# *The Bayesian approach to poverty measurement*

# Lecture 4: Restricted stochastic dominance

Michel Lubrano

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## Contents

# 1 Introduction

The notion of stochastic dominance is very important in poverty analysis. Depending on the shape of the income distribution, poverty indices can give quite different results when varying the poverty line $z$, as underlined in Foster and Shorrocks (1988). More precisely, there can be more poverty in country $A$ than in country $B$ for poverty line $z_1$ and the reverse result for poverty line $z_2$. We would like to have poverty comparisons which are valid whatever the level of the poverty line. This is the notion of stochastic dominance.

Stochastic dominance is a mathematical notion that allows to compare distributions. It comes from the theory of probability with Blackwell (1953), was used to solve decision problems under uncertainty in various contexts. However and most importantly for us, it was used by Atkinson (1970) to compare income distributions, replacing the utility function in individual decision problems by a social welfare function, making the parallel between risk aversion in an individual decision problem and aversion for inequality in a collective decision problem at the level of a society.

# 2 Poverty deficit curves and stochastic dominance

There is a nice relation between FGT poverty indices and the notion of stochastic dominance. It is useful to detail this relationship in order to explain what is exactly stochastic dominance.

## 2.1 FGT indices and poverty deficit curves

Let us recall that if $F(.)$ is the income distribution and $z$ the poverty line, then for a given $\alpha$ this family of poverty indices is defined by:

$$P_\alpha(z) = \int_0^z (1 - x/z)^\alpha \, \mathrm{d}F(x). \tag{1}$$

If we now let $z$ varying in the domain of definition of $x$, we get the *poverty incidence curve* for $\alpha = 0$, using a terminology due to Ravallion (1996). Poverty can be measured by counting the poor with $F(z)$. We might like to measure the *severity* of poverty by measuring the surface under the poverty incidence curve given by:

$$\int_0^z F(x)\mathrm{d}x.$$

We can decompose this surface, using integration by parts with $u = dx$, $v = F(x)$ and $z = \int_0^z dx$:

$$\int_0^z F(x)dx = z\int_0^z f(x)dx - \int_0^z xf(x)dx = z\int_0^z (1 - x/z)f(x)dx. \quad (2)$$

The surface below the incidence curve is thus equal to the poverty line times the truncated mean of the relative poverty gap, the latter being defined by:

$$1 - x/z.$$

If we divide on both sides of (2) by $z$, we get the second poverty index of Foster et al. (1984) noted $P_1(z)$: It was called the normalised poverty deficit by Atkinson:

$$\frac{1}{z}\int_0^z F(x)dx = \int_0^z (1 - x/z)f(x)dx = P_1(z).$$

If we now let $z$ vary over the domain of $x$, we get the *poverty deficit curve*. Calling $\mu_p$ the average standard of living of the poor and using some integral calculus we get:

$$P_1 = F(z)\left[1 - \frac{\mu_p}{z}\right] = P_0\left[1 - \frac{\mu_p}{z}\right]. \quad (3)$$

These two curves, *poverty incidence curve* and *poverty deficit curve* are directly related to the notion of stochastic dominance at the order one and at the order two.

## 2.2 Mathematical definition of stochastic dominance

The usual and simplified definition of stochastic dominance at the order one (or first degree stochastic dominance) is (see e.g. Hadar and Russell 1969):

**Definition 1** *The probability distribution $F$ stochastically dominates the probability distribution $G$ at the order one if and only if:*

$$F(z) < G(z) \qquad \forall z \in [0, +\infty[.$$

This definition means that the probability of getting $z$ or less is not larger with $F$ than it is with $G$, whatever the value of $z$. The usual definition make use of loose inequality, but add the restriction that there are at least one point where the inequality is strict.

This definition allows us to compare two distributions only when they do not intersect. If they intersect, we cannot conclude. In this case, it might be useful to use a second notion, which is stochastic dominance at the second order. Second order (or second degree) stochastic dominance is based on the comparison of the surface under the cumulative distribution functions and may remove this indeterminacy. We have:

**Definition 2** *The probability distribution $F$ stochastically dominates the probability distribution $G$ at the order two if and only if*

$$\int_0^z [F(t) - G(t)]dt < 0 \qquad \forall z \in [0, +\infty[.$$

We can define stochastic dominance for any order because there is a strict relation between each order. It is useful to consider a sequence of integrals for a density $f$ that we define as follows:

$$\begin{aligned} F_0(x) &= f(x), & (4) \\ F_1(x) &= \int_0^x F_0(t)dt, & (5) \\ F_2(x) &= \int_0^x F_1(t)dt, & (6) \\ &\cdots & (7) \end{aligned}$$

that we can generalise in the following recurrence relation:

$$F_s(x) = \int_0^x F_{s-1}(t)dt = \frac{1}{(s-1)!}\int_0^x (x-t)^{s-1} f(t)dt.$$

Because distributions are positive and increasing functions of $x$, stochastic dominance at the order $s$, which can be written as:

$$F_s(z) \leq G_s(z) \qquad \forall z \in [0, +\infty[$$

implies stochastic dominance at any higher order. In particular, stochastic dominance at the order two:

$$F_2(x) \leq G_2(x), \forall x$$

implies

$$F_{2+j}(x) \leq G_{2+j}(x), \forall j \geq 1,$$

but does not rely on stochastic dominance at the order 1:

$$F_1(x) \leq G_1(x), \qquad \forall x.$$

## 2.3 Ordering income distributions and poverty indices

Let us start from the general recurrence relation:

$$F_s(x) = \int_0^x F_{s-1}(t)dt = \frac{1}{(s-1)!}\int_0^x (x-t)^{s-1} f(t)dt.$$

This last writing is particularly interesting as it directly links the Foster et al. (1984) poverty indices to the notion of stochastic dominance. As a matter of fact, if we set $x$ equal to the poverty line $z$, we discover that the dominance function $F_s(z)$ is identical to the poverty incidence curve $P_{s-1}(z)$ modulo a proportional factor that depends only on $s$. Stochastic dominance thus correspond to the generalisation of these indices when we let the poverty line $z$ vary over the whole segment $[0, +\infty[$. This is the point of view developed in Atkinson (1987) and in Foster and Shorrocks (1988). Let us note that the notion of poverty deficit curve is obtained when we let $zP_1(z)$ be a function of $z$.

The link with poverty indices is even more direct if we consider a notion of restricted dominance instead of a notion of full dominance. We no longer consider inequalities for all $x$, but inequalities for a restricted interval $[z_*, z^*]$. We thus consider:

$$F_s(z) = \frac{1}{(s-1)!}\int_0^z (z-t)^{s-1} f(t)dt \qquad \forall z \in [z_*, z^*].$$

This writing allows us to compare two income distributions when the poverty line varies between two boundaries. This leads to a robust comparison which will no longer be strictly depend on the definition chosen for the poverty line.

# 3 Testing for stochastic dominance

When we are interested in poverty, the meaningful concept is restricted stochastic dominance. Whenever we speak about poverty, we have to define a poverty line, using for instance half the mean or half the median. If we want to make robust comparisons, it is better to select a rather wide interval instead of just a point. We thus consider the interval $[z_*, z^*]$ which corresponds to two extreme reasonable values for the poverty line.

## 3.1 Hypotheses

We have two samples $A$ and $B$ for which we have computed two dominance curves at the order $s$ that we note $F_s^A(x)$ and $F_s^B(x)$ for the two samples.

For comparing the two distributions in term of stochastic dominance, we can distinguish three different type of hypothesis that can be in turn the null and the alternative in a classical framework. We have first to define the dominance function $\delta_s(x)$:

$$\delta_s(x) = F_s^A(x) - F_s^B(x),$$

which is a distance function between two distributions. Then:

1. $H_0 : \delta_s(x) = 0 \qquad \forall x \in [z_*, z^*]$. The two distributions corresponding to samples $A$ and $B$ cannot be distinguished.

2. $H_1 : \delta_s(x) =\geq 0 \qquad \forall x \in [z_*, z^*]$. The two distributions are ranked, distribution $B$ clearly dominates distribution $A$.

3. $H_2 : \delta_s(x)$ can be anything. There is no possibility to rank the two distributions.

If we were in an uni-dimensional framework, $H_0$ would correspond to a point hypothesis, $H_1$ would lead to a unilateral test and $H_2$ to a bilateral test. But here these hypotheses have to be verified for a fixed grid of equidistant points covering the interval $[z_*, z^*]$.

## 3.2 A classical test

Let us consider the following grid of $K$ equidistant points

$$z = [z_k] = z_*, z_2, \cdots, z_{K-1}, z^*,$$

and two independent samples from two independent populations $A$ and $B$. We want to compare these two populations. We have to consider the distribution of the estimated difference between the two dominance curves:

$$\hat{\delta}(x) = \hat{F}_s^A(x) - \hat{F}_s^B(x).$$

Davidson and Duclos (2000) have derived the asymptotic distribution of $\hat{\delta}(x)$ which is normal with zero mean and variance-covariance matrix the sum of the two variances when $A$ and $B$ are independent:

$$\sqrt{n}(\hat{\delta}_s(z) - \delta_s(z)) \sim \mathrm{N}(0, \Omega = \Sigma_A + \Sigma_B). \qquad (8)$$

Several dominance tests can be built on this result, depending on the choice of the null hypothesis. They however lead to complicated distributions reviewed in Dardanoni and Forcina (1999). However, a much simpler test can be built by testing $H_2$ of no restriction against $H_1$ of $\delta_s(x) \geq 0$. It consists in computing separately the $K$ values of the Student statistics of a significant difference and then to take their minimum. Consequently, our test statistics is:

$$T_{21} = \min_{z_i} \hat{\delta}(z_i)/\omega_{ii}.$$

This statistics has two advantages. It is easy to compute. Its asymptotic distribution is simple as it is a $\mathrm{N}(0,1)$ under the null. Davidson and Duclos (2013) recommend to use this test. They however show that it produces coherent results only if we truncate the tails of the distributions, regions where we have not enough observations. This trimming operation becomes natural when we test for restricted dominance provided the bounds $z_*$ et $z^*$ are adequately chosen.

### 3.3 A Bayesian test

There is a number of domains where classical and Bayesian procedures provide identical inference results. For instance the usual regression model under normality assumption for the error term and a non-informative prior. However, tests is the domain where the two approaches significantly differ. One of the reasons is that in a Bayesian framework we compute the posterior probability of an hypothesis and that there is no privileged hypothesis like the null hypothesis in the classical test theory of Neyman and Pearson.

In a Bayesian framework, testing for restricted stochastic dominance means first defining a grid of $np$ points over $[z_*, z^*]$ and then computing the posterior probability of:

$$\Pr(\delta(z|\theta)) \geq 0,$$

where $\theta$ is a parameter and

$$\delta(x|\theta) = F_s^A(x|\theta_A) - F_s^B(x|\theta_B).$$

The crucial point is of course to obtain an analytical expression for the parametric form $\delta(x|\theta)$. Then, the condition $\delta(z|\theta) \geq 0$ defines a logical vector of zeros and ones. It is equivalent to check any of the three conditions:

$$\prod_{i=1}^{np} \mathbb{1}[\delta(z_i|\theta) > 0] = 1 \tag{9}$$

$$\max_i \mathbb{1}[\delta(z_i|\theta) > 0] = 1, \tag{10}$$

$$\min_i \mathbb{1}[-\delta(z_i|\theta) < 0] = 1, \tag{11}$$

where $\mathbb{1}$ is the indicator function. Let us now suppose that we have managed to derive the posterior density of $\theta$, say $\varphi(\theta|x)$ and obtained $m$ draws from this posterior density by any simulation method. We have for instance:

$$\begin{aligned} \Pr\left(\max_z d(z|y) > 0\right) &= \int_\theta \mathbb{1}\left[\max_z \delta(z|\theta) > 0\right] \varphi(\theta|x)\, d\theta \\ &\simeq \frac{1}{m}\sum_{j=1}^{m} \mathbb{1}\left[\max_z \delta(z|\theta^{(j)}) > 0\right]. \end{aligned} \quad (12)$$

Once we have computed this posterior probability, we have all the information needed to compare two distribution by means of restricted stochastic dominance, which means that we have the posterior probability that there is less poverty in distribution $B$ than in distribution $A$ for a poverty range $[z_*, z^*]$.

For practical implementation, Lander et al. (2020) have used mixtures of gamma densities to model the income distribution in Indonesia. They develop Bayesian tests of stochastic dominance and restricted stochastic dominance. They compute posterior probabilities for stochastic dominance for the poorest 10% of the population, to assess whether their situation has improved over time. We shall see that comparing two GICs relates to first order stochastic dominance while comparing two TIP curves is another way for testing restricted stochastic dominance at the second order.

# 4 GIC dominance

Because a GIC represents the difference between two quantiles functions, it corresponds to the p-approach to dominance of Davidson and Duclos (2000). We have first-order stochastic if $g_t(p) > 0$ for all $p$. Growth has been welfare-improving in terms of first-order stochastic dominance if $g_t(p) \geq 0$ for all $p$ with strict inequality holding at least for one point $p$. We have restricted stochastic dominance if the range of $p$ is limited to $p \in [0, F(z)]$.

## 4.1 Pro-poor growth

For each point $p$ of a grid, Fourrier-Nicolaï and Lubrano (2021) evaluate:

$$\Pr(g_t(p) > 0) \simeq \frac{1}{m}\sum_{j=1}^{m} \mathbb{1}[g_t(p|\theta^{(j)}) > 0], \quad (13)$$

which allows us to see for which part of the income distribution the situation has been improved. The probability of dominance defined as:

$$\Pr(g_t(p) > 0) \simeq \frac{1}{m}\sum_{j=1}^{m} \mathbb{1}[\min_p(g_t(p|\theta^{(j)})) > 0]. \tag{14}$$

A further requirement is that growth has been favourable to the poor, leading to the vector corresponding to $p \in [0, F(z)]$:

$$\Pr(g_t(p) > \gamma) \simeq \frac{1}{m}\sum_{j=1}^{m} \mathbb{1}[g_t^{(j)}(p) > \gamma^{(j)}], \tag{15}$$

where

$$\gamma^{(j)} = \log\sum_k \eta_{k,2}^{(j)} e^{\mu_{k,2}^{(j)}+\sigma_{k,2}^{2(j)}} - \log\sum_k \eta_{k,1}^{(j)} e^{\mu_{k,1}^{(j)}+\sigma_{k,1}^{2(j)}} \tag{16}$$

is the $j^{th}$ draw of the average growth rate between $t$ and $t-1$ when the two income distributions are modelled as a mixture of lognormals.

## 4.2 Trickle down theory and the UK case

In the previous chapter, we have derived various GIC to analyse the impact of economic growth in the UK over the period 1979-1996 under the government of Margaret Thatcher. We now report probabilistic judgements coming from Fourrier-Nicolaï and Lubrano (2021).

Table 1 provides probabilities of pro-poor growth computed using (15) for each decile for the periods 1979-1988 and 1992-1996, using three different parametric models. We keep in mind that the mixture models should be the closest one to the distribution free approach as it corresponds to a semi-parametric approach and thus represent the case with minimum bias. The three models deliver the same probability of anti-poor growth up to the $0.60^{th}$ quantile and the same probability of pro-rich growth from the $0.90^{th}$ quantile for 1979-1988. There are however differences for the $0.70^{th}$ quantile where the log-normal model over-evaluates the probability that this quantile has benefited more from growth, nearly crossing the mythical value of 0.50. When we consider the second period (1992-1996), Kakwani (1980)'s form is fairly well in accordance with the mixture model. The log-normal model provides slightly different probabilities for quantiles greater than 0.60, but do not contradict the essential message when 0.50 is taken as the reference to decide if growth is pro-poor or not. In summary, the extra parameter of

Table 1: Probability of pro-poor growth, whole sample

| $p$ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1979-1988 | | | | |
| Log-normal | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.48 | 0.97 | 1.00 |
| Kakwani | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.75 | 1.00 |
| Mixture | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.29 | 0.93 | 1.00 |
| | | | | | 1992-1996 | | | | |
| Log-normal | 1.00 | 1.00 | 1.00 | 1.00 | 0.99 | 0.93 | 0.64 | 0.18 | 0.00 |
| Kakwani | 1.00 | 1.00 | 1.00 | 0.97 | 0.91 | 0.76 | 0.53 | 0.28 | 0.06 |
| Mixture | 1.00 | 1.00 | 1.00 | 0.99 | 0.93 | 0.77 | 0.58 | 0.39 | 0.19 |

the Kakwani (1980)'s model allows to provide more reliable conclusions than the simple log-normal model.

Table 1 can also be read in more restrictive way. Pro-poor growth is defined in (15) as the probability that the lower quantiles increase more than average up to the quantile defined by the poverty rate in the first period. Poverty rates for 1979 and 1992 are respectively 0.135 and 0.196, using the poverty line defined as 60% of the median income. This implies that Table 1 can be limited to the first two columns to decide if growth was pro-poor or not. The answer becomes unambiguous for all models.

Table 2: Probability of pro-poor growth for 1979-1988 using the Kakwani (1980)'s GIC

| $p$ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1979-1988 | | | | |
| Retired | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.26 | 0.78 | 0.99 | 1.00 |
| Working | 0.24 | 0.66 | 0.89 | 0.97 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |
| Unemployed | 0.04 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Lone parents | 0.06 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| | | | | | 1992-1996 | | | | |
| Retired | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Working | 0.67 | 0.47 | 0.45 | 0.47 | 0.51 | 0.54 | 0.56 | 0.54 | 0.43 |
| Unemployed | 1.00 | 1.00 | 0.99 | 0.95 | 0.74 | 0.35 | 0.09 | 0.01 | 0.00 |
| Lone parents | 1.00 | 0.99 | 0.99 | 0.99 | 0.99 | 0.98 | 0.97 | 0.95 | 0.86 |

We complete this picture by computing pro-poor growth probabilities for different subgroups as reported in Table 2. During the first period, retired people gained more than average after the $0.70^{th}$ decile while the working group gained more than average starting from the $0.20^{th}$ decile. Unemployed and lone parents gained less than average for all deciles. During the second period, all the deciles gained more than average for retired and lone parents. Unemployed gained more than average up to the median. The probabilities for the working group are un-conclusive.

# 5 TIP dominance

There is a close relation between GIC dominance and first order stochastic dominance simply because a GIC is the difference between two quantile functions. We have the same type of relation, but at a lesser extend, between TIP dominance and second order stochastic dominance, but this time restricted stochastic dominance, just because a TIP curve is essentially a poverty deficit curve. Remember what we said about the relation between poverty deficit curves and stochastic dominance.

## 5.1 Stochastic dominance and TIP dominance

Robust poverty ranking can be obtained using the poverty deficit curve obtained when (1) is seen as a function of $z$ and letting $z$ vary within a given interval. This corresponds to the notion of restricted stochastic dominance of Atkinson (1987), at the order 2 when $\alpha = 1$. This is the primal approach to stochastic dominance. The dual approach to stochastic dominance consider quantiles and the order 2 corresponds to Generalised Lorenz ordering. As it is related to the Generalised Lorenz curve, the TIP curve provides a natural framework for testing restricted second order stochastic dominance. We have however to show how. We first propose a definition of TIP dominance and then explore what it implies in term of stochastic dominance.

**Definition 3** *Let us consider two income distributions corresponding to populations $A$ and $B$ and a common poverty line. Population $A$ TIP dominates $B$ if $TIP_A(p,z) \leq TIP_B(p,z)$ $\forall p \in [0,1]$, and the strict inequality holds at least for one $p$. The strict TIP dominance requires that this inequality is strict for all $p$.*

**Remark:**

> TIP dominance according to the definition given in the 4th footnote of Jenkins and Lambert (1997) implies that there is more poverty in $A$ than in $B$ if $A$ TIP dominates $B$. This might appear counter-intuitive when confronted to stochastic dominance. Thus, in our context, TIP dominance will mean less poverty as in Thuysbaert (2008).

Jenkins and Lambert (1998), with their theorem 1 provide a relation between TIP dominance and poverty ordering. Their theorem could be rephrased as follows (see also Thuysbaert 2008):

**Theorem 1** *Let us consider two TIP curves and a common poverty line. The following two conditions are equivalent:*

1. $TIP_A(p, z) \leq TIP_B(p, z)$ *for all* $p \in [0, 1]$
2. $P_A(\lambda z) \leq P_B(\lambda z)$ *for all* $\lambda \in [0, 1]$,

*where* $P(z)$ *is the poverty deficit curve defined in (3).*

This theorem means that TIP dominance is equivalent to restricted stochastic dominance at the order 2 over the range $[0, z]$, which means for all poverty lines lower or equal to $z$. In other words, if $TIP_A(p, z)$ is always below $TIP_B(p, z)$ with a common $z$, then there is less poverty intensity and less inequality among the poor in $A$ than in $B$ *for all common poverty lines smaller than or equal to* $z$. However, we cannot say anything about poverty headcount. Furthermore and like for Lorenz curves, when TIP curves intersect there is indeterminacy since the poverty ranking can be reversed for some values of $p$. So no ranking can be provided in this case.

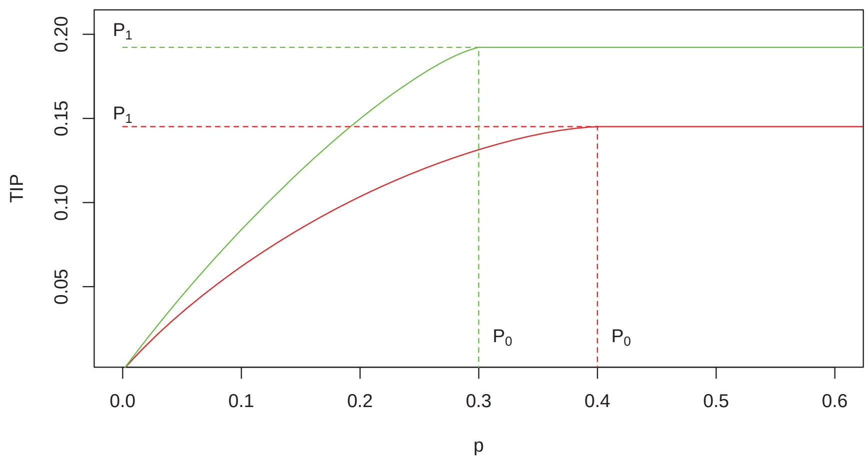


Figure 1: TIP dominance and no first order stochastic dominance

A situation of TIP dominance is illustrated in Figure 1. The red curve TIP dominates the green curve *using a common* $z$. Then the green curve exhibits more poverty intensity $P_1$ than the red curve *for any poverty line smaller than* $z$ and the ranking cannot be reversed for a smaller value of $z$. However, there is more poverty incidence $P_0$ in the red curve, so that we cannot rank the two distributions in term of global poverty even if there is restricted stochastic dominance at the second order.

## 5.2 A Bayesian test

Testing for TIP dominance in a Bayesian frameworks leads first to compute for each draw of $\theta$ a vector $\delta(p|\theta)$ of dimension $k$ corresponding to the grid over $p$:

$$\delta(p|\theta) = TIP_A(p, z|\theta_A) - TIP_B(p, z|\theta_B).$$

The condition $\delta(x, p|\theta) \leq 0$ defines a logical vector of zeros and ones. It is then equivalent to check any of the three conditions:

1. $\prod_{i=1}^{k} \mathbb{1}[\delta(p_i|\theta) < 0] = 1,$
2. $\max_i \mathbb{1}[\delta(p_i|\theta) < 0] = 1,$
3. $\min_i \mathbb{1}[-\delta(p_i|\theta) > 0] = 1.$

So for instance:

$$\begin{aligned}\Pr\left(\max_p d(p|y) < 0\right) &= \int_\theta \mathbb{1}\left[\max_p \delta(p|\theta) < 0\right] \varphi(\theta|y) d\theta \\ &\simeq \frac{1}{m}\sum_{j=1}^{m} \mathbb{1}\left[\max_p \delta(p|\theta^{(j)}) < 0\right], \end{aligned} \tag{17}$$

where $\varphi(\theta|y)$ is the posterior density of $\theta$. The range of $p$ has to be slightly restricted because all TIP curves are zero at $p = 0$. So the practical range for the test should be something like $p \in [0.01, F(z)]$, values adopted in e.g. Davidson and Duclos (2013).

Because TIP dominance corresponds to restricted second order stochastic dominance, TIP dominance does not imply less poverty incidence. Using $H(z|\theta) = P_0(z)$, we have to check the additional condition $H(z|\theta_A^{(j)}) < H(z|\theta_B^{(j)})$ and evaluate the proportion of draws when it is verified.

Finally, when can we say that the situation in $A$ is not statistically different from the situation in $B$? Equality is rejected if, for at least one value of $p_s$, $\delta(p_s|\theta)$ is statistically different from zero. This means that we have to compute a credible interval for $\delta(p_s|\theta)$ and see if zero is included in this interval. If we find a single $p_s$ for which zero does not belong to a say 90% credible interval for $\delta(p_s|\vartheta)$, then we can reject at the 90% level that the two TIP curves are equal.

## 5.3 Child poverty in Germany

We have detailed in the previous chapter a picture of child poverty in Germany, distinguishing between two periods 2007-2011 and 2002-2006. It increased a lot between 2002 and 2006 to finally decrease between 2007 and 2011. Formal dominance tests confirm this diagnostic. The probability that 2007-2011 TIP dominates 2002-2006 is equal to 0.997. For current child poverty, Table 3, shows that 2011 dominates all the other years, which means that child poverty is lowest for this year. Child poverty has significantly increased between 2002 and 2006 as 2002 TIP dominates 2006 at 91% while it decreases after that date because 2007 TIP dominates 2006 at 94%.

Table 3: Probability of TIP dominance for current child poverty

| Year | 2002 | 2006 | 2007 | 2011 |
|---|---|---|---|---|
| 2002 | 0.000 | 0.907 | 0.260 | 0.000 |
| 2006 | 0.003 | 0.000 | 0.002 | 0.000 |
| 2007 | 0.525 | 0.939 | 0.000 | 0.000 |
| 2011 | 1.000 | 1.000 | 1.000 | 0.000 |

Each line represents the probability that the corresponding year TIP dominates the year given in column.

## 5.4 The difference between child and adult poverty

So the evolution of poverty over the two subperiods concerned mainly children with an increase and a decrease, while there was no significant effect on the population of adults. This is confirmed by TIP dominance tests reported in

Table 4: Probability of TIP dominance for current adult poverty

| Year | 2002 | 2006 | 2007 | 2011 |
|---|---|---|---|---|
| 2002 | 0.000 | 0.241 | 0.228 | 0.215 |
| 2006 | 0.496 | 0.000 | 0.435 | 0.370 |
| 2007 | 0.194 | 0.150 | 0.000 | 0.135 |
| 2011 | 0.493 | 0.403 | 0.424 | 0.000 |

Each line represents the probability that the corresponding year TIP dominates the year given in column.

Table 4. There is no convincing probability of Tip dominance of any year for adult poverty. So child poverty has changed a lot over the period, while adult poverty remained at a comparative level.

# 6 Conclusion and discussion

Stochastic dominance and more precisely restricted stochastic dominance, allows us to make unambiguous judgements on poverty comparisons. We have not the same result when using simple poverty indices, because they can be very sensitive to the chosen poverty line. We have understood that testing for stochastic dominance is a complicated topic in a classical framework. This is due to the fact that we have to make simultaneous tests on a whole range of values, so defining the null hypothesis can be cumbersome. On the contrary, in a Bayesian framework we simply have to compute a posterior probability and there is no privileged hypothesis. Computing a posterior probability is particularly simple in a parametric framework, once we have a MCMC output for the parameters of the income distribution.

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