 1960–1990	Average inflation rate between 1960 and 1990. Levine and Renelt (1992).	13.1298	14.9899
55	Oil-producing country dummy	Dummy for oil-producing country. Barro (1999).	0.0568	0.2328
56	Population growth rate 1960–1990	Average growth rate of population between 1960 and 1990. Barro and Lee (1993).	0.0215	0.0095
57	Timing of independence	Timing of national independence measure: 0 if before 1914; 1 if between 1914 and 1945; 2 if between 1946 and 1989; and 3 if after 1989. From Gallup et al. (2001).	1.0114	0.9767
58	Fraction of land area near navigable water	Proportion of country's land area within 100 km of ocean or ocean-navigable river. From Gallup et al. (2001).	0.4722	0.3802
59	Square of inflation 1960–1990	Square of average inflation rate between 1960 and 1990.	394.5368	1,119.6992
60	Fraction spent in war 1960–1990	Fraction of time spent in war between 1960 and 1990. Barro and Lee (1993).	0.0695	0.1524
61	Land area	Area in km^2. Barro and Lee (1993).	867,188.52	1,814,688.29
62	Tropical climate zone	Fraction tropical climate zone. From Gallup et al. (2001).	0.19	0.2687
63	Terms of trade ranking	Barro (1999).	0.2813	0.1904
64	Capitalism	Degree Capitalism index. From Hall and Jones (1999).	3.4659	1.3809
65	Fraction Orthodox	Fraction of population Orthodox in 1960. Barro (1999).	0.0187	0.0983
66	War participation 1960–1990	Indicator for countries that participated in external war between 1960 and 1990. Barro and Lee (1993).	0.3977	0.4922
67	Interior density	Interior (more than 100 km from coastline) population per interior area in 1965. From Gallup et al. (2001).	43.3709	88.0626

we mean an equal number of observations for all regressions. Since different variables miss observations for different countries, we selected the 67 explanatory variables that maximized the product of the number of countries with observations for all variables and the number of variables.

With these restrictions, the total size of the data set becomes 68 variables (including the dependent variable, the annualized growth rate of GDP per capita between 1960 and 1996) for 88 countries. The variable names, their means, and standard deviations are depicted in Table 1. Appendix Table A1 provides a list of the included countries.

III. Results

We are now ready to conduct our BACE estimation. We calculate the posterior distributions for all of the β's as well as the posterior means and variances given by equations (8) to (9), using the posterior model weights from equation (7). We also calculate the posterior inclusion probability, discussed in Section I, subsection B. Figure 3 shows the posterior densities (approximated by histograms) of the coefficient estimates for four selected variables (the *investment price,* the *initial level of GDP per capita, primary schooling,* and the *number*

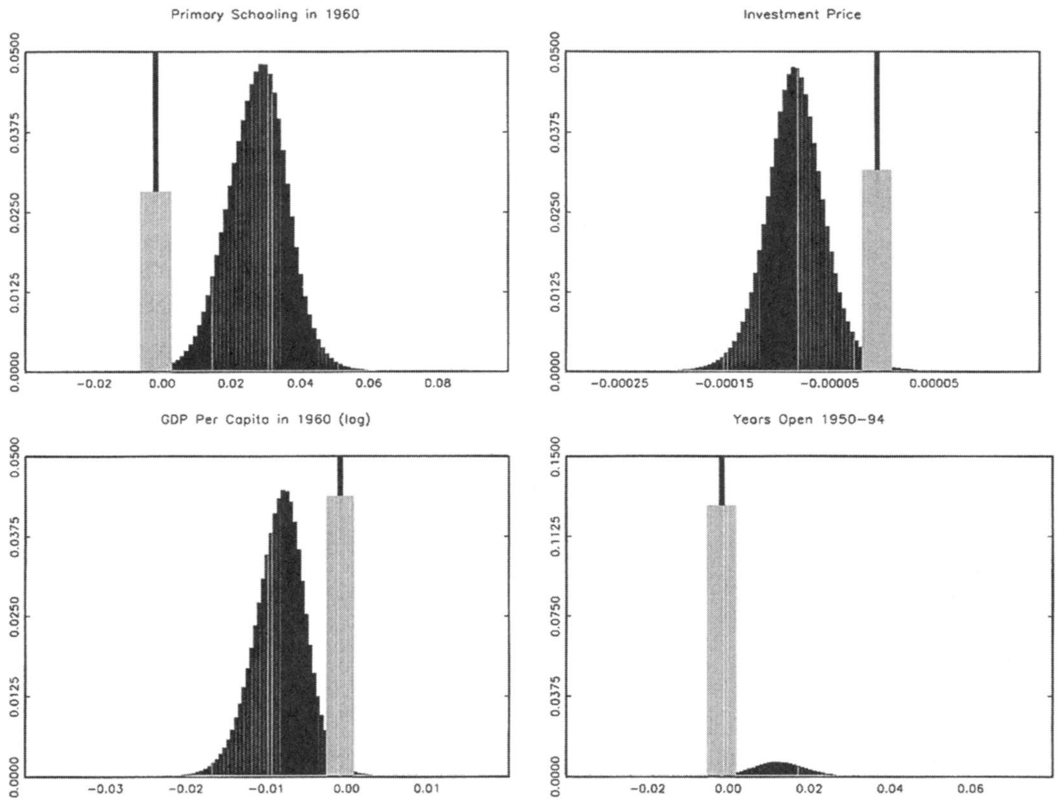

FIGURE 3. POSTERIOR DISTRIBUTION OF MARGINAL RELATIONSHIP OF SELECTED VARIABLES WITH LONG-RUN GROWTH

Notes: The distribution consists of two parts: (1) the "hump-shaped" part of the distribution represents the distribution of estimates conditional on the variable being included in the model; (2) the "lump" at zero shows the posterior probability that a variable is *not* included in the regression, which equals one minus the posterior inclusion probability. Note that the scale of the vertical axis for the "Years open" variable is larger to reflect the smaller posterior inclusion probability.

of years an economy has been "open").[8] Note that, in Figure 3 each distribution consists of two parts: first, a continuous part that is the posterior density conditional on inclusion in the model, and second, a discrete mass at zero representing the probability that the variable does not belong in the model; this is given by one minus the posterior inclusion probability.[9] As described in Section I, these densities are weighted averages of the posterior densities conditional on each particular model with the weights given by the posterior model probabilities. A standard result from Bayesian theory (see, e.g., Leamer, 1978, or Poirier, 1995) is that if priors are taken as diffuse by taking the limit of a Student-Gamma prior[10] then the posterior can be represented by:

$$t_i = \frac{\beta_i - \hat{\beta}_i}{s[(X'X)^{-1}]_{ii}^{1/2}} \sim t(T-k) \tag{10}$$

[8] The figures for the remaining variables are available in the Appendix on the AER Web site (http://www.aeaweb.org/aer/contents).

[9] The probability mass at zero is split into seven bins around zero to make the area of the mass comparable with areas under the conditional density. The figure for *years open* is plotted with a different vertical axis scaling reflecting the lower posterior inclusion probability.

[10] That is, a prior in which the marginal prior for the slope coefficients is multivariate student and the marginal prior for the regression error variance is inverted Gamma.

where s is the usual OLS estimate of the standard deviation of the regression residual. In other words, with the appropriate diffuse prior, the posterior distribution conditional on the model is identical to the classical sampling distribution. Thus, the marginal posterior distribution for each coefficient is a *mixture-t* distribution. In principle these distributions could be of almost any form, but most of the densities in Figure 3 look reasonably Gaussian.

A. *Baseline Estimation*

This section presents the baseline estimation results with a prior expected model size, $\bar{k} = 7$. The choice of the baseline model size is motivated by the fact that most empirical growth studies include a moderate number of explanatory variables. The posterior model size for the baseline estimation is 7.46, which is very close to the prior model size. In Section III, subsection B, we check the robustness of our results to changes in the prior mean model size. The results are based on approximately 89 million randomly drawn regressions.[11]

Table 2 shows the results for the 67 variables: Column (1) reports the posterior inclusion probability of a variable in the growth regression. Variables are sorted in descending order of this posterior probability. The posterior inclusion probability is the sum of the posterior probabilities of all of the regressions including that variable. Thus, computationally, the posterior inclusion probability is a measure of the weighted average goodness-of-fit of models including a particular variable, relative to models not including the variable. The goodness-of-fit measure is adjusted to penalize highly parameterized models in the fashion of the Schwarz model selection criterion. Thus, variables with high inclusion probabilities are variables that have high marginal contribution to the goodness-of-fit of the regression model.

We can divide the variables according to whether seeing the data causes us to increase or decrease our inclusion probability relative to the prior probability. Since our expected model size $\bar{k}$ equals 7, the prior inclusion probability is $7/67 = 0.104$. There are 18 variables for which the posterior inclusion probability increases (these are the first 18 variables in Table 2). For these variables, our belief that they belong in the regression is strengthened once we see the data and we call these variables "significant." The remaining 49 variables have little or no support for inclusion: seeing the data further reduces our already modest initial assessment of their inclusion probability.

Columns (2) and (3) show the posterior mean and standard deviation of the distributions, conditional on the variable being included in the model. That is, these are the means and standard deviations of the "hump-shaped" part of the distribution shown in Figure 3. The true (unconditional) posterior mean is computed according to equation (8) while the posterior standard deviation is the square root of the variance formula in equation (9). The true posterior mean is a weighted average of the OLS estimates for all regressions, including regressions in which the variable does not appear and thus has a coefficient of zero. Hence, the unconditional posterior mean can be computed by multiplying the conditional mean in column (2) times the posterior inclusion probability in column (1).[12]

If one has the prior with which we began the estimation, then the unconditional posterior mean is the "right" estimate of the marginal effect of the variable in the sense that it is the coefficient that would be used for forecasting.[13] The conditional mean and variance are also

[11] The total number of possible regression models equals 2^{67}, which is approximately equal to 1.48×10^{20} models. However, convergence of the estimates is attained relatively quickly; after 33 million draws the maximum change of coefficient estimates normalized by the standard deviation of the regressors relative to the dependent variable is smaller than 10^{-3} per 10,000, and after 89 million draws the maximum change is smaller than 10^{-6}. The latter tolerance was used as one of the convergence criteria for the reported estimates. See technical Appendix (available at the AER Web site: http://www.aeaweb.org/aer/contents/) for further details.

[12] Similarly, the unconditional variance can be calculated from the conditional variance as follows:

$$\sigma^2_{uncond} = [\sigma^2_{cond} + \beta^2_{cond}] * PosteriorInclusionProb. - \beta^2_{uncond}. \tag{14}$$

[13] In a pure Bayesian approach there is not really a notion of a single estimate. However, for many purposes the posterior mean is reasonable, and it is what would be used for constructing unbiased, minimum mean-squared-error predictions.

TABLE 2—BASELINE ESTIMATION FOR ALL 67 VARIABLES

Rank	Variable	Posterior inclusion probability (1)	Posterior mean conditional on inclusion (2)	Posterior s.d. conditional on inclusion (3)	BACE sign certainty probability (4)	OLS p-value (5)	OLS sign certainty probability (6)	Fraction of regressions with $\lvert t\text{stat}\rvert > 2$ (7)
1	East Asian dummy	0.823	0.021805	0.006118	0.999	0.505	0.999	0.99
2	Primary schooling 1960	0.796	0.026852	0.007977	0.999	0.155	0.999	0.96
3	Investment price	0.774	−0.000084	0.000025	0.999	0.032	0.999	0.99
4	GDP 1960 (log)	0.685	−0.008538	0.002888	0.999	0.387	0.999	0.30
5	Fraction of tropical area	0.563	−0.014757	0.004227	0.997	0.466	0.997	0.59
6	Population density coastal 1960's	0.428	0.000009	0.000003	0.996	0.767	0.996	0.85
7	Malaria prevalence in 1960's	0.252	−0.015702	0.006177	0.990	0.515	0.010	0.84
8	Life expectancy in 1960	0.209	0.000808	0.000354	0.986	0.761	0.014	0.79
9	Fraction Confucian	0.206	0.054429	0.022426	0.988	0.377	0.988	0.97
10	African dummy	0.154	−0.014706	0.006866	0.980	0.589	0.980	0.90
11	Latin American dummy	0.149	−0.012758	0.005834	0.969	0.652	0.969	0.30
12	Fraction GDP in mining	0.124	0.038823	0.019255	0.978	0.305	0.978	0.07
13	Spanish colony	0.123	−0.010720	0.005041	0.972	0.507	0.028	0.24
14	Years open	0.119	0.012209	0.006287	0.977	0.826	0.023	0.98
15	Fraction Muslim	0.114	0.012629	0.006257	0.973	0.478	0.973	0.11
16	Fraction Buddhist	0.108	0.021667	0.010722	0.974	0.460	0.974	0.90
17	Ethnolinguistic fractionalization	0.105	−0.011281	0.005835	0.974	0.991	0.974	0.52
18	Government consumption share 1960's	0.104	−0.044171	0.025383	0.975	0.344	0.025	0.77
19	Population density 1960	0.086	0.000013	0.000007	0.965	0.815	0.965	0.01
20	Real exchange rate distortions	0.082	−0.000079	0.000043	0.966	0.835	0.034	0.92
21	Fraction speaking foreign language	0.080	0.007006	0.003960	0.962	0.474	0.962	0.43
22	(Imports + exports)/GDP	0.076	0.008858	0.005210	0.949	0.951	0.949	0.67
23	Political rights	0.066	−0.001847	0.001202	0.939	0.664	0.939	0.35
24	Government share of GDP	0.063	−0.034874	0.029379	0.935	0.329	0.935	0.58
25	Higher education in 1960	0.061	−0.069693	0.041833	0.946	0.379	0.946	0.10
26	Fraction population in tropics	0.058	−0.010741	0.006754	0.940	0.657	0.940	0.85
27	Primary exports in 1970	0.053	−0.011343	0.007520	0.926	0.752	0.926	0.75
28	Public investment share	0.048	−0.061540	0.042950	0.922	0.115	0.922	0.00
29	Fraction Protestant	0.046	−0.011872	0.009288	0.909	0.715	0.091	0.29
30	Fraction Hindu	0.045	0.017558	0.012575	0.915	0.790	0.915	0.07
31	Fraction population less than 15	0.041	0.044962	0.041100	0.871	0.545	0.871	0.24
32	Air distance to big cities	0.039	−0.000001	0.000001	0.888	0.572	0.888	0.18
33	Government consumption share deflated with GDP prices	0.036	−0.033647	0.027365	0.893	0.565	0.893	0.05
34	Absolute latitude	0.033	0.000136	0.000233	0.737	0.484	0.263	0.37
35	Fraction Catholic	0.033	−0.008415	0.008478	0.837	0.939	0.163	0.16
36	Fertility in 1960's	0.031	−0.007525	0.010113	0.767	0.697	0.767	0.46
37	European dummy	0.030	−0.002278	0.010487	0.544	0.768	0.456	0.19
38	Outward orientation	0.030	−0.003296	0.002727	0.886	0.882	0.886	0.01
39	Colony dummy	0.029	−0.005010	0.004721	0.858	0.770	0.858	0.44
40	Civil liberties	0.029	−0.007192	0.007122	0.846	0.740	0.846	0.15
41	Revolutions and coups	0.029	−0.007065	0.006089	0.877	0.276	0.877	0.07
42	British colony	0.027	0.003654	0.003626	0.844	0.660	0.844	0.09
43	Hydrocarbon deposits in 1993	0.025	0.000307	0.000418	0.773	0.813	0.773	0.01
44	Fraction population over 65	0.022	0.019382	0.119469	0.566	0.540	0.566	0.20
45	Defense spending share	0.021	0.045336	0.076813	0.737	0.327	0.737	0.26
46	Population in 1960	0.021	0.000000	0.000000	0.806	0.581	0.806	0.07
47	Terms of trade growth in 1960's	0.021	0.032627	0.046650	0.752	0.982	0.248	0.00
48	Public education spending/GDP in 1960's	0.021	0.129517	0.172847	0.777	0.322	0.777	0.11
49	Landlocked country dummy	0.021	−0.002080	0.004206	0.701	0.951	0.701	0.04
50	Religion measure	0.020	−0.004737	0.007232	0.751	0.910	0.249	0.18
51	Size of economy	0.020	−0.000520	0.001443	0.661	0.577	0.661	0.18
52	Socialist dummy	0.020	0.003983	0.004966	0.788	0.916	0.212	0.00
53	English-speaking population	0.020	−0.003669	0.007137	0.686	0.543	0.314	0.07
54	Average inflation 1960–1990	0.020	−0.000073	0.000097	0.784	0.774	0.216	0.01
55	Oil-producing country dummy	0.019	0.004845	0.007088	0.751	0.933	0.751	0.00
56	Population growth rate 1960–1990	0.019	0.020837	0.307794	0.533	0.924	0.467	0.21
57	Timing of independence	0.019	0.001143	0.002051	0.716	0.846	0.284	0.11

TABLE 2—*Continued.*

Rank	Variable	Posterior inclusion probability (1)	Posterior mean conditional on inclusion (2)	Posterior s.d. conditional on inclusion (3)	BACE sign certainty probability (4)	OLS p-value (5)	OLS sign certainty probability (6)	Fraction of regressions with $\lvert t\text{stat}\rvert > 2$ (7)
58	Fraction land area near navig. water	0.019	−0.002598	0.005864	0.657	0.730	0.657	0.35
59	Square of inflation 1960–1990	0.018	−0.000001	0.000001	0.736	0.668	0.264	0.00
60	Fraction spent in war 1960–1990	0.016	−0.001415	0.009226	0.555	0.904	0.445	0.00
61	Land area	0.016	0.000000	0.000000	0.577	0.950	0.577	0.01
62	Tropical climate zone	0.016	−0.002069	0.006593	0.616	0.614	0.616	0.16
63	Terms of trade ranking	0.016	−0.003730	0.009625	0.647	0.845	0.647	0.23
64	Capitalism	0.015	−0.000231	0.001080	0.589	0.313	0.589	0.06
65	Fraction Orthodox	0.015	0.005689	0.013576	0.660	0.900	0.660	0.00
66	War participation 1960–1990	0.015	−0.000734	0.002983	0.593	0.868	0.593	0.02
67	Interior density	0.015	−0.000001	0.000016	0.532	0.768	0.468	0.00

Notes: The left-hand-side variable in all regressions is the growth rate from 1960–1996 across 88 countries. Apart from the final column, all statistics come from a random sample of approximately 89 million of the possible regressions including any combination of the 67 variables. Prior mean model size is seven. Variables are ranked by the first column, the posterior inclusion probability. This is the sum of the posterior probabilities of all models containing the variable. The next two columns reflect the posterior mean and standard deviations for the linear marginal effect of the variable: the posterior mean has the usual interpretation of a regression β. The conditional mean and standard deviation are conditional on inclusion in the model. The "sign certainty probability" is the posterior probability that the coefficient is on the same side of zero as its mean conditional on inclusion. It is a measure of our posterior confidence in the sign of the coefficient. The final column is the fraction of regressions in which the coefficient has a classical t-test greater than two, with all regressions having equal sampling probability.

of interest, however. From a Bayesian point of view these have the interpretation of the posterior mean and variance for a researcher who has a prior inclusion probability equal to one for the particular variable while maintaining the 7/67 inclusion probability for all the other variables. In other words, if one is certain that the variable belongs in the regression, this is the estimate to consider. It is also comparable to coefficient estimates in standard regressions not accounting for model uncertainty. The conditional standard deviation provides one measure of how well estimated the variable is conditional on its inclusion. It averages both the standard errors of each possible regression as well as the dispersion of estimates across models.[14]

From the posterior density we can also estimate the posterior probability that conditional on a variable's inclusion a coefficient has the same sign as its posterior mean reported in column (1).[15] This "sign certainty probability," reported in column (4), is another measure of the significance of the variables. As noted above, for each individual regression the posterior density is equal to the classical sampling distribution of the coefficient. In classical terms, a coefficient would be 5-percent significant in a two-sided test if 97.5 percent of the probability in the sampling distribution were on the same side of zero as the coefficient estimate. So if, for example, it just happened that a coefficient were exactly 5-percent significant in every single regression its sign certainty probability would be 97.5 percent. Applying a 0.975 cutoff to this quantity identifies a set of 13 variables, all of which are also in the group of 18 "significant" variables for which the posterior inclusion probability is larger than the prior inclusion probability. The remaining five have very large sign certainty probabilities (between 0.970 and

[14] Note that one cannot interpret the ratio of the posterior mean to the posterior standard deviation as a t-statistic for two reasons. Firstly the posterior is a mixture t-distribution and secondly it is not a sampling distribution. However, for most of the variables which we consider the posterior distributions are not too far from being normal. To the extent to which these are approximately normal, having a ratio of posterior conditional mean to standard deviation around two in absolute value indicates an approximate 95-percent Bayesian coverage region that excludes zero.

[15] This "sign certainty probability" is analogous to the area under the normal CDF(0) calculated by Sala-i-Martin (1997a, b).

0.975). Note that there is in principle no reason why a variable could not have a very high posterior inclusion probability and still have a low sign certainty probability.

The results can also be contrasted with the estimates of including all 67 explanatory variables resulting from diffuse prior OLS. First, note from the *p*-values reported in column (5) that all but one variable fail to be statistically significant at the 10-percent level. Second, the rank correlation between diffuse prior OLS (ranked by *p*-values) and BACE (ranked by posterior inclusion probability) equals only 0.43. This implies that BACE and diffuse prior OLS disagree about the relative importance of several important variables. Third, one can also calculate the posterior "sign certainty" that the coefficient in column (2) takes on the diffuse prior OLS sign. Column (6) of Table 2 shows this probability and we can see that for six of the top 21 variables, BACE gives a very different prediction of the sign of the effect of a variable than OLS. The disagreement between BACE and OLS concerning the relative ranking of variables and their predicted signs could be due to low number of degrees of freedom and some correlation among explanatory variables. The limited number of observations does not allow us to make reliable inference on the relative sign and importance of the explanatory variables. BACE, on the other hand, considers models of smaller expected size and allows explanatory variables to capture different aspects of the variation in the dependent variable.

The final column (7) in Table 2 shows the fraction of regressions in which the variable is classically significant at the 95-percent level, in the sense of having a *t*-statistic with an absolute value greater than two. This statistic was calculated separately from the other estimates.[16] This is reported partly for the sake of comparison with extreme bounds analysis results. Note that for all the variables, many individual regressions can be found in which the variable is not significant, but even the top variables would still be labeled fragile by an extreme bounds test.

Variables Significantly Related to Growth.— We are now ready to analyze the variables that are "significantly" related to growth. Not surprisingly, the top variable is the *dummy for East Asian countries,* which is positively related with economic growth. This, of course, reflects the exceptional growth performance of East Asian countries between 1960 and the mid-1990's. Notice that the dummy is present despite the significant positive relationship between the *fraction of population Confucian* (which is ranked 9th in the table). Although the Confucian variable can be interpreted as a dummy for East Asian economies, it gains relative to the East Asia dummy variable as more regressors are included in the regression with larger prior model sizes (this is seen in Tables 3 and 5). The posterior mean coefficient is very precisely estimated to be positive. The sign certainty probability in column (4) shows that the probability mass of the density to the right of zero equals to 0.9992. Notice that the fraction of regressions for which the East Asian dummy has a *t*-statistic greater than two in absolute value is 99 percent.

The second variable is a measure of human capital: the *primary schooling enrollment rate in 1960.* This variable is positively related to growth and the inclusion probability is 0.80. The posterior distribution of the coefficient estimates is shown in the first panel of Figure 3. Since the inclusion probability is relatively high, the mass at zero (which shows one minus this inclusion probability) is relatively small. Conditional on being included in the model, a 10-percentage-point increase of the primary school enrollment rate is associated with a 0.27-percentage-point increase of the growth rate. The sign certainty probability for this variable is also 0.999 and the fraction of regressions with a *t*-statistic larger than two is 96 percent.

The third variable is the *average price of investment goods between 1960 and 1964.* Its inclusion probability is 0.77. This variable is also depicted graphically in the second panel of Figure 3. The posterior mean coefficient is very precisely estimated to be negative, which indicates that a relative high price of investment

[16] This column was calculated based on a run of 72.5 million regression. It was calculated separately so that the sampling could be based solely on the prior inclusion probabilities. The other baseline estimates were calculated by oversampling "good" variables for inclusion and thus produce misleading results for this statistic.

goods at the beginning of the sample is strongly and negatively related to subsequent income growth.[17] The sign certainty probability in column (4) shows that the probability mass of the density to the left of zero equals 0.99: this can also be seen in Figure 3 by the fact that almost all of the continuous density lies below zero.

The next variable is the *initial level of per capita GDP,* a measure of conditional convergence. The inclusion probability is 0.69. The third panel in Figure 3 shows the posterior distribution of the coefficient estimates for initial income. Conditional on inclusion, the posterior mean coefficient is -0.009 (with a standard deviation of 0.003). The sign certainty probability in column (4) shows that the probability mass of the density to the left of zero equals 0.999. The fraction of regressions in which the coefficient for initial income has a *t*-statistic greater than two in absolute value is only 30 percent, so that an extreme bounds test very easily labels the variable as not robust. Nonetheless, the tightly estimated coefficient and the very high sign certainty statistic show that initial income is indeed robustly partially correlated with growth. The explanation is that the regressions in which the coefficient on initial income is poorly estimated are regressions with very poor fit, so they receive little weight in the averaging process. Furthermore, the high inclusion probability suggests that regressions that omit initial income are likely to perform poorly.

The next variables reflect the poor economic performance of tropical countries: The *proportion of a country's area in the tropics* has a negative relationship with income growth. Similarly, the *index of malaria prevalence* has a negative relationship with growth. Table 3 shows that the posterior inclusion probability of the malaria index falls as models become larger indicating that it could act as a catch-all variable when few explanatory variables are included, but drops in significance as more variables explaining different steady states enter. Similarly Table 5 shows the sign certainty probability falling with model size for this variable.

Another geographical variable that performs well is the *density of the population in coastal areas,* which has a positive relationship with growth suggesting that areas that are densely populated and are close to the sea have experienced higher growth rates.

Another measure of health is *life expectancy in 1960* (which is related to things like nutrition, health care, and education). Countries with high life expectancy in 1960 tended to grow faster over the following four decades. The inclusion probability for this variable is 0.28.

Dummies for *Sub-Saharan Africa* and *Latin America* are negatively related to income growth. The posterior means imply that the growth rate for Latin American and Sub-Saharan African countries were 1.47 and 1.28 percentage points below the level that would be predicted by the countries' other characteristics. The African dummy is significant in 90 percent of the regressions and the sign certainty probability is 98 percent. Although the Latin American dummy is only significant in 33 percent of the regressions, its sign certainty is 97 percent.

The *fraction of GDP in mining* has a positive relationship with growth and inclusion probability of 0.12. This variable captures the success of countries with a large endowment of natural resources. While many economists expect that the large rents are associated with more political instability or rent-seeking and low growth, our study shows that economies with a larger mining sector tend to perform better.[18]

Former Spanish colonies tend to grow less whereas the *number of years an economy has been open* has a positive sign. Both of these variables have inclusion probabilities that increase only moderately with larger models in Table 3 indicating that they capture steady-state variations in smaller models, but are relatively less important in explaining growth when more variables are included in the regression models.

Both the *fraction of the population Muslim*

[17] Once the relative price of investment goods is included among the pool of explanatory variables, the share of investment to GDP in 1961 becomes insignificant and has the "wrong sign" while the other results are unaffected. The estimation results including investment share are available from the authors upon request.

[18] The regressions that include the fraction of mining tend to have an outlier: Botswana, which is a country that discovered diamonds in its territory in the 1960's and has managed to exploit them successfully (which implies a large share of mining in GDP) and has experienced extraordinary growth rates over the last 40 years.

TABLE 3—POSTERIOR INCLUSION PROBABILITIES WITH DIFFERENT PRIOR MODEL SIZES

Rank	Variable	kbar5	kbar7	kbar9	kbar11	kbar16	kbar22	kbar28
	Prior inclusion probability	0.075	0.104	0.134	0.164	0.239	0.328	0.418
1	East Asian dummy	0.891	0.823	0.757	0.711	0.585	0.481	0.455
2	Primary schooling 1960	0.709	0.796	0.826	0.862	0.890	0.924	0.950
3	Investment price	0.635	0.774	0.840	0.891	0.936	0.968	0.985
4	GDP 1960 (log)	0.526	0.685	0.788	0.843	0.920	0.960	0.978
5	Fraction of tropical area	0.536	0.563	0.548	0.542	0.462	0.399	*0.388*
6	Population density coastal 1960's	0.350	0.428	0.463	0.473	0.433	0.389	*0.352*
7	Malaria prevalence in 1960's	0.339	0.252	0.203	0.176	*0.145*	*0.131*	*0.138*
8	Life expectancy in 1960	0.176	0.209	0.262	0.278	0.368	0.440	0.467
9	Fraction Confucian	0.140	0.206	0.272	0.333	0.501	0.671	0.777
10	African dummy	0.095	0.154	0.223	0.272	0.406	0.519	0.565
11	Latin American dummy	0.101	0.149	0.205	0.240	0.340	0.413	0.429
12	Fraction GDP in mining	*0.072*	0.124	0.209	0.275	0.478	0.659	0.761
13	Spanish colony	0.130	0.123	*0.119*	*0.116*	*0.124*	*0.148*	*0.182*
14	Years open	0.090	0.119	*0.124*	*0.132*	*0.145*	*0.155*	*0.177*
15	Fraction Muslim	0.078	0.114	0.150	0.178	0.267	0.366	0.450
16	Fraction Buddhist	*0.073*	0.108	0.152	0.190	0.320	0.465	0.563
17	Ethnolinguistic fractionalization	0.080	0.105	*0.131*	*0.140*	*0.155*	*0.160*	*0.184*
18	Government consumption share 1960's	0.090	0.104	0.135	*0.147*	*0.213*	*0.262*	*0.297*
19	Population density 1960	0.043	0.086	**0.137**	**0.175**	**0.257**	0.295	0.316
20	Real exchange rate distortions	0.059	0.082	0.117	0.134	0.205	0.263	0.319
21	Fraction speaking foreign language	0.052	0.080	0.110	0.149	**0.247**	**0.374**	**0.478**
22	(Imports + exports)/GDP	0.063	0.076	0.085	0.099	0.131	0.181	0.240
23	Political rights	0.042	0.066	0.082	0.095	0.114	0.130	0.154
24	Government share of GDP	0.044	0.063	0.087	0.112	0.186	0.252	0.291
25	Higher education in 1960	0.059	0.061	0.066	0.070	0.079	0.103	0.131
26	Fraction population in tropics	0.047	0.058	0.061	0.074	0.099	0.132	0.157
27	Primary exports in 1970	0.047	0.053	0.065	0.072	0.104	0.137	0.162
28	Public investment share	0.023	0.048	0.096	0.151	**0.321**	**0.525**	**0.669**
29	Fraction Protestant	0.035	0.046	0.055	0.061	0.083	0.120	0.156
30	Fraction Hindu	0.028	0.045	0.059	0.077	0.126	0.179	0.227
31	Fraction population less than 15	0.035	0.041	0.045	0.050	0.067	0.093	0.123
32	Air distance to big cities	0.024	0.039	0.054	0.072	0.097	0.115	0.141
33	Government consumption share deflated with GDP prices	0.021	0.036	0.056	0.075	0.137	0.225	0.310
34	Absolute latitude	0.029	0.033	0.040	0.042	0.059	0.086	0.115
35	Fraction Catholic	0.019	0.033	0.042	0.056	0.104	0.163	0.223
36	Fertility in 1960's	0.020	0.031	0.043	0.063	0.108	0.170	0.232
37	European dummy	0.020	0.030	0.043	0.049	0.094	0.148	0.201
38	Outward orientation	0.019	0.030	0.043	0.054	0.085	0.134	0.178
39	Colony dummy	0.022	0.029	0.039	0.049	0.075	0.105	0.146
40	Civil liberties	0.021	0.029	0.037	0.044	0.069	0.106	0.155
41	Revolutions and coups	0.019	0.029	0.038	0.056	0.106	0.188	0.282
42	British colony	0.022	0.027	0.034	0.041	0.057	0.085	0.119
43	Hydrocarbon deposits in 1993	0.015	0.025	0.035	0.048	0.089	0.143	0.196
44	Fraction population over 65	0.020	0.022	0.029	0.038	0.069	0.119	0.169
45	Defense spending share	0.016	0.021	0.027	0.033	0.049	0.073	0.102
46	Population in 1960	0.016	0.021	0.040	0.041	0.063	0.092	0.118
47	Terms of trade growth in 1960's	0.015	0.021	0.026	0.033	0.051	0.068	0.104
48	Public education spending/GDP in 1960's	0.014	0.021	0.027	0.037	0.063	0.102	0.141
49	Landlocked country dummy	0.012	0.021	0.029	0.033	0.055	0.080	0.109
50	Religion measure	0.012	0.020	0.025	0.037	0.048	0.068	0.092
51	Size of economy	0.016	0.020	0.026	0.033	0.051	0.076	0.104
52	Socialist dummy	0.012	0.020	0.024	0.032	0.054	0.091	0.144
53	English-speaking population	0.015	0.020	0.025	0.028	0.043	0.063	0.087
54	Average inflation 1960–1990	0.015	0.020	0.024	0.030	0.043	0.064	0.100
55	Oil-producing country dummy	0.012	0.019	0.025	0.033	0.050	0.071	0.095
56	Population growth rate 1960–1990	0.014	0.019	0.023	0.029	0.046	0.074	0.098
57	Timing of independence	0.014	0.019	0.024	0.031	0.048	0.076	0.099

TABLE 3—*Continued.*

Rank	Variable	kbar5	kbar7	kbar9	kbar11	kbar16	kbar22	kbar28
58	Fraction land area near navig. water	0.013	0.019	0.024	0.031	0.055	0.092	0.142
59	Square of inflation 1960–1990	0.013	0.018	0.022	0.027	0.041	0.063	0.105
60	Fraction spent in war 1960–1990	0.010	0.016	0.019	0.024	0.039	0.060	0.087
61	Land area	0.010	0.016	0.022	0.026	0.043	0.071	0.103
62	Tropical climate zone	0.012	0.016	0.020	0.028	0.042	0.067	0.100
63	Terms of trade ranking	0.011	0.016	0.019	0.026	0.039	0.063	0.086
64	Capitalism	0.010	0.015	0.020	0.026	0.047	0.084	0.128
65	Fraction Orthodox	0.011	0.015	0.020	0.025	0.036	0.059	0.083
66	War participation 1960–1990	0.010	0.015	0.019	0.025	0.040	0.060	0.089
67	Interior density	0.010	0.015	0.019	0.023	0.039	0.062	0.085

Notes: The left-hand-side variable in all regressions is the growth rate from 1960–1996 across 88 countries. Each column contains the posterior probability of all models including the given variable. These are calculated with the same data but with different prior mean model sizes as labeled in the column headings. They are based on different random samples of all possible regressions using the same convergence criterion for stopping sampling. Samples range from around 63 million regressions for $\bar{k} = 5$ to around 38 million for $\bar{k} = 28$.

and Buddhist have a positive association with growth where the conditional mean of the latter variable is almost twice as large (0.022) than for the fraction Muslim (0.013), but both are relatively small compared to the fraction Confucian (0.054). The index of *ethnolinguistic fractionalization* is negatively related to growth.

Finally, the *share of government consumption in GDP* has a negative association with economic growth. This could be expected because public consumption does not tend to contribute to growth directly, but it needs to be financed with distortionary taxes which hurt the growth rate. Perhaps the real surprise is the negative coefficient of the *public investment share.* Table 2 shows that this variable is not robust when the prior model size is $\bar{k} = 7$. However, we will see later that this is one of the variables that becomes important in larger models and the sign remains *negative.*

Variables Marginally Related to Growth.— There are three variables that have posterior probabilities somewhat lower than their prior probabilities but nonetheless are fairly precisely estimated if they are included in the growth regression (that is, their sign certainty probability is larger than 95 percent). These variables are: the overall *density* in 1960 (which is positively related to growth), *real exchange rate distortions* (negative) and the *fraction of population speaking a foreign language* (positive).

Variables Weakly or Not Related to Growth.— The remaining 46 variables show little evidence of any sort of robust partial correlation with growth. They neither contribute importantly to the goodness-of-fit of growth regressions, as measured by their posterior inclusion probabilities, nor have estimates that are robust across different sets of conditioning variables. It is interesting to notice that some political variables such as the *number of revolutions and coups* or the *index of political rights* are not robustly related to economic growth. Similarly the *degree of capitalism* measure or a *Socialism dummy* measuring whether a country was under Socialism for a significant period of time, have no strong relationship with growth between 1960 and 1996.[19] This could be due to the fact that other variables, capturing political or economic instability such as the relative price of investment goods, real exchange rate distortions, the number of years an economy has been open, and life expectancy or regional dummies, capture most of the effect.

We also notice that some macroeconomic variables such as the *inflation rate* do not appear to be strongly related to growth. Other surprisingly weak variables are the *spending in public*

[19] We should remind the reader that most of the economies Eastern Europe and the former Soviet bloc are not included in our data set (see Appendix Table A1 for a list of included countries).

TABLE 4—POSTERIOR MEANS CONDITIONAL ON INCLUSION WITH DIFFERENT PRIOR MODEL SIZES

Rank	Variable	kbar5	kbar7	kbar9	kbar11	kbar16	kbar22	kbar28
1	East Asian dummy	0.023633	0.021805	0.020595	0.019716	0.017942	0.015867	0.014105
2	Primary schooling 1960	0.025899	0.026852	0.027235	0.027311	0.026740	0.025984	0.025605
3	Investment price	−0.000083	−0.000084	−0.000084	−0.000084	−0.000085	−0.000087	−0.000089
4	GDP 1960 (log)	−0.008245	−0.008538	−0.008871	−0.009044	−0.009535	−0.009856	−0.009979
5	Fraction of tropical area	−0.014760	−0.014757	−0.014528	−0.014217	−0.013048	−0.011437	−0.010086
6	Population density coastal 1960's	0.000009	0.000009	0.000009	0.000008	0.000008	0.000007	0.000006
7	Malaria prevalence in 1960's	−0.017303	−0.015702	−0.014432	−0.013080	−0.010334	−0.007332	−0.005397
8	Life expectancy in 1960	0.000812	0.000808	0.000790	0.000760	0.000716	0.000657	0.000608
9	Fraction Confucian	0.055184	0.054429	0.053324	0.052695	0.051453	0.050820	0.050184
10	African dummy	−0.014162	−0.014706	−0.015152	−0.014983	−0.014910	−0.014426	−0.013907
11	Latin American dummy	−0.012025	−0.012758	−0.013340	−0.013379	−0.013278	−0.012873	−0.012067
12	Fraction GDP in mining	0.036381	0.038823	0.043653	0.044337	0.048799	0.052185	0.053946
13	Spanish colony	−0.011022	−0.010720	−0.010177	−0.009504	−0.008203	−0.007034	−0.006406
14	Years open	0.012831	0.012209	0.011490	0.010951	0.009711	0.008253	0.007235
15	Fraction Muslim	0.012361	0.012629	0.012720	0.012848	0.012909	0.012944	0.012863
16	Fraction Buddhist	0.022501	0.021667	0.020501	0.020428	0.019962	0.019933	0.019788
17	Ethnolinguistic fractionalization	−0.011923	−0.011281	−0.011020	−0.010656	−0.009759	−0.008492	−0.007597
18	Government consumption share 1960's	−0.046997	−0.044171	−0.043073	−0.041697	−0.040810	−0.038192	−0.035443
19	Population density 1960	0.000012	0.000013	0.000013	0.000014	0.000014	0.000014	0.000013
20	Real exchange rate distortions	−0.000080	−0.000079	−0.000081	−0.000077	−0.000075	−0.000070	−0.000066
21	Fraction speaking foreign language	0.006864	0.007006	0.007083	0.007203	0.007404	0.007447	0.007500
22	(Imports + exports)/GDP	0.009305	0.008858	0.008523	0.008183	0.007582	0.007169	0.007118
23	Political rights	−0.001774	−0.001847	−0.001908	−0.001872	−0.001760	−0.001589	−0.001414
24	Government share of GDP	−0.034446	−0.034874	−0.033147	−0.035389	−0.035999	−0.035820	−0.035114
25	Higher education in 1960	−0.073730	−0.069693	−0.062763	−0.061209	−0.050170	−0.042404	−0.040126
26	Fraction population in tropics	−0.012008	−0.010741	−0.009761	−0.009386	−0.008367	−0.007683	−0.006675
27	Primary exports in 1970	−0.012145	−0.011343	−0.010929	−0.010536	−0.009957	−0.008988	−0.008147
28	Public investment share	−0.049229	−0.061540	−0.071923	−0.076999	−0.082566	−0.085304	−0.086961
29	Fraction Protestant	−0.010969	−0.011872	−0.010327	−0.010297	−0.010552	−0.009979	−0.009726
30	Fraction Hindu	0.016548	0.017558	0.017274	0.017600	0.018583	0.017887	0.017318
31	Fraction population less than 15	0.045099	0.044962	0.040324	0.038032	0.030408	0.027761	0.022600
32	Air distance to big cities	−0.000001	−0.000001	−0.000001	−0.000001	−0.000001	−0.000001	−0.000001
33	Government consumption share deflated with GDP prices	−0.030646	−0.033647	−0.036617	−0.037569	−0.040057	−0.041811	−0.043023
34	Absolute latitude	0.000169	0.000136	0.000114	0.000069	0.000014	−0.000023	−0.000054
35	Fraction Catholic	−0.006630	−0.008415	−0.007745	−0.008401	−0.009127	−0.009569	−0.009948
36	Fertility in 1960's	−0.005933	−0.007525	−0.008816	−0.009633	−0.010719	−0.011290	−0.011172
37	European dummy	−0.001383	−0.002278	−0.001497	0.000997	0.002930	0.004671	0.006152
38	Outward orientation	−0.003317	−0.003296	−0.003320	−0.003385	−0.003269	−0.003229	−0.003178
39	Colony dummy	−0.005196	−0.005010	−0.005038	−0.005109	−0.004862	−0.004553	−0.004135
40	Civil liberties	−0.007263	−0.007192	−0.007343	−0.007012	−0.006871	−0.006876	−0.006791
41	Revolutions and coups	−0.007255	−0.007065	−0.006969	−0.007443	−0.008232	−0.008711	−0.009015
42	British colony	0.004059	0.003654	0.003352	0.003181	0.002753	0.002593	0.002480
43	Hydrocarbon deposits in 1993	0.000220	0.000307	0.000352	0.000390	0.000423	0.000455	0.000461
44	Fraction population over 65	0.011873	0.019382	0.040908	0.053298	0.088785	0.113289	0.118943
45	Defense spending share	0.052439	0.045336	0.044461	0.038554	0.026337	0.017661	0.013697
46	Population in 1960	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
47	Terms of trade growth in 1960's	0.035425	0.032627	0.032750	0.030418	0.021860	0.009712	−0.001535
48	Public education spending/GDP in 1960's	0.127413	0.129517	0.128488	0.139824	0.150386	0.156571	0.159089
49	Landlocked country dummy	−0.001509	−0.002080	−0.002576	−0.002740	−0.002913	−0.002855	−0.002725
50	Religion measure	−0.004409	−0.004737	−0.004924	−0.004759	−0.003540	−0.001831	−0.000414
51	Size of economy	−0.000733	−0.000520	−0.000323	−0.000161	0.000014	0.000109	0.000166
52	Socialist dummy	0.004140	0.003983	0.003976	0.004071	0.004318	0.004591	0.004706
53	English-speaking population	−0.004839	−0.003669	−0.002854	−0.002243	−0.001229	−0.000485	0.000311
54	Average inflation 1960–1990	−0.000082	−0.000073	−0.000064	−0.000055	−0.000031	−0.000007	0.000008
55	Oil-producing country dummy	0.003559	0.004845	0.003855	0.003995	0.003127	0.001992	0.000995
56	Population growth rate 1960–1990	0.059271	0.020837	0.021521	−0.023353	−0.035501	−0.036410	0.001145
57	Timing of independence	0.001258	0.001143	0.001034	0.000963	0.000630	0.000360	0.000121
58	Fraction land area near navig. water	−0.002874	−0.002598	−0.002666	−0.002705	−0.002881	−0.003575	−0.003687
59	Square of inflation 1960–1990	−0.000001	−0.000001	−0.000001	−0.000001	0.000000	0.000000	0.000000
60	Fraction spent in war 1960–1990	−0.002555	−0.001415	−0.001315	−0.000751	−0.000211	0.000251	0.000801
61	Land area	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
62	Tropical climate zone	−0.003258	−0.002069	−0.001395	−0.001280	−0.000922	−0.001455	−0.001403
63	Terms of trade ranking	−0.004179	−0.003730	−0.003079	−0.002481	−0.001011	0.000251	0.000894
64	Capitalism	−0.000090	−0.000231	−0.000323	−0.000384	−0.000564	−0.000743	−0.000892
65	Fraction Orthodox	0.007559	0.005689	0.004019	0.003189	0.001995	0.001679	0.001914
66	War participation 1960–1990	−0.000746	−0.000734	−0.000810	−0.000874	−0.001009	−0.001144	−0.001091
67	Interior density	0.000000	−0.000001	−0.000002	−0.000002	−0.000003	−0.000003	−0.000004

education, some measures of *higher education,* some geographical measures such as the *absolute latitude* (the distance from the equator), and various proxies for "scale effects" such as the total *population, aggregate GDP,* or the total area of a country.

B. *Robustness of the Results*

Up until now we have concentrated on results derived for a prior model size $\bar{k} = 7$. As discussed earlier, while we feel that this is a reasonable expected model size it is in some sense arbitrary. We need to explore the effects of the prior on our conclusions. Tables 3, 4, and 5 do precisely this, reporting the posterior inclusion probabilities, the conditional posterior means, and sign certainty probabilities for $\bar{k}$ equal to 5, 9, 11, 16, 22, and 28, as well as repeating the benchmark numbers for easy comparison. Note that each $\bar{k}$ has a corresponding value of the prior probability of inclusion, which is reported in the first row of Table 3. Thus, to see whether a variable improves its probability of inclusion relative to the prior, we need to compare the posterior probability to the corresponding prior probability. The variables that are important in the baseline case of $\bar{k} = 7$ and are not important for other prior model sizes are shown in italics in Table 3. Variables that are not important for $\bar{k} = 7$ but become important with other sizes are shown in boldface.

"Significant" Variables That Become "Weak".—The most significant variables show very little sensitivity to the choice of prior model size, either in terms of their inclusion probabilities or their coefficient estimates. Some of the important variables seem to improve substantially with the prior model size. For example, for the *fraction of GDP in mining,* the posterior inclusion probability rises from 7 percent with $\bar{k} = 5$ to 66 percent with $\bar{k} = 22$. This suggests that mining is a variable that requires other conditioning variables in order to display its full importance.[20] Both the *fraction of Confucians* and the *Sub-Saharan Africa dummy* are also variables that appear to do better with more conditioning variables and have stable coefficient estimates.

Although most of the significant variables remain so, five of them tend to lose significance as we increase the prior model size. That is, for larger models, the posterior probability declines to levels below the prior size. These variables are the *index of malaria prevalence,* the *former Spanish colonies,* the number of *years an economy has been open,* the index of *ethnolinguistic fractionalization* and the *government consumption share.* This suggests that these variables could be acting as a "catch-all" for various other effects. For example, the openness index captures various aspects of the openness of a country to trade (tariff and nontariff barriers, black market premium, degree of Socialism, and monopolization of exports by the government). Interestingly, Table 5 shows the sign certainty probabilities of these five variables dropping below 0.95 for relatively large models ($\bar{k} = 28$). The other 13 variables that were significant in the baseline model also appear to be robust to different prior specifications.

"Insignificant" Variables That Become "Significant."—At the other end of the scale, most of the 46 variables that showed little partial correlation in the baseline estimation are not helped by alternative priors.[21] Their posterior inclusion probabilities rise as $\bar{k}$ increases, which is hardly surprising as their prior inclusion probabilities are rising. But their posterior inclusion probabilities remain below the prior so we are forced to think of them as "insignificant."

There are three variables that are insignificant in the baseline study but become "significant" with some prior model sizes. These are the *population density,* the *fraction of population that speak a foreign language* (a measure of international social capital and openness) and the *public investment share.* As mentioned above, the public investment share is particularly interesting because it becomes strong for larger prior model sizes, but the sign of the correlation is *negative.* That is, a larger public

[20] Notice that the *fraction of GDP in mining* is largely driven by the success of Botswana. Once other control variables such as a Sub-Saharan dummy are included, the Mining variable captures Botswana's unusual performance.

[21] The exception is the *public investment share* which has a posterior inclusion probability greater than the prior in Table 3 and sign certainty probability exceeding 0.95 in Table 5 for relatively large models with $\bar{k} \geq 16$.

TABLE 5—SIGN CERTAINTY PROBABILITIES WITH DIFFERENT PRIOR MODEL SIZES

Rank	Variable	kbar5	kbar7	kbar9	kbar11	kbar16	kbar22	kbar28
1	East Asian dummy	1.000	0.999	0.998	0.997	0.992	0.979	0.964
2	Primary schooling 1960	0.999	0.999	0.999	0.999	0.999	0.999	0.998
3	Investment price	0.999	0.999	0.999	0.999	0.999	0.999	0.999
4	GDP 1960 (log)	0.998	0.999	0.999	0.999	0.999	0.999	0.999
5	Fraction of tropical area	0.998	0.997	0.996	0.995	0.988	0.974	0.955
6	Population density coastal 1960's	0.996	0.996	0.995	0.994	0.988	0.975	0.954
7	Malaria prevalence in 1960's	0.996	0.990	0.982	0.972	0.936	0.873	0.810
8	Life expectancy in 1960	0.988	0.986	0.986	0.985	0.984	0.980	0.974
9	Fraction Confucian	0.987	0.988	0.989	0.989	0.991	0.992	0.993
10	African dummy	0.975	0.980	0.983	0.983	0.985	0.983	0.980
11	Latin American dummy	0.965	0.969	0.973	0.972	0.974	0.969	0.957
12	Fraction GDP in mining	0.972	0.978	0.984	0.986	0.990	0.992	0.992
13	Spanish colony	0.983	0.972	0.959	0.945	0.916	0.885	0.867
14	Years open	0.980	0.977	0.970	0.963	0.940	0.903	0.869
15	Fraction Muslim	0.966	0.973	0.974	0.974	0.973	0.970	0.967
16	Fraction Buddhist	0.972	0.974	0.976	0.977	0.980	0.982	0.981
17	Ethnolinguistic fractionalization	0.977	0.974	0.972	0.966	0.945	0.907	0.873
18	Government consumption share 1960's	0.980	0.975	0.970	0.967	0.962	0.948	0.930
19	Population density 1960	0.948	0.965	0.972	0.971	0.972	0.961	0.945
20	Real exchange rate distortions	0.967	0.966	0.969	0.964	0.965	0.958	0.949
21	Fraction speaking foreign language	0.958	0.962	0.965	0.967	0.972	0.974	0.975
22	(Imports + exports)/GDP	0.962	0.949	0.938	0.928	0.914	0.908	0.907
23	Political rights	0.927	0.939	0.941	0.935	0.905	0.860	0.828
24	Government share of GDP	0.934	0.935	0.920	0.937	0.940	0.935	0.921
25	Higher education in 1960	0.960	0.946	0.920	0.915	0.870	0.834	0.818
26	Fraction population in tropics	0.951	0.940	0.924	0.918	0.899	0.880	0.848
27	Primary exports in 1970	0.945	0.926	0.920	0.912	0.906	0.890	0.872
28	Public investment share	0.869	0.922	0.953	0.965	0.979	0.986	0.989
29	Fraction Protestant	0.917	0.909	0.883	0.855	0.843	0.820	0.796
30	Fraction Hindu	0.904	0.915	0.911	0.914	0.919	0.903	0.896
31	Fraction population less than 15	0.865	0.871	0.827	0.798	0.704	0.641	0.598
32	Air distance to big cities	0.868	0.888	0.897	0.899	0.875	0.835	0.795
33	Government consumption share deflated with GDP prices	0.870	0.893	0.912	0.921	0.933	0.941	0.943
34	Absolute latitude	0.786	0.737	0.697	0.628	0.541	0.520	0.576
35	Fraction Catholic	0.782	0.837	0.840	0.851	0.885	0.891	0.898
36	Fertility in 1960's	0.718	0.767	0.819	0.855	0.891	0.909	0.909
37	European dummy	0.520	0.544	0.531	0.560	0.626	0.686	0.740
38	Outward orientation	0.883	0.886	0.893	0.894	0.892	0.895	0.892
39	Colony dummy	0.866	0.858	0.852	0.853	0.840	0.821	0.805
40	Civil liberties	0.860	0.846	0.843	0.829	0.834	0.843	0.846
41	Revolutions and coups	0.874	0.877	0.876	0.895	0.917	0.931	0.935
42	British colony	0.871	0.844	0.827	0.810	0.781	0.767	0.759
43	Hydrocarbon deposits in 1993	0.684	0.773	0.816	0.844	0.873	0.893	0.900
44	Fraction population over 65	0.547	0.566	0.632	0.677	0.787	0.847	0.864
45	Defense spending share	0.773	0.737	0.730	0.697	0.623	0.568	0.541
46	Population in 1960	0.799	0.806	0.762	0.809	0.811	0.795	0.786
47	Terms of trade growth in 1960's	0.771	0.752	0.753	0.730	0.661	0.564	0.529
48	Public education spending/GDP in 1960's	0.768	0.777	0.779	0.800	0.818	0.830	0.836
49	Landlocked country dummy	0.646	0.701	0.753	0.761	0.777	0.776	0.767
50	Religion measure	0.728	0.751	0.758	0.757	0.690	0.604	0.529
51	Size of economy	0.727	0.661	0.600	0.559	0.507	0.517	0.526
52	Socialist dummy	0.788	0.788	0.788	0.795	0.810	0.826	0.829
53	English-speaking population	0.745	0.686	0.646	0.613	0.570	0.527	0.521
54	Average inflation 1960–1990	0.809	0.784	0.754	0.720	0.625	0.529	0.534
55	Oil-producing country dummy	0.704	0.751	0.718	0.726	0.675	0.610	0.552
56	Population growth rate 1960–1990	0.595	0.533	0.530	0.532	0.562	0.573	0.528
57	Timing of independence	0.738	0.716	0.687	0.679	0.611	0.559	0.514
58	Fraction land area near navig. water	0.666	0.657	0.668	0.675	0.700	0.748	0.761

TABLE 5—*Continued.*

Rank	Variable	kbar5	kbar7	kbar9	kbar11	kbar16	kbar22	kbar28
59	Square of inflation 1960–1990	0.766	0.736	0.705	0.675	0.572	0.530	0.552
60	Fraction spent in war 1960–1990	0.603	0.555	0.553	0.531	0.508	0.511	0.531
61	Land area	0.527	0.577	0.544	0.625	0.634	0.643	0.658
62	Tropical climate zone	0.673	0.616	0.583	0.576	0.560	0.599	0.597
63	Terms of trade ranking	0.663	0.647	0.620	0.595	0.534	0.518	0.545
64	Capitalism	0.540	0.589	0.620	0.639	0.699	0.758	0.803
65	Fraction Orthodox	0.707	0.660	0.618	0.593	0.561	0.553	0.561
66	War participation 1960–1990	0.593	0.593	0.606	0.616	0.637	0.658	0.651
67	Interior density	0.509	0.532	0.542	0.544	0.578	0.595	0.607

investment share tends to be associated with lower growth rates. Our interpretation of these results is that our baseline results are very robust to alternative prior size specifications.

IV. Conclusions

In this paper we propose a Bayesian Averaging of Classical Estimates (BACE) method to determine what variables are significantly related to growth in a broad cross section of countries. The method introduces a number of improvements relative to the previous literature. For example, we use an averaging method that is fully justified on Bayesian grounds and we do not restrict the number of regressors in the averaged models. Our approach provides an alternative to a standard Bayesian Model Averaging since BACE does not require the specification of the prior distribution of the parameters, but has only one hyper-parameter, the expected model size, $\bar{k}$. This parameter is easy to interpret, easy to specify, and easy to check for robustness. The interpretation of the BACE estimates is straightforward, since the weights are analogous to the Schwarz model selection criterion. Finally, our estimates can be calculated using only repeated applications of OLS which makes the approach transparent and straightforward to implement.

Our main results support Sala-i-Martin (1997a, b) rather than Levine and Renelt (1992): we find that a good number of economic variables have robust partial correlation with long-run growth. In fact, we find that about one-fifth of the 67 variables used in the analysis can be said to be significantly related to growth while several more are marginally related. The strongest evidence is found for primary schooling enrollment, the relative price of investment goods and the initial level of income where the latter reflects the concept of conditional convergence. Other important variables include regional dummies (such as East Asia, Sub-Saharan Africa, or Latin America), some measures of human capital and health (such as life expectancy, proportion of a country lying in the tropics, and malaria prevalence), religious dummies, and some sectoral variables such as mining. The public consumption and public investment shares are negatively related to growth, although the results are significant only for certain prior model sizes.

Finally, we show that our results are quite robust to the choice of the prior model size: most of the "significant" variables in the baseline case remain significant for other prior model sizes. Nonlinear relationships between explanatory variables and the dependent variable can be readily estimated within the BACE framework.

TABLE A1—LIST OF 88 COUNTRIES INCLUDED IN THE REGRESSIONS

Algeria	Mexico	Netherlands
Benin	Panama	Norway
Botswana	Trinidad & Tobago	Portugal
Burundi	United States	Spain
Cameroon	Argentina	Sweden
Central African Republic	Bolivia	Turkey
Congo	Brazil	United Kingdom
Egypt	Chile	Australia
Ethiopia	Colombia	Fiji
Gabon	Ecuador	Papua New Guinea
Gambia	Paraguay	
Ghana	Peru	
Kenya	Uruguay	
Lesotho	Venezuela	
Liberia	Hong Kong	
Madagascar	India	
Malawi	Indonesia	
Mauritania	Israel	
Morocco	Japan	
Niger	Jordan	
Nigeria	Korea	
Rwanda	Malaysia	
Senegal	Nepal	
South Africa	Pakistan	
Tanzania	Philippines	
Togo	Singapore	
Tunisia	Sri Lanka	
Uganda	Syria	
Zaire	Taiwan	
Zambia	Thailand	
Zimbabwe	Austria	
Canada	Belgium	
Costa Rica	Denmark	
Dominican Republic	Finland	
El Salvador	France	
Guatemala	Germany, West	
Haiti	Greece	
Honduras	Ireland	
Jamaica	Italy	

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