The Bayesian approach to poverty measurement

Lecture 6: Poverty dynamics

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1 Introduction

Up to now, we have considered individual survey data for different years, but we never assumed that the same individual was observed over time. Poverty dynamics means that we should be able to observe individual trajectories. For this we need panel data surveys. There are not many of such surveys. In Europe, the most well known panels are the BHPS for the UK and the GSOEP for Germany. We should also mention the Euro-SILC which is a program led by the European Commission (Eurostat) to have a European panel in order to be able to compare living conditions in Europe. The cross-section part of the Euro-SILC is very complete. But the panel version of this survey is upsetting.

It is difficult to maintain a panel over time, just because individuals are born, grow up, get married and leave the parent's house, have children and then die. Sometimes they refuse to answer the survey, either because they have moved, or because they are ashamed of their sudden fall into poverty. This phenomenon is known under the name of attrition. It can be at random, when individuals move for instance. In the case of missing at random, there is no specific statistical problem. However, in some cases, for instance being ashamed of having fallen into poverty, they are not missing at random. In this case, a model should explain why they are missing in order to avoid a selection bias.

When speaking about poverty dynamics, we can think about a two-states dynamic process, being poor or not, running over time. Then imagine a transition matrix depicted in Table 1. Each line represents poverty status in $t-1$ and each column the status at time t (the next period). Ravallion

Table 1: Poverty dynamics

	Poor	non Poor
Poor	Poverty persistence	Poverty exit
non Poor	Poverty entry	-

(1988) the transition into and out of poverty and the duration of spells of poverty. transient and persistent poverty;

Cappellari and Jenkins (2004) propose a model for explaining transition between states, taking into account attrition, over two periods. Ravallion (1988) propose a decomposition of observed poverty on a longer term in chronic and transitory poverty. Suppose that we have a panel of ten years. Bane and Ellwood (1986) say that chronic poverty is when an individual stays a long time in the state of poverty, in other terms the individuals experience long spells of poverty. Transitory poverty is when those spells are rather short. Another aspect of poverty dynamics is discussed in Rodgers and Rodgers (1993) where transitory poverty is explained by the possibility of transferring income from one period to the other.

The Bayesian literature is rather scarce for studying poverty dynamics. Hasegawa and Ueda (2007) propose to model individual incomes as a stationary process and derive the distribution of Ravallion (1988) decomposition of poverty into total, chronic and transitory poverty, using panel data. Panel data sets are seldom available in developing countries where the analysis of poverty should be of prime importance. Sadeq and Lubrano (2018) develop a pseudo panel approach to analyse the impact of the Wall on poverty entry and poverty persistence in the West Bank.

Statistical inference for measuring mobility relies first on the definition of income classes. There are three ways of doing this as detailed in Formby et al. (2004). Depending on the chosen way, mobility measurement may present opposed characteristics: absolute, relative to the mean or the median and finally transition matrices based on quantiles. Matrices can be estimated directly using panel or can be the result of an ordered multinomial probit. Important concepts with Bane and Ellwood (1986), Kuchler and Goebel (2003). Long panel are needed. With Cappellari and Jenkins (2004), two years are enough.

2 Poverty decomposition

Poverty means having an income lower than the poverty line. However, which income should we take into account? An individual can be in a transient state of poverty if her income temporally falls below the poverty line. But if her average income over a longer period is permanently lower than the poverty line, then she is in a chronic state of poverty.

2.1 Ravallion decomposition of poverty

Ravallion (1988) was the first to consider a decomposition of poverty. He was interested in measuring the impact of risk on poverty in India, and there risk was essentially weather variation and dryness, measured by η. Ravallion wants to address the question to know if variability is good or bad for individual welfare and poverty. The level of poverty is supposed to be an increasing function $P(\eta)$ of a climatic index. Welfare y is measured as a function of a constant income x or steady state income while $\nu(\eta)$ measures the deviation of real income to the steady state income. So welfare is supposed to be an additive function:

$$y = \phi(x + \nu(\eta)).$$

The impact of variability of income on poverty is measured by

$$P(\eta) = \int_0^z p(y, z)\, f(y)\, dy.$$

The chosen and most convenient poverty measure is the FGT because of its separability properties:

$$p(y, z) = (1 - y/z)^\alpha.$$

This is the theoretical framework that allow Ravallion (1988) to make the distinction between chronic and transitory poverty.

In his application on Indian data, he makes use of a panel with n individuals and T period, with incomes y_{it}. Every year poverty is measured by:

$$P_{\alpha,t} = \frac{1}{n} \sum_{i=1}^{q_t} (1 - y_{it}/z)^\alpha$$

where the y_{it} have been ordered and q_t is such that for $i = q_t$ the income is equal to z. Total poverty corresponds to:

$$\overline{P}_\alpha = \frac{1}{T} \sum_t P_{\alpha,t},$$

while chronic poverty corresponds to poverty measured at the value of average income over the period

$$P_\alpha^* = \frac{1}{n} \sum_{i=1}^{q^*} (1 - \bar{y}_i/z)^\alpha.$$

In fact, Ravallion is interested in measuring the cost of the variability of income, which means by how much the average income $\bar{y}$ would have to be decreased in order to match total poverty. This value is τ in the following equation

$$\overline{P}_\alpha = \frac{1}{n} \sum_{i=1}^{q^*} (1 - \bar{y}_i/z - \tau/z)^\alpha.$$

We implicitly a decomposition of total poverty $\overline{P}_\alpha$ into chronic poverty P_α^* and transient poverty obtained by difference. The distinction between transient and chronic poverty is essential for implementing public policies.

2.2 Smoothed income poverty

Rodgers and Rodgers (1993) give the following example which is quite illuminating for understanding the difference between chronic and transient poverty.

Example 1 *Let us suppose that the poverty line is $z = 100$.*

Person	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Year 6*
A	*300*	*300*	*300*	*99*	*99*	*300*
B	*101*	*101*	*101*	*10*	*10*	*101*
C	*300*	*300*	*99*	*99*	*99*	*300*

If we measure the duration of poverty spells, person C has the longest poverty spell compared to A and B. But if we suppose that a person can smooth his income over the years, spare money and transfer money to the next period, then clearly person B is in a state of chronic poverty.

We shall consider a linear decomposable poverty index and a period of length T. The most common choice is the FGT index. We first compute that index for every period and note it P_t for a given poverty line z_t. Then we have the following definitions of average, chronic and transitory poverty rates:

1. *Average annual poverty rate.* It is defined as a weighted sum of annual poverty measures. Most of the time the weights w_t are equal to $1/T$. This average is possible because the index P is linearly decomposable:

$$A_P(T) = \sum_{t=1}^{T} w_t P_t.$$

   There is no possible inter year income transfers. This is in a way the maximum poverty rate as said in Hill and Jenkins (2001).

2. *Chronic poverty.* We now assume that it is possible to transfer income between the years. We call Y_i^* the permanent income of person i. There are n individuals in the sample. So chronic poverty is defined as a poverty index applied to the series of the n smoothed incomes:

$$C_P(T) = P(Y_1^*, \cdots, Y_n^*).$$

   Rodgers and Rodgers (1993) have a complicated way of computing the permanent income. Hill and Jenkins (2001) and Kuchler and Goebel (2003) use a much simpler formula.

3. *Transitory poverty.* As the poverty index P is supposed to be linearly decomposable, transitory poverty can be found using a difference

$$T_P(T) = A_P(T) - C_P(T).$$

A positive $T_P(T)$ represents the amount of poverty which is not chronic for an average year. Negative values are possible according to Rodgers and Rodgers (1993), depending on the chosen P and the way permanent income is computed.

2.3 Poverty dynamics in Europe

Kuchler and Goebel (2003) start from the relative income position of individual i in the sample of size n at time t which is

$$y_{it}^r = \frac{y_{it}}{\bar{y}_t}, \quad \bar{y}_t = \frac{1}{n}\sum_{i=1}^{n} y_{it}.$$

Dividing by the sample mean allows to avoid having to divide by a price index. Incomes are made comparable using the modified OECD scale. Using these data, it is possible to compute average annual poverty, the index $A_{FGT}(T)$ of Rodgers and Rodgers (1993), using the data displayed in Table 1 of Kuchler and Goebel (2003). The poverty line is 50% of the average y_{it}^r. This is called total poverty in Hill and Jenkins (2001). So Table 2 represents total maximum poverty. Quite different pictures of total poverty are obtained when considering incidence or intensity. Denmark and the Netherlands have the smallest poverty incidence. At the other extreme, Spain, Ireland Greece and Portugal have the highest poverty incidence. However, when considering the intensity of poverty, Denmark and the Netherlands remain in the group where intensity is the smallest, but they are joined by France and by Ireland. Greece and Portugal remain in the group where poverty is highest. But they are joined by Italy, the UK and Spain.

Let us now turn to chronic poverty which aims at measuring poverty when we allow for inter-temporal income transfers. Smoothed or permanent income can be computed in different ways. Rodgers and Rodgers (1993) adopt a complicated mechanism based on borrowing and lending which might lead to apparent incoherencies (negative transitory poverty). The later literature adopted some kind of smoothing. We could imagine exponential smoothing, non-parametric smoothing following the time series literature where a topic is the decomposition of a time series in permanent and cyclical components. Kuchler and Goebel (2003) adopt the simplest way to define permanent income, using in fact just the mean income, resulting in a single value for each

Table 2: Average annual income poverty: 1994-1997

Total poverty

Country	Incidence P_0	Intensity $P_1 \times 10$	Inequality $P_2 \times 100$
Denmark	5.63	1.28	0.54
Netherlands	9.98	3.83	2.43
Germany	13.85	4.98	3.01
France	14.68	3.83	1.84
Italy	16.95	6.40	3.96
Belgium	16.45	4.85	2.55
UK	18.25	6.73	4.08
Spain	19.10	6.58	3.68
Ireland	20.10	3.95	1.47
Greece	21.38	7.63	4.04
Portugal	24.53	8.43	4.55

Source: Kuchler and Goebel (2003), Table 1 and own calculations.

individual. So the time dimension is compressed. In a panel of size T, the smoothed relative income position of individual i is:

$$\bar{y}_i^r = \frac{1}{T} \sum_{t=1}^{T} y_{it}^r.$$

The poverty line will be defined now as 50% of the mean smoothed relative income position. It results the following picture of chronic poverty as depicted in Table 4. Chronic income poverty is a minor phenomenon in Denmark, and also in the Netherlands; while Portugal and Greece are at the other extreme. Between total and chronic poverty, the ranking does not change, except for France which has a higher chronic poverty as measured by P_0. It is interesting to analyse and compare three countries which can look similar: France, Germany and the UK. We have already compared France and the UK in the previous section, using different data and periods. For Germany, we are well before 2003, the time when Gerhard Schröder launched his cuts in the social welfare system. And for the UK, we are well after the Thatcher's period. For total poverty, the UK is well above Germany and France while France has the lowest intensity and inequality. For chronic poverty, France and the UK have very similar incidence, well above that of Germany. For chronic poverty severity and inequality, the UK is in the least favourable position, while France and Germany become comparable.

By inspecting the proportion of chronic poverty over total poverty, we can

Table 3: Smoothed or chronic income poverty: 1994-1997

Country	Incidence P_0	Intensity $P_1 \times 10$	Inequality $P_2 \times 100$
Denmark	2.4	0.2	0.04
Netherlands	6.1	1.0	0.38
Germany	8.2	2.0	0.91
France	13.8	2.2	0.55
Ireland	17.1	2.3	0.48
UK	13.5	2.8	1.03
Belgium	13.1	3.1	1.28
Italy	12.4	3.2	1.38
Spain	14.8	3.6	1.43
Greece	17.5	4.6	1.77
Portugal	21.6	6.7	3.11

Source: Kuchler and Goebel (2003), Table 2.

Table 4: Proportion of chronic poverty over total poverty: 1994-1997

Country	Incidence P_0	Intensity $P_1 \times 10$	Inequality $P_2 \times 100$
Denmark	43.67	15.69	07.37
Netherlands	61.16	26.14	15.62
Germany	59.21	40.20	30.28
UK	73.97	41.64	25.23
Italy	73.16	50.00	34.87
Spain	73.40	54.75	38.89
France	94.04	57.52	29.97
Ireland	85.08	58.23	32.77
Greece	81.87	60.33	43.81
Belgium	79.64	63.92	50.30
Portugal	88.07	79.53	63.32

Source: Kuchler and Goebel (2003), Table 1 and 2 and own calculations.

have an idea of social mobility among the poor. Table 4 shows that again Denmark and the Netherland have the best position according to intensity. Portugal and Greece are at the bottom, but paradoxically with Belgium.

3 A first Bayesian approach

In the paper of Ravallion (1988), nothing is said about the choice of α and about income formation. Hasegawa and Ueda (2007) tackled one of these issues in a Bayesian framework. They assume that:

$$y_{it} = \mu_i + u_{it}, \qquad i = 1, \dots, n, \quad t = 1, \dots, T, \tag{1}$$

where y_{it} be income for individual i at time t, μ_i represents the steady-state or long term income while u_{it} denotes its transient component. For FGT poverty indices expressed as the discrete counterpart of the poverty incidence curve depicted in the previous chapters:

$$\pi(y_{it}, z) = \sum_{i=1}^{n} (1 - y_{it}/z)^{\alpha} \mathbb{1}(y_{it} < z),$$

total, chronic and transient poverty are measured by:

$$\begin{aligned} \text{Total poverty} \quad \pi_F(z) &= \frac{1}{T} \sum_t \pi(y_{it}, z) & (2) \\ \text{Chronic poverty} \quad \pi_C(z) &= \frac{1}{n} \sum_i (1 - \mu_i/z)^{\alpha} \mathbb{1}(y_{it} < z) & (3) \\ \text{Transient poverty} \quad \pi_T(z) &= \pi_F(z) - \pi_C(z). & (4) \end{aligned}$$

To go from a descriptive point of view to an inferential point of view, Hasegawa and Ueda (2007) model income by a mixture of k lognormal distributions for each individual i, assuming μ_i constant over time, but adding an error-in-variable mechanism. They derive the posterior predictive distribution of y_{it}, $p(\tilde{y}|y)$ and use simulations of $\tilde{y}$ to estimate poverty indices with $\hat{\mu}_i = \sum_t \tilde{y}_{it}/T$. In this way, for each draw of $\tilde{y}$we get a draw of total, chronic and transient poverty. We can use these draws to form a posterior density of each of these poverty components.

3.1 An alternative income model

An alternative possibility would be to consider (1) as a panel data model with random individual effects. Let us define the vector of observations for an individual $y_i = [y_{i1}, \dots, y_{iT}]$, the basic panel data model with random effects of Chib (1996) writes:

$$\begin{aligned} y_i &= \iota\mu_i + X_i\beta + u_i, \qquad u_i \sim N(0, \sigma^2 I_T), & (5) \\ \mu_i &\sim N(0, \omega^2), & (6) \end{aligned}$$

where ι is a T vector of ones. With a common random effect μ_i, the T incomes of individual i become correlated around the individual effect with:

$$\text{Var}(y_i|\beta, \sigma^2, \omega^2) = \sigma^2 I_T + \iota\iota'\omega^2 = V,$$

so that:

$$y_i \sim N(X_i\beta, V).$$

Bayesian inference on β, σ^2 and ω^2 is obtained with a Gibbs sampler corresponding to algorithm 2 of Chib and Carlin (1999) with an informative prior on σ^2 and ω^2 to ease convergence.

With a MCMC output for $\beta^{(j)}$, $\sigma^{2(j)}$ and $\omega^{(j)}$, we can simulate m random draws for y_i using:

$$y_i^{(j)} \sim N(X_i\beta^{(j)}, \sigma^{2(j)} I_T + \iota\iota'\omega^{2(j)}).$$

We then transform each nT vector $y^{(j)} = [y_i^{(j)}]$ together with the by-product $\mu_i^{(j)}$ into:

$$\pi_F^{(j)}(z) = \frac{1}{nT}\sum_{i,T}(1 - y_{it}^{(j)}/z)^\alpha \mathbb{1}(y_{it}^{(j)} < z), \tag{7}$$

$$\pi_C^{(j)}(z) = \frac{1}{n}\sum_{i}(1 - \mu_i^{(j)}/z)^\alpha \mathbb{1}(\mu_i^{(j)} < z). \tag{8}$$

We have thus m posterior draws of the three poverty indices and compute standard error for each of them.

3.2 Chib's algorithm for panel with random effects

The idea of algorithm 2 of Chib and Carlin (1999) is to simulate jointly β and μ_i because we have for β:

$$\beta|y, \sigma^2, \omega^2 \sim N(\hat\beta, M), \tag{9}$$

with

$$\hat\beta = \sigma^2 M \sum_i X_i' V_i^{-1} y_i, \tag{10}$$

$$M = \sum_i X_i' V_i^{-1} X_i)^{-1}/\sigma^2. \tag{11}$$

Once we have a conditional draw of β, the conditional posterior density of μ_i is easily obtained because conditionally on β we have in (5) a simple regression model, with however the sum of two error terms with variance σ^2 and ω^2. The conditional posterior density of μ_i is thus a normal density. Then we can sample alternatively:

$$\omega^2|y,\beta,\mu,\sigma^2 \quad (12)$$

$$\sigma^2|y,\beta,\mu,\omega^2 \quad (13)$$

3.3 A different approach using mixtures

We have seen in Chapter 3 how to model the income distribution using mixtures of lognormal densities and in Chapter 4 how to draw random numbers from the FGT indices. The procedure is rather different than before. Instead of simulating values of y_{it}, we use random draws from the parameters of the estimated distribution of two types of income: the annual observed income y_{it} for which we thus need to make inference on T different income distributions; the average income $\bar{y}_i$ which s taken as a proxi for the permanent income. Using these $T+1$ MCMC output, we can simulate poverty rates.

For the average annual poverty rate representing total poverty:

$$A_P(T|\theta^{(j)}) = \sum_{t=1}^{T} FGT(z|\theta_t^{(j)}),$$

where $\theta_t^{(j)}$ is the j^{th} draw from the posterior density of the estimated mixture for period t.

For chronic poverty, we use the MCMC output from the mixture estimated on $\bar{y}_i$. So:

$$C_P(T|\theta^{(j)}) = FGT(z|\theta^{(j)}),$$

where $\theta_t^{(j)}$ is the j^{th} draw from the posterior density of the estimated mixture over $\bar{y}_i$.

Finally, for transitory poverty,

$$T_P(T|\theta^{(j)}) = A_P(T|\theta^{(j)}) - C_P(T|\theta^{(j)}).$$

We finally get m draws for the indices of total, chronic and by substraction transitory poverty. Having obtained these draws, we can plot the posterior density of each of these indices, compute posterior means and standard deviations. This approach goes through the analytical derivation of the FGT poverty indices in the lognormal case. So a draw from the posterior density of a poverty index is seen as the transformation of a draw from the posterior density of the modelled income distribution.

The approach depicted in the literature that we have reported above proceed in a different way. It derives the predictive density of y_{it} and then transforms a draw of a predicted $y_{it}|\theta^{(j)}$, using the usual empirical formulae for the poverty index.

4 TIP curves and poverty decomposition

We have shown how to decompose total poverty in chronic and transitory poverty. However, the empirical results concerning poverty decomposition in Europe have shown that we could obtain different rankings, depending on the chosen value of α for computing FGT indices. We should look for a tool which manages to present the three I's of poverty at the same time. This is the TIP curves introduced by Jenkins and Lambert (1997), curves that we have detailed in chapter 4. Thuysbaert (2008) was the first to develop statistical inference for TIP curves in a classical framework. For instance, he provides TIP curves together with a confidence bands for Belgium and tests of TIP dominance.

4.1 TIP curves and chronic poverty

Considering a panel of n individuals and T periods, we observe current income y_{it}. The usual TIP curves is defined as:

$$TIP_t(p,z) = \int_0^{F_t^{-1}(p)} (1 - y/z)\mathbb{1}(y \le z) f_t(y) dy,$$

where $f_t(y)$ is the distribution of individual income for period t. Without defining it explicitly, Kuchler and Goebel (2003) consider the individual permanent income to compute the TIP curve corresponding to chronic poverty:

$$TIP(p,z) = \int_0^{F^{-1}(p)} (1 - \bar{y}/z)\mathbb{1}(\bar{y} \le z) f(\bar{y}) dy,$$

where $f(\bar{y})$ is the distribution of average income of each individual.

4.2 Bayesian inference for TIP curves

Here again, we can have two solutions. We could model the permanent income distribution, that of $\bar{y}_i$, derive its predictive density and then evaluate a draw of the TIP curve using a draw $\tilde{y}|\theta^{(j)}$ of size n from the predictive density:

$$TIP(p, z|\theta^{(j)}) = \frac{1}{n}\sum_{i=1}^{n}(1 - \tilde{y}_i|\theta^{(j)})\mathbb{1}(\tilde{y}_i|\theta^{(j)} < z).$$

We have chosen a different route which consists in finding a parametric representation of the TIP curve explained in Chapter 4, section 4.2, and which is:

$$\begin{aligned} TIP(p, z|\theta^{(j)}) \quad = \quad & \sum_{k=1}^{K}\eta_k^{(j)}\left[\Phi\left(\frac{\log q^{(j)} - \mu_k^{(j)}}{\sigma_k^{(j)}}\right)\right. \\ & \left. - \quad \frac{1}{z}e^{\mu_k^{(j)}+\sigma_k^{2(j)}/2}\Phi\left(\frac{\log q^{(j)} - \mu_k^{(j)} - \sigma_k^{2(j)}}{\sigma_k^{(j)}}\right)\right]. \end{aligned} \quad (14)$$

where μ_k, σ_k and η_k are the parameters of the mixture of lognormals explaining the permanent income $\bar{y}_i$.

The difficulty is that the left-hand side is a function of p while the right-hand side is a function of q. For each draw of θ, we have to solve numerically the equation:

$$F(q^{(j)}|\theta^{(j)}) = p, \quad (15)$$

for each point of a predefined grid on p. This is a feasible problem because it is of dimension one on a finite interval defined by the range of x. Brent (1971) algorithm is very efficient in this case.

5 Child poverty using the GSOEP

The GSOEP is a socio-economic panel provided by the German Institute for Economic Research in Berlin (DIW). It is a representative sample of households living in Germany since 1983 and including former East German households after the reunification. For the period 2000-2012, each wave covers on average 11 623 households. Among these, on average 3 154 households have children and their average number of children is 1.48 so that on average, the sample contains 4 680 children in each wave.

Two types of income can be reconstructed: an annual *market income* which represents labour income, capital income, in fact all incomes coming from a market activity; a *disposable income* which is the market income minus taxes and plus redistribution including unemployment benefits, social security pensions, family allowances and all remaining forms of social redistribution. We consider the real disposable income obtained by dividing the current income by the Consumer Price Index (2005) provided in the GSOEP.

In order to keep coherency with the paper of Corak et al. (2008), we define the poverty line as 50% of the sample median disposable income (taking into account all households, those having children and those without children). Remark that the disposable income is normalised by the new OECD equivalence scale and expressed in real terms. The average poverty line is 8 455 euros per year and per equivalent adult over the whole period. The annual poverty line is slightly greater than this value before 2006 and slightly below after that date. But the fluctuation is less than 2%, so we have decided to keep the same poverty line over the entire period, which is more convenient to implement tests of TIP dominance.

5.1 Child and adult poverty in East Germany

Using the data set of Fourrier-Nicolaï and Lubrano (2020), we analyse how social transfers were alleviating child poverty compared to adult poverty in East Germany over the period (2002-2006), just before the most important social and redistributive reforms introduced by the Hartz plan in 2006. We consider both disposable and market incomes (after taxes and transfers including family allowances or before taxes and allowances, divided by the new OECD equivalence scale) to build a five year balanced panel. We had 500 children and 1 466 adults without children. The poverty line is defined as 50% of the corresponding median income.

We adjusted a panel data model on the log of the income-to-need ratio, explained by an intercept, the household size and the number of children in the household (without the number of children for the adult sample). We used an informative inverted gamma2 prior on ω^2 with prior mean 0.25, 1 000 draws plus 100 for warming the chain. Posterior draws were then used to simulate incomes and poverty indices with results reported in Table 5. Before taxes and transfers, there is much more poverty among adults as if poor adults had decided not to have children. Poverty among adults is mostly chronic when it is mainly transitory among children. Poverty intensity is also stronger among adults while being mostly transient. When taxes and transfers are introduced, total poverty is much reduced, but the reduction is more important among adults than among children. With transfers, child and adult poverty become mainly transient while chronic poverty intensity is reduced to very low levels. We have thus a contrasted impact of social transfers on the dynamic of poverty in East Germany for that period. As underlined in Fourrier-Nicolaï and Lubrano (2020), the major changes introduced after the Hartz plan reforms in the German redistributive system after 2006 contributed a lot to reduce child poverty.

Table 5: Poverty rate and intensity in East Germany 2002-2006

	Poverty rate			Poverty intensity		
	Total	Chronic	Transient	Total	Chronic	Transient
Child disposable	0.218	0.071	0.147	0.063	0.012	0.051
	(0.012)	*(0.013)*	*(0.011)*	*(0.005)*	*(0.003)*	*(0.004)*
Child market	0.496	0.208	0.289	0.372	0.116	0.255
	(0.030)	*(0.033)*	*(0.015)*	*(0.027)*	*(0.022)*	*(0.012)*
Adult disposable	0.185	0.052	0.134	0.050	0.008	0.042
	(0.007)	*(0.006)*	*(0.005)*	*(0.003)*	*(0.001)*	*(0.002)*
Adult market	0.649	0.355	0.294	0.557	0.253	0.304
	(0.022)	*(0.032)*	*(0.012)*	*(0.023)*	*(0.026)*	*(0.008)*

Standard errors are given between parentheses. Market income represent total income before taxes and redistribution. Disposable income includes taxes and redistribution and in particular family allowances. Child corresponds to the population between 1 and 18 years. Adult are over 18 years and have no child.

5.2 TIP curves and the East-West contrast

TIP curves can be used to compare situations of chronic poverty, so it enters by this way the analysis of poverty dynamics. The basic ingredients of a TIP curve are a poverty line z and an income series y_t. When analysis chronic poverty, a TIP curve can be estimated using a smoothed income series. Fourrier-Nicolaï and Lubrano (2020) have used this approach to analyse the impact of the Hartz plan on child poverty in Germany, using the GSOEP.

Analysing child poverty means that we consider households with children having an equivalent disposable income below the poverty line. We define a child as a person under 18 years old and consider it as the unit of observation, following Hill and Jenkins (2001), Jenkins and Schluter (2003), Corak et al. (2008) and many others. This means having possibly several observations coming from the same household. The child poverty incidence rate corresponds thus to the ratio between the number of children living in poor households over the total number of children.

It is much worse for a household to be in a state of chronic poverty over a long period (here five years) than being temporarily in poverty. East and West Germany have been reunified in 1990. However, the convergence between these two regions is slow and the economic differences are still important. Corak et al. (2008) concluded that child poverty incidence was much more important in the East part of Germany.

This is confirmed for child chronic poverty during period I (2002-2006) as seen when comparing the two panels of Figure 1. However because the credible intervals are large, there is no TIP dominance of the West over the

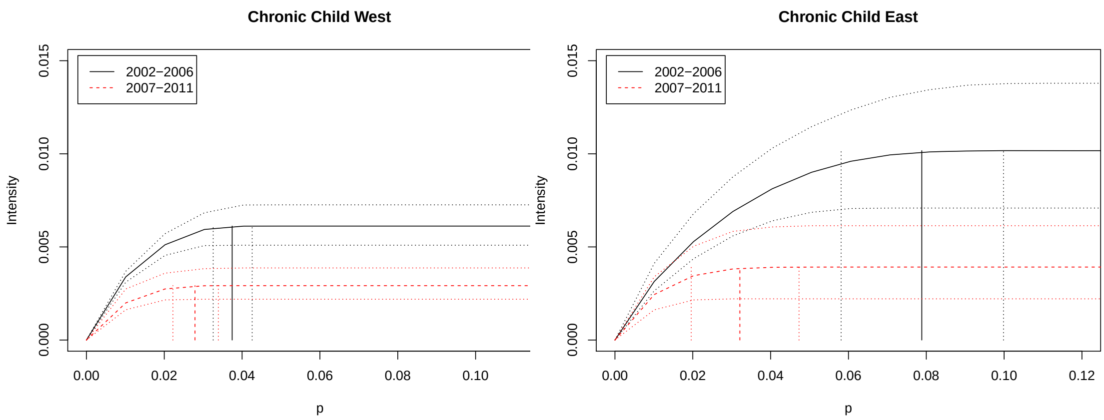

Figure 1: The West-East contrast of chronic child poverty

East for the first period.

There is a massive reduction of chronic child poverty during period II (2007-2011), both in East and West Germany as seen from the two panels of Figure 1. This massive reduction of chronic child poverty has erased the differences between the two regions concerning chronic child poverty. The two TIP curves were tested not to be statistically different between the two regions (the two red dashed curves in the two panels of Figure 1). We conclude that the redistributive system has been very efficient during the second period for fighting against chronic child poverty and after the Hartz reformed came into force.

6 A model of poverty transitions

Cappellari and Jenkins (2004) have proposed a model of poverty transitions based on two equations corresponding to period $t-1$ and t plus a third equation dealing with individuals observed at time $t-1$, but absent the next period in order to cope with attrition, a problematic phenomenon with panel data. Let us discard this third equation in this chapter. The first equation explains the marginal probability of being in poverty at time $t-1$. The second equation explains the conditional probability of being in poverty at time t when in poverty at time $t-1$. A key parameter is the correlation ρ of the error terms between $t-1$ and t. This model is essentially a dynamic probit model with selection bias. It serves to explain *poverty persistence* and *exit from poverty*. We shall detail a similar model, but based on the income-to-needs ratio $\log(y_{i1}/z_1)$ when Cappellari and Jenkins (2004) explain the dichotomous variable $\mathbb{1}(y_{i1} < z)$. We shall use this model in order to detect a particular treatment effect on poverty dynamics. This model was introduced in Sadeq and Lubrano (2018) in the very specific context of pseudo panel, a context that we shall not develop here, sticking to the much more simple case of balanced panels.

6.1 Modelling poverty dynamics using panel data

Let consider a balanced panel over two periods, 1 and 2 for explaining the log of the income-to-needs ratio $\log(y_i/z)$ where y_i is the income of individual or household i and z the poverty line, with thus the same number of observations n in the two periods. A household i will be said to be in a state of poverty if $\log(y_i/z)$ is negative, which means that its income y_i is lower than the poverty line z. We explain the log of the income-to-needs ratio at period 1 by following regression which provides information on the initial state of poverty:

$$\log(y_{i,1}/z_1) = x'_{i,1}\beta_1 + u_{i,1}, \tag{16}$$

where $x_{i,1}$ is a set of k exogenous variables observed during the first period. If the error term is Gaussian with zero mean and variance σ_1^2, the marginal probability of being poor at the initial period for household i is equal to:

$$\Phi(-x'_{i,1}\beta_1/\sigma_1) = 1 - \Phi(x'_{i,1}\beta_1/\sigma_1),$$

where Φ is the Gaussian CDF. Let us now suppose that a treatment $w_{i,1}$ was introduced during the first period and which is supposed to have a direct impact on poverty during the second period. The income-to-needs ratio for the second period can now be explained both by the treatment effect variable $w_{i,1}$ and by other exogenous variables observed at time 2, $x_{i,2}$. We suppose that the treatment can have a different impact on the income-to-needs ratio, depending on the initial poverty status, being poor or not. For this, we define a dummy variable $d_{i,1}$ which indicate if individual i is in a state of poverty in period 1 and the same variable $d_{i,2}$ for period 2:

$$d_{i,1} = \mathbb{1}(\log(y_{i,1}/z_1) < 0), \qquad d_{i,2} = \mathbb{1}(\log(y_{i,2}/z_2) < 0). \tag{17}$$

In this writing, $\mathbb{1}(a)$ is the indicator function equal to 1 if a is true and 0 otherwise. So the equation explaining the income-to-needs ratio in period 2, conditionally on the poverty status in period 1 is:

$$\log(y_{i,2}/z_2) = d_{i,1}w'_{i,1}\gamma_1 + (1 - d_{i,1})w'_{i,1}\gamma_2 + x'_{i,2}\beta_2 + u_{i,2}. \tag{18}$$

The error term $u_{i,2}$ is assumed to be Gaussian with zero mean and variance σ^2. The error terms of the two equations, $u_{i,1}$ and $u_{i,2}$ are correlated over time with $\text{Cor}(u_{i,1}, u_{i,2}) = \rho$ for the same individual i and independent between two different individuals. Poverty persistence is defined as the state of being poor in period 2 while having being poor in period 1. Letting θ represent the vector of parameter of our model, Cappellari and Jenkins (2004) provide the following analytical expression for poverty persistence:

$$\begin{aligned} s_{i,2}(\theta) &= \Pr(d_{i,2} = 1 | d_{i,1} = 1) && (19) \\ &= \Phi_2\left(-\frac{w'_{i,1}\gamma_1 + x_{i,2}\beta_2}{\sigma_2}, -\frac{x_{i,1}\beta}{\sigma_1}; \rho\right) \Big/ \Phi\left(-\frac{x_{i,1}\beta}{\sigma_1}\right). && (20) \end{aligned}$$

It corresponds to the ratio between a joint probability and a marginal probability, Φ_2 being the bivariate Gaussian cumulative distribution. Poverty entry is defined in a similar way as:

$$\begin{aligned} e_{i,2}(\theta) &= \Pr(d_{i,2} = 1 | d_{i,1} = 0) && (21) \\ &= \Phi_2\left(-\frac{w'_{i,1}\gamma_2 + x_{i,2}\beta_2}{\sigma_2}, \frac{x_{i,1}\beta}{\sigma_1}; -\rho\right) \Big/ \Phi\left(\frac{x_{i,1}\beta}{\sigma_1}\right). && (22) \end{aligned}$$

What would happen in the absence of treatment? We simply have to impose the restriction $\gamma_1 = \gamma_2 = 0$ in the model in order to single out the absence of treatment. Consequently the impact of the treatment on poverty dynamics is obtained by computing the difference:

$$\begin{aligned} s_{i,2}(\theta | \gamma_1 = 0) &- s_{i,2}(\theta), && (23) \\ e_{i,2}(\theta | \gamma_2 = 0) &- e_{i,2}(\theta). && (24) \end{aligned}$$

$s_{i,2}(\theta | \gamma_1 = 0)$ represent the natural evolution of poverty dynamics in the absence of treatment, which is corrected by the treatment with $s_{i,2}(\theta)$.

6.2 Bayesian inference

Estimating model (16)-(18) is quite simple if we observe the same individual over the two periods. And poverty persistence and poverty exit are just simple transformations of the estimated parameters. So let us recall how the complete model looks like:

$$\begin{aligned} \log(y_{i,1}/z_1) &= x'_{i,1}\beta_1 + u_{i,1}, && (25) \\ \log(y_{i,t}/z_2) &= d_{i,1}w'_{i,1}\gamma_1 + (1 - d_{i,1})w'_{i,1}\gamma_2 + x'_{i,2}\beta_2 + u_{i,2}, && (26) \\ d_{i,1} &= \mathbb{1}(\log(y_{i,1}/z_1) < 0). && (27) \end{aligned}$$

We have in fact what is called in the literature a Seemingly Unrelated Regression model or SURE. It is convenient to note the model in a matrix form.

For this purpose, let us define the following matrices:

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & 0 & 0 & 0 \\ 0 & d_1 W_1 & (1-d_1)W_1 & X_2 \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \gamma_1 \\ \gamma_2 \\ \beta_2 \end{pmatrix},$$

and

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix},$$

so that the model becomes:

$$y = X\beta + u, \quad u \sim N(0, \Sigma).$$

Bauwens et al. (1999, Chap. 9, pp. 267-269) detail Bayesian inference in a SURE model. Because the matrix X contains restrictions, there is no direct expression for the posterior density of β and Σ. However, we can derive the conditional posterior densities of $\beta|\Sigma$ and of $\Sigma|\beta$ so that we can implement a Gibbs sampler. Under the non-informative prior:

$$\varphi(\beta, \Sigma) \propto |\Sigma|^{-(n+1)/2},$$

the conditional posterior density of β is a multivariate Gaussian:

$$\beta|\Sigma \sim N(\hat{\beta}, [X'(\Sigma^{-1} \otimes I_n)X]^{-1}) \qquad (28)$$

$$\hat{\beta} = [X'(\Sigma^{-1} \otimes I_n)X]^{-1} X'(\Sigma^{-1} \otimes I_n)y. \qquad (29)$$

$$(30)$$

For deriving the conditional posterior density of Σ, we have to adopt a different matrix notation for the model. Let us define the following matrices:

$$\mathcal{Y} = (y_1, y_2), \quad \mathcal{X} = (X_1, d_1W_1, (1-d_1)W_1, X_2),$$

and

$$\mathcal{B} = \begin{pmatrix} \beta_1 & 0 & 0 & 0 \\ 0 & \gamma_1 & 0 & 0 \\ 0 & 0 & \gamma_2 & 0 \\ 0 & 0 & 0 & \beta_2 \end{pmatrix}$$

so that the model becomes

$$\mathcal{Y} = \mathcal{X}\mathcal{B} + E.$$

Using that notation, the conditional posterior density of $\Sigma|\beta$ is an inverted Wishart density with parameters:

$$\begin{aligned} \Sigma|\beta &\sim IW(S, n), && (31) \\ S &= (\mathcal{Y} - \mathcal{X}\mathcal{B})'(\mathcal{Y} - \mathcal{X}\mathcal{B}). && (32) \end{aligned}$$

The proof is given in Bauwens et al. (1999, Chap. 9, page 269).

Let us now suppose that we have obtained m draws from the previous Gibbs sampler, resulting in a MCMC output for β and Σ. Let us call $\theta^{(j)}$ the j^{th} draw of a parameter. We can evaluate the posterior probabilities of poverty entry and poverty persistence, using:

$$\begin{aligned} s_{i2}(\theta^{(j)}) &= \Phi_2\left(-\frac{w'_{i,1}\gamma_1^{(j)} + x_{i,2}\beta_2(j)}{\sigma_2^{(j)}}, -\frac{x_{i,1}\beta_1^{(j)}}{\sigma_1^{(j)}}; \rho^{(j)}\right) \Big/ \Phi\left(-\frac{x_{i,1}\beta_1^{(j)}}{\sigma_1^{(j)}}\right), \\ e_{i2}(\theta^{(j)}) &= \Phi_2\left(-\frac{w'_{i,1}\gamma_2^{(j)} + x_{i,2}\beta_2(j)}{\sigma_2^{(j)}}, \frac{x_{i,1}\beta_1^{(j)}}{\sigma_1^{(j)}}; -\rho^{(j)}\right) \Big/ \Phi\left(\frac{x_{i,1}\beta_1^{(j)}}{\sigma_1^{(j)}}\right). \end{aligned}$$

Computing these two probabilities is rather time consuming as each draw requires a two dimensional numerical integration in order to evaluate a bivariate Gaussian CDF $\Phi_2(\cdot)$. Using the same draw, we can impose $\gamma_1^{(j)} = 0$ and $\gamma_2^{(j)} = 0$ in order to compute the net effect of the treatment variable $w_{i,1}$, using (23)-(24).

6.3 Poverty dynamics in the West Bank

A similar model was used in Sadeq and Lubrano (2018) to measure the impact of the building of a Wall on the occupied West Bank on the dynamics of poverty. The model was in fact much more complicated as the available data were not in the form of a panel, but simply two cross sections collected in 2004 and 2011 by the Palestinian Central Bureau of Statistics. A dummy variable (our $w_{i,1}$) documented if a household had lost lands because of the building of the Wall in 2002. This Wall does not follow the *Green Line*, the international separation between the State of Israel and the occupied West Bank. With a total length of 708 kilometers, the Wall is more than double the length of the Green Line and at times runs 18 kilometers deep inside the West Bank. Many academic studies were led to measure the consequences of the wall. But those were mainly concerned about law and politics. Sadeq and Lubrano (2018) focuss on the economic consequences of the Wall on the Palestinian society, particularly in term of poverty dynamics.

We shall not detail here the full econometric model of Sadeq and Lubrano (2018), but just report their empirical results. The first remark we have to issue is that a naive and simple approach would be totally misleading. Measured poverty is lower in places impacted by the Wall than in places that were not impacted. This is simply due to the fact that the Wall was not built at random, but in the initially richest parts of the West Bank. Let us present in Table 6 a decomposition of a poverty headcount for 2004 and 2011 for the impacted and un-impacted regions, using the official poverty line of the Palestinian Central Bureau of Statistics. Poverty decreased between

Table 6: Poverty rates decomposition

		Poverty rates		
Date	Total	Jerusalem	Impacted	Un-impacted
2004	18.0	1.1	15.6	22.9
2011	14.7	1.1	12.2	18.0
		Sample sizes		
2004	1934	272	486	1176
2011	2909	271	847	1791

The official poverty line was used for computing the poverty rates. The variable `wall` can take three values: 0 Jerusalem, 2 impacted, 3 not impacted. Total population can be exactly decomposed according to these three characteristics.

2004 and 2011. Poverty is very low in Jerusalem, much higher in the West Bank, but significantly lower in the region where the wall was built and said to have impacted the population. This simply means that the wall was built in the richer part of the West Bank. If we use a simple differences-in-differences approach, poverty has diminished by 22.9 - 18.0 = 4.9 percentage points in the un-impacted region while it diminished by only 15.6 - 12.2 = 3.4 percentage points in the impacted region, which makes an excess of 4.9 - 3.4 = 1.5 percentage point. But this result is not very precise.

We first start by estimating our model, imposing the restriction $\gamma_1 = \gamma_2 = 0$. We have then two marginal equations, explaining for the two periods the income-to-needs ratio, using the same explanatory variables. The results are given in Table 7. From these equations, we compute the marginal probability of poverty entry which is equal to 0.109 and the marginal probability of poverty persistence which is 0.319. Both are rather low values and they do not take into account the effect of the Wall.

We then relax the constraint to inference on the full dynamic model where $w_{i,t-1}$ is the dummy variable representing the impact of the Wall. We report the estimation results in Table 8 for the second equation. The state dependence effect, which corresponds to the wall effect while being in a poverty

Table 7: Marginal models over the two periods

	2004			2011		
	Mean	SD	Ratio	Mean	SD	Ratio
Intercept	0.682	0.131	5.194	0.634	0.117	5.421
Sex	0.151	0.044	3.399	0.041	0.033	1.219
Age	-0.013	0.005	-2.423	-0.007	0.005	-1.461
Age2	0.014	0.005	2.600	0.009	0.004	2.098
Urban	0.080	0.027	2.961	0.037	0.024	1.511
Camp	-0.087	0.036	-2.401	-0.172	0.031	-5.601
Jerusalem	0.712	0.035	20.253	0.711	0.037	19.373
σ	0.519	0.008	61.670	0.542	0.007	76.531
ρ	0.399	0.0174	23.00			

These results were obtained with the informative prior $E(\rho) = 0.50$ and $SD(\rho) = 0.50$ and a non-informative prior on the remaining parameters. Mean is the sample average of the draws, SD means standard deviation of the draws and Ratio is the mean divided by the SD.

Table 8: Conditional transition model with state dependence (wall effect) and an informative prior

	Mean	S.d.	Ratio
Intercept	0.609	0.119	5.129
γ_1	-0.254	0.049	-5.132
γ_2	0.109	0.024	4.492
Sex	0.035	0.033	1.036
Age	-0.006	0.005	-1.291
Age^2	0.008	0.004	1.910
Urban	0.023	0.025	0.922
Camp	-0.164	0.031	-5.271
Jerusalem	0.727	0.038	19.298
σ	0.537	0.007	74.066

These results were obtained with the informative prior $E(\rho) = 0.50$ and $SD(\rho) = 0.50$ and a non-informative prior on the remaining parameters. Mean is the sample average of the draws, S.d. means standard deviation of the draws and Ratio is the mean divided by the S.d.

state in period 1, has a marked negative effect. The value of γ_2 is positive and strongly significant. Both are going to alter the probabilities of poverty persistence and poverty entry. The main determinants of poverty dynamics are being in a camp and living in Jerusalem, which are too opposed situations.

In Table 9, we recall in the top panel the marginal probabilities of poverty entry and persistence as obtained from the two marginal models. The second panel provides the same quantities obtained using (**??**) and (**??**) using the estimated values of γ_1 and γ_2. In the bottom panel of Table 9, we compute the difference between the two types of probabilities which provides a measure of the impact of the Wall on poverty dynamics. Taking into account the

Table 9: Wall effect on poverty dynamics

	Mean	S.d.	Ratio
Marginal Persistence	0.320	0.017	19.38
Marginal Entry	0.109	0.005	21.11
Conditional Persistence	0.897	0.026	34.91
Conditional Entry	0.287	0.007	43.10
Diff. in persistence	0.578	0.029	20.17
Diff. in entry	0.178	0.007	24.80

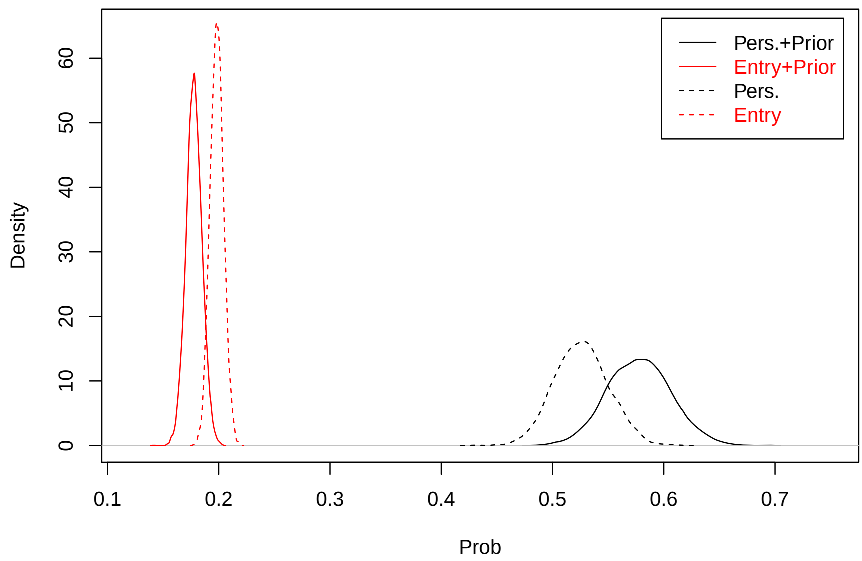

Figure 2: Posterior density of poverty persistence and poverty entry differentials due to the Wall

wall has a large effect on poverty dynamics. For those who were already poor in period 1, the wall increases their probability of staying poor by 58 percentage points. For those who were not in poverty, the probability of entering into poverty during the second period is increased by 18 percentages points. These are important values, much higher than those that we reported at the beginning of this section, using a simple diff-in-diff strategy.

We have reproduced in Figure 2 the posterior densities of these probabilities, using plain lines. We compare these probabilities to those obtained under a non-informative prior on ρ, using dashed lines. With a non-informative prior on ρ, the differential in probability of poverty entry is slightly increased while the differential in poverty persistence is slightly decreased. But these differences are mild. So the prior information we gave had a sizable influence on the posterior density of ρ, but not on the posterior density of poverty entry and persistence differentials.

7 Conclusion

A poverty head-count index is simple to understand. It has been widely criticised because it led to a very crude description of poverty. Nevertheless, **?** underlined its importance in the framework of a targeting policy. In the previous chapter, we have detailed other aspects of poverty such as poverty intensity and inequality among the poor. In this last chapter, we adopted a dynamic point of view because the situation of the poor is very different if poverty is transitory of if poverty is chronic. Using the TIP curve, we could cross the gap between static and dynamic poverty decomposition.

Studying the dynamics of poverty is particularly interesting when a targeted policy is implemented. We detailed in the last section a controversial experiment which in fact increased poverty. A more consensual targeted experiment was the Bolsa Família in Brazil, which became the largest conditional cash-transfer program in the world. The competitor to targeted poverty programs is the belief that growth will benefit to the poor and so would be an excellent tool for banishing poverty. Empirical work in the previous chapter has shown, using GIC and British data over the Margaret Thatcher period, that this belief was a lie. Without a voluntary redistributive policy, growth is just making the rich richer and leave the poor in the same state of poverty. With Bolsa Familia, benefits were paid mostly to women via a chip card for people who had an income lower than international poverty line of \$1.25 a day, conditionally on their acceptance to send kids to school and to get them vaccinated. More than thirty million Brazilians escaped from extreme poverty between 2003 and 2014, thanks to this targeted conditional program.

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