Detecting and modelling non-linearities in official time series

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Abstract

This paper reviews two types of non-linear threshold regression models: structural break models based on a time index and self exiting threshold regressions models. It investigates how they can be applied to detect and model breaks in the french consumption function and asymmetries in the US business cycle. The approach is Bayesian because in this type of models it is far better to average using numerical integration than to maximize a possibly discontinuous likelihood function. The final question which is addressed is the possible presence of spurious non-linearity and the tools which are available to detect them.

JEL Classification: C11, C12, D63, I29
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1 Introduction

Two debated empirical issues:

- the stability of the consumption function
- asymmetries in the business cycle.

Which type of models are adequate to investigate these two questions, is a common model feasible and how can we point out spurious non-linearities.

One particularly interesting way of introducing non-linearities is to suppose that the conditional expectation of the endogenous variable belongs to two or more regimes

$$\mathrm{E}(y_t|x_t) = \begin{cases} x_t'\,\beta_1, & \text{first regime,} \\ x_t'\,\beta_2, & \text{second regime.} \end{cases} \tag{1}$$

The choice between the two regimes is represented by a nonlinear transition function $F(\tilde{z}_t, \tilde{\theta})$ which takes values between zero and one. We can write

$$y_t = (1 - F(\tilde{z}_t, \tilde{\theta}))\, x_t'\beta_1 + F(\tilde{z}_t, \tilde{\theta})\, x_t'\beta_2 + \epsilon_t, \quad \mathrm{Var}(\epsilon_t) = \sigma^2, \tag{2}$$

with ϵ_t being normally distributed with zero mean and constant variance σ^2 over the two regimes. Extension to unequal variances would mean

$$\mathrm{Var}(\epsilon_t) = [1 - F(\tilde{z}_t, \tilde{\theta})]\sigma_1^2 + F(\tilde{z}_t, \tilde{\theta})\sigma_2^2. \tag{3}$$

The change of regime is made according to the sign of a linear combination of the elements of $\tilde{z}_t$.

- Dynamics.
- Tong (1983, 1990) and Teräsvirta (1994).
- At odds with Lindgren (1978), Hamilton (1989)

  $$y_t = (x_t'\,\beta_1 + \epsilon_{1t})s_t + (x_t'\,\beta_2 + \epsilon_{2t})(1 - s_t), \tag{4}$$

  where s_t is a Bernouilli random variable being one with probability p and zero with probability $1 - p$.

Starting from (**??**) or (**??**), we have to specify a transition function $F(.)$ and the switching variables contained in $\tilde{z}_t$:

- an abrupt transition function and a time index produces the structural break model where the breaking point is unknown. Used to investigate the stability of a french consumption function. Previous exemples: DeJong (1996) for unit roots with a structural break in the US GNP, Lubrano (1995) for cointegrating relationships applied to a wage equation.

- Threshold regression models : a continuous variable for $\tilde{z}_t$. Geweke and Terui (1993) and Chen and Lee (1995) and found applications in finance with e.g. Pfann, Schotman, and Tschernig (1996) or Forbes, Kalb, and Kofman (1997). This model will be used to test asymmetries in the US business cycle.

Numerical integration is reduced to the parameters of the transition function $F(\tilde{z}_t, \tilde{\theta})$.

2 Bayesian analysis of the general two regime model

2.1 Notation

For practical computations it is useful to reparameterise the model so as to obtain

$$y_t = x_t'\beta_1 + F(\tilde{z}_t, \tilde{\theta})\, x_t'\delta + \epsilon_t \tag{5}$$

with $\delta = \beta_2 - \beta_1$. β and the regime shift parameter $\tilde{\theta}$ have no element in common. Thus, it is possible to separate the parameter set into

- nonlinear parameters $\tilde{\theta}$ and linear parameters β.
- $\tilde{\theta} = [\gamma, \tilde{\theta}_2]$, where γ is a scalar parameter which measures the degree of smoothness of the transition.
- Sign of $z_t'\tilde{\theta}_2$. $\tilde{\theta}_2$ has to be normalised.

  $\tilde{z}_t = [z_t, 1]$.

  Implies $\tilde{\theta}_2 = [1, -c]$.

  Switching rule = sign of $z_t - c$

  c threshold parameter.

  $\theta = [\gamma, c]$.

The separability between β and $\theta = [\gamma, c]$ make it useful to consider the following reparameterisation:

$$\begin{aligned} x_t'(\gamma, c) &= [x_t', F(\gamma, z_t - c)\, x_t'], \\ \beta' &= [\beta_1', \beta_2' - \beta_1'], \end{aligned} \tag{6}$$

so that the model can be written in a compact form:

$$y_t = x_t'(\gamma, c)\beta + \epsilon_t. \tag{7}$$

One variance for the error term, two variances or even function form.

2.2 Posterior analysis

Theorem 1 *Under a noninformative prior for β and σ^2, the conditional posterior densities of β and σ^2 are given by*

$$\begin{aligned} \varphi(\beta|\gamma,c,y) &= f_t(\beta|\beta_*(\gamma,c)\,,\, M_*(\gamma,c)\,,\, s_*(\gamma,c)\,,\, T-k), \\ \varphi(\sigma^2|\gamma,c,y) &= f_{IG}(\sigma^2|s_*(\gamma,c)\,,\, T-k), \end{aligned} \tag{8}$$

where

$$\begin{aligned} M_*(\gamma,c) &= \textstyle\sum x_t(\gamma,c)\, x_t'(\gamma,c), \\ \beta_*(\gamma,c) &= M_*^{-1}(\gamma,c) \textstyle\sum x_t(\gamma,c)\, y_t, \\ s_*(\gamma,c) &= \textstyle\sum y_t^2 - \beta_*'(\gamma,c)\, M_*(\gamma,c)\, \beta_*(\gamma,c). \end{aligned} \tag{9}$$

The corresponding posterior density of γ and c is

$$\varphi(\gamma,c|y) \propto |s_*(\gamma,c)|^{-(T-k)/2}\, |M_*(\gamma,c)|^{-1/2}\, \varphi(\gamma)\, \varphi(c). \tag{10}$$

The posterior density of γ and c has to be analysed numerically to compute its integrating constant and its moments, using a two dimensional numerical integration routine. The marginal posterior densities of β and σ^2 follow with

$$\varphi(\beta|y) = \int \varphi(\beta|\gamma,c,y)\, \varphi(\gamma,c|y)\, d\gamma\, dc, \tag{11}$$

$$\varphi(\sigma^2|y) = \int f_{IG}(\sigma^2|s_*(\gamma,c)\,,\, T-k)\, \varphi(\gamma,c|y)\, d\gamma\, dc. \tag{12}$$

The posterior density (**??**) has pathological behaviour in the directions of both γ and c.

2.3 Identification and nonlinearity

The model becomes linear when $F(\gamma, z_t - c)$ is zero or constant. Happens for $\gamma = 0$.

- $M_*(\gamma = 0, c)$ becomes singular, the joint posterior density of $\theta = [\gamma, c]$ is not integrable.
- δ is not identified
- c is also not identified

Davies' problem (1977) in the classical literature when testing for linearity, not a problem for inference.

Bayesian framework, the problem is more serious as we have to integrate over the whole domain of definition of γ. Introduce adequate prior information on δ. Consider the following conditional normal prior for δ:

$$\varphi(\delta|\sigma^2,\gamma) = f_N(\delta|0, \sigma^2 \exp(\gamma)\, N_0^{-1}). \tag{13}$$

Becomes informative when $\gamma \to 0$ and says that δ is zero in this case.

2.4 Integrability of the posterior density of γ

Theorem 2 *With a flat prior, the posterior density of γ is not integrable.*

A convenient prior is the truncated Cauchy density with

$$\varphi(\gamma) = \begin{cases} (1+\gamma^2)^{-1} & \text{if } \gamma > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The scale of γ depends on the scale of the variable z_t. Thus it is wise to standardise the difference $z_t - c$ by the standard deviation of z_t.

2.5 Testing for linearity

The linear model

$$y_t = x_t'\beta + u_t \tag{15}$$

can be tested for various misspecification directions, one of which is the presence of nonlinearity in the conditional expectation. Augmented regressions of Pagan (1984), reinterpreted in a Bayesian framework by Aprahamian, Lubrano, and Marimoutou (1994). A linearisation of $F(.)$ around $\gamma = 0$ yields the additional variables looked for:

$$F(\gamma, z_{t-d} - c) \simeq \alpha_0 + \alpha_1 z_{t-d} + \alpha_2 z_{t-d}^2 + \alpha_3 z_{t-d}^3. \tag{16}$$

The augmented regression for detecting a nonlinear direction becomes

$$y_t = x_t'\beta + x_t'\delta_1 z_{t-d} + x_t'\delta_2 z_{t-d}^2 + x_t'\delta_3\alpha_3 z_{t-d}^3 + \epsilon_t. \tag{17}$$

Schwarz criterion.

3 Transition models based on a time index

Structural breaks or segmented trends at an unknown point of time. The regression coefficient β changes according to an unknown date τ and is treated as a discrete parameter.

3.1 Step change

The change between the two regimes is made via a step transition function $ID(t-\tau)$ defined by

$$ID(t-\tau) = \begin{cases} 0 \text{ if } t \geq \tau, \\ 1 \text{ if } t < \tau. \end{cases} \tag{18}$$

$ID(.)$ is thus an indicator function (or Heaviside function).

Departure and return to equilibrium. The step function

$$ID(\tau_1, \tau_2) = ID(t - \tau_2) - ID(t - \tau_1) \text{ with } \tau_1 < \tau_2, \tag{19}$$

is equal to zero when t is outside the interval $[\tau_1, \tau_2]$, and one in between.

3.2 Smooth transition

A smooth transition between the two regimes is obtained by replacing the step indicator function (**??**) by a smooth, monotonically increasing, odd function $F(.)$ with $F(-\infty) = 0$ and $F(\infty) = 1$

$$F(\gamma, t - \tau) = \frac{1}{1 + e^{-\gamma(t-\tau)}}, \tag{20}$$

The double step function (**??**) can be generalised to the smooth transition case with

$$F(\gamma, \tau_1, \tau_2) = \frac{1}{1 + e^{\gamma(t-\tau_1)(t-\tau_2)}}. \tag{21}$$

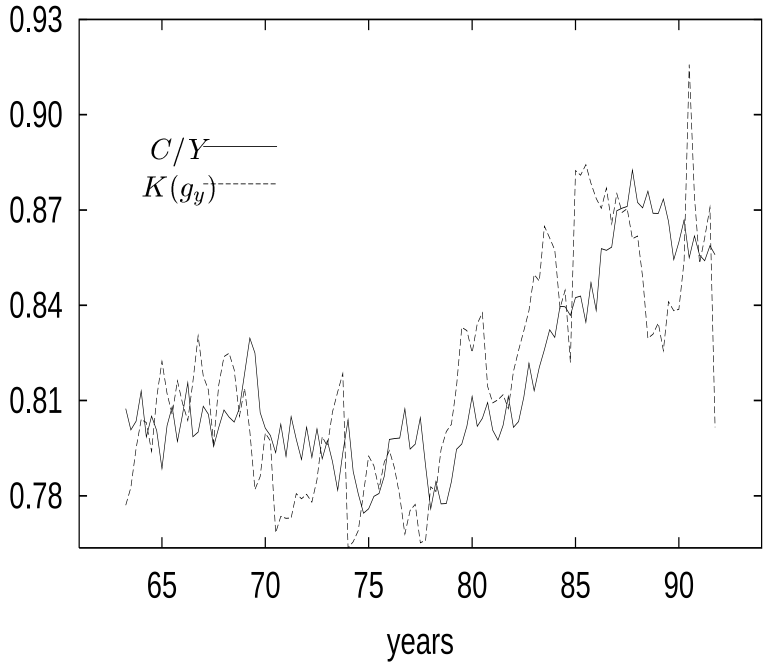
3.3 The French consumption function

A traditional consumption function, as reported for instance in Davidson, Hendry, Sbra, and Yeo (1978), relates the logarithm of real consumption to the logarithm of real disposable income:

$$\begin{aligned}\Delta \log CP_t &= \delta + \delta_0 \ d69.2 + \gamma \Delta \log CP_{t-4} + \beta_0 \Delta \log RDP_t \\ &\quad + (\alpha - 1)[\log CP_{t-1} - \nu \log RDP_{t-1}] + \epsilon_t,\end{aligned} \tag{22}$$

where $d69.2$ is a dummy variable. This formulation implies that in the long run the saving ratio is constant (provided the long run elasticity equals unity). The French

Figure 1: Mean propensity to consume:
data and simulation with the double threshold model.



economy has experienced a fall in its saving ratio; witness Figure **??**, which displays the ratio between CP (real consumption) and RDP (real disposable income). This is the mean propensity to consume displayed for quarterly data covering the period 1963:1-1991:4. It remained fairly stationary till 1977, despite the first oil shock of 1974. As noted by Villa (1996), profits were first to be affected by the oil shock of 1974, which was transmitted only later to wages. The share of wages in real GNP rose till 1977 and eventually till 1982. Income distribution and consumption played a countercyclical role. Adjustments operated later with the conjugate effects of wage and fiscal policies.

The steady state means

$$\log CP = \log K + \nu \log RDP$$

K being the mean propensity to consume. In a dynamic equilibrium where $\Delta \log RDP = g_y$ and $\Delta \log CP = g_c = \nu g_y$, the log of the mean propensity to consume is expressed as

$$\log K = \frac{\delta}{1-\alpha} + g_y \frac{\beta_0 - (1-\gamma)\nu}{1-\alpha}. \tag{23}$$

- First estimation of this equation: Schwarz criterion -4.926.
- Augmented regression with trend, a squared trend, and a cubic trend: Schwarz criterion of -4.931.

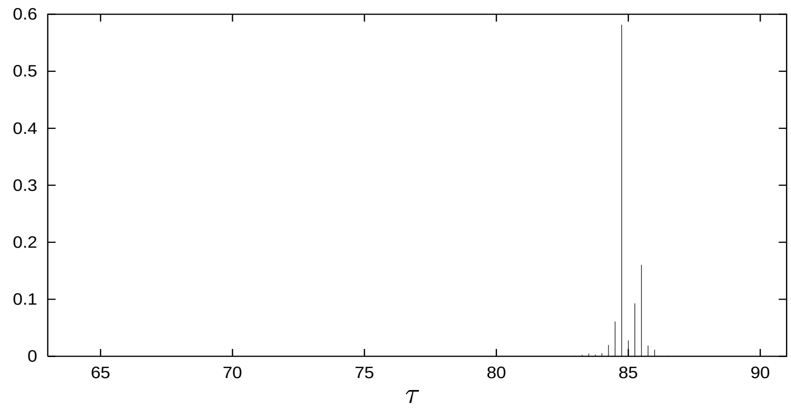
Let me now consider a step transition function for the constant term. The model is reparameterised in such a way so as to allow for the possibility of a unit long term elasticity. Using a noninformative prior, Bayesian inference produces the following output

$$\begin{aligned}\Delta \log CP_t \;=\; & \underset{[0.041]}{0.11} + \underset{[0.0035]}{0.013}\, ID(t-\tau) + \underset{[0.0067]}{0.021}\, d69.2 - \underset{[0.079]}{0.23}\, \Delta \log CP_{t-4} \\ & + \underset{[0.067]}{0.25}\, \Delta \log RDP_t - \underset{[0.040]}{0.16} \log \left(\frac{CP}{RDP}\right)_{t-1} - \underset{[0.0031]}{0.011} \log RDP_{t-1},\end{aligned}$$

$\mathrm{E}(\sigma^2|y) = 0.000423$,

Schwarz $= -5.030$.

Figure 2: Posterior density of τ
for the French consumption function with a step transition.

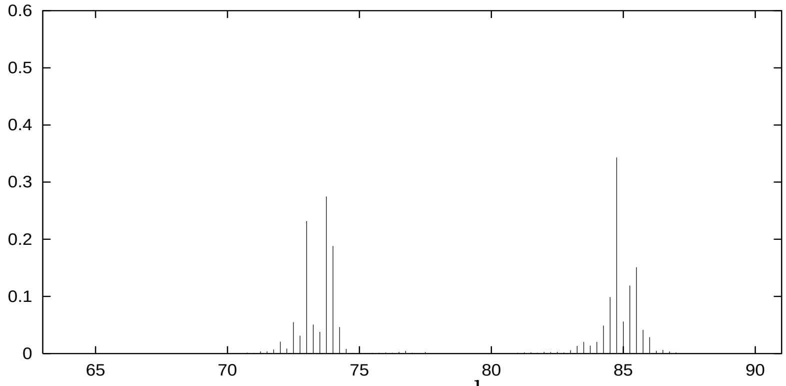


Note that this fully nonlinear model receives more evidence from the sample than the augmented regression, due to a lower value of its Schwarz criterion. The long term elasticity ν of consumption to income is lower than one, precisely 0.93 (computed with posterior means) while the marginal propensity to consume is $0.25/(1+0.23) = 0.20$.

The expectation of τ is 1984.1, while the mode is 84.4. The probability of the posterior mode is 0.58. Figure **??** shows that the break point seems to be unique around this date, but 42% of the probability is scattered around this point.

The shift in the constant term after 1984.1 means that the mean propensity to consume increased from $1.99 \times \exp[-0.89 \times gx]$ to $2.16 \times \exp[-0.89 \times gx]$. These figures are unrealistic, whatever the value of g_y. They cast some doubt on the specification of the equation.

Figure 3: Double break for the French consumption function with a step transition.



As an everlasting increase in the mean propensity to consume, as suggested by the single break model, is improbable, let me try the double break model (**??**), still with a diffuse prior. With this model, the unit elasticity hypothesis is accepted. The final results are

$$\begin{aligned} \Delta \log CP_t \;=\; & \underset{[0.0041]}{-0.0088} - \underset{[0.0017]}{0.0071} ID(\tau_1, \tau_2) + \underset{[0.0067]}{0.019}\, d692 \\ & - \underset{[0.078]}{0.26}\, \Delta \log CP_{t-4} + \underset{[0.069]}{0.23}\, \Delta \log RDP_t - \underset{[0.022]}{0.11} \log \left(\frac{CP}{RDP} \right)_{t-1}, \end{aligned}$$

$\mathrm{E}(\sigma^2|y) = 0.000432,$

Schwarz $= -5.051.$

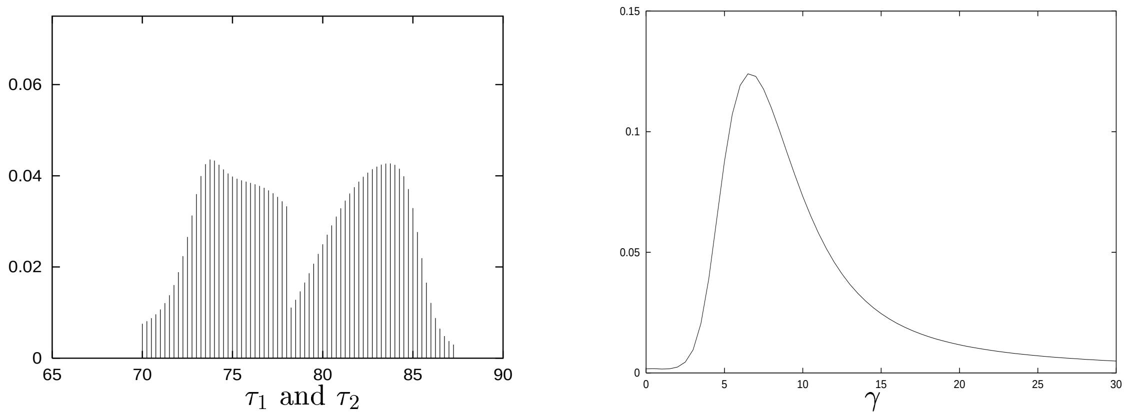
This model is slightly preferred to the one break model on the basis of the Schwarz criterion. Moreover, it has many remarkable features

- The long run elasticity is now equal to one.

- There is one break with expectation at 1984.1, as before, but also a second break earlier, in 1973.3. These breaks are well marked, with posterior standard deviations of 3 quarters.

- The mean propensity to consume is $0.92\exp(-9.36 g_y)$ in the long term equilibrium, but seems lower, $0.87 \times \exp(-9.36 g_y)$, during the period of adjustment. With $g_y \in [0, 0.02]$, these figures are within the order of magnitude of the data. Figure **??** reproduces the data on C/Y and $K(g_y)$ with a g_y smoothed by a moving average of order 5. The two series follow similar paths. Even if the constant term is lower during the intermediate regime between 1973 and 1984, the resulting dynamic mean propensity to consume, $K(g_y)$, is slightly decreasing till 1975, but rises between 1978 and 1986, following the path shown by the ratio CP/RDP. The model thus manages to cope with the change in the structure of earning.

Figure 4: Double break for the French
consumption function with a smooth transition.



The final question is what will yield a smooth transition function. With the conditional normal and truncated Cauchy priors, the inference results are as follows:

$$
\begin{aligned}
\Delta \log CP_t \;=\; & -\underset{[0.0047]}{0.0092} - \underset{[0.0043]}{0.012}\, F(\gamma, \tau_1, \tau_2) + \underset{[0.0069]}{0.019}\, d692 \\
& - \underset{[0.079]}{0.24}\, \Delta \log CP_{t-4} + \underset{[0.070]}{0.26}\, \Delta \log RDP_t - \underset{[0.025]}{0.10} \log \left(\frac{CP}{RDP} \right)_{t-1},
\end{aligned}
$$

$\mathrm{E}(\sigma^2 | y) = 0.000453$,

Schwarz $= -5.003$.

The posterior densities of τ_1 and τ_2 in Figure **??** indicate the same modes for the breaking dates. But the message becomes muddled.

The posterior density of γ indicates that high values are not rejected with a mean of 10 and a posterior 95% confidence interval of [0, 23]. Schwarz criterion favours the step function.

4 Transition models based on a continuous variable

SETAR (self exiting transition autoregressive) models can be viewed as an alternative to linear ARMA models. The transition function depends on the lagged value of the endogenous variable which is compared to a threshold.

Keynes : the dynamics of the business cycle behaves differently in expansion and in recession phases, which in general are shorter but violent.

Keynes also stated that positive changes in the money supply are neutral, while negative changes have real effects on aggregate output. Ravn and Sola (1996) found on the contrary that big changes in nominal demand are neutral, while small changes have a real effect, indicating the relevance of menu cost models and nominal rigidities.

4.1 Step transition regression models

A generic model is

$$y_t = x_t'\beta_1 + ID(z_{t-d} - c)\, x_t'(\beta_2 - \beta_1) + \epsilon_t \tag{24}$$

with

$$ID(z_{t-d} - c) = \begin{cases} 0 & \text{if } z_{t-d} - c \leq 0, \\ 1 & \text{otherwise} \end{cases} \tag{25}$$

and

$$x_t'(c) = [x_t',\, ID(z_{t-d} - c)\, x_t']. \tag{26}$$

Identification problem. Basically, there must be enough observations per regime. Let us denote by H the empirical distribution of z_t. As c belongs to R, a possible prior is

$$\varphi(c) = \begin{cases} 1 & \text{if } c \in [c_a, c_b], \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

with c_a and c_b such that $H(c_a) = 0.15$ and $H(c_b) = 0.85$. A similar suggestion can be found in the classical literature (Andrews 1993).

Chen and Lee (1995) define a Gibbs sampler for TAR models, but the posterior analysis of this model enters the general formulation of Section **??**. In the SETAR model, c is the sole nonlinear parameter, and thus inference is a one dimensional numerical integration exercise. The posterior density of c given by

$$\varphi(c|y) \propto |s_*(c)|^{-(T-k)/2} \, |M_*(c)|^{-1/2} \, \varphi(c) \tag{28}$$

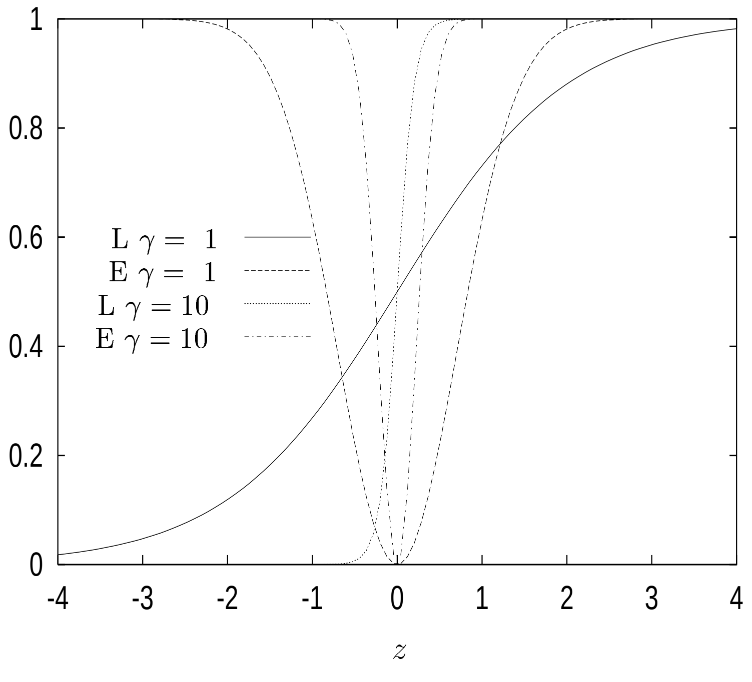
may be extremely peculiar. It may be discontinuous for a finite number of points given by the sample values of z_t, but is continuous (and constant) between two consecutive values of z_t.

4.2 Smooth transition regression models

$$y_t = x_t'\beta_1 + F(\gamma, z_{t-d} - c)\, x_t'(\beta_2 - \beta_1) + \epsilon_t \tag{29}$$

c and γ are in general the sole nonlinear parameters of the model, leading to numerical integration of dimension two. As there is a smooth transition between the two regimes, the posterior density of c no longer presents the same discontinuities as in the step transition case.

Figure 5: Two transition functions.



The logistic transition function is an odd function with $F(-\infty) = 0$, $F(+\infty) = 1$. In business cycle studies, it aims at detecting asymmetries between positive and negative shocks.

On the contrary, an even function ($F(\pm\infty) = 1$, $F(0) = 0$), such as the exponential function used for instance by Haggan and Ozaki (1981), aims at discriminating between the effects of big shocks compared to small ones. The exponential transition function is:

$$F(\gamma, z_{t-d} - c) = 1 - e^{-\gamma\,(z_{t-d}-c)^2}. \qquad (30)$$

4.3 Business cycle and nonlinear asymmetries

There is a debate about the presence or absence of nonlinearities in the US GNP.

- Hamilton's (1989) model explains the changes in the log of real quarterly US GNP over 1951:2 to 1984:4 using a discrete shift between two regimes, where the shift obeys a Markov process. His model reproduces the usual turning points of the US business cycle as determined by the NBER.

- Boldin (1996) showed that Hamilton found only a local maximum of the likelihood function and that his results were not robust to an extension of the sample period.

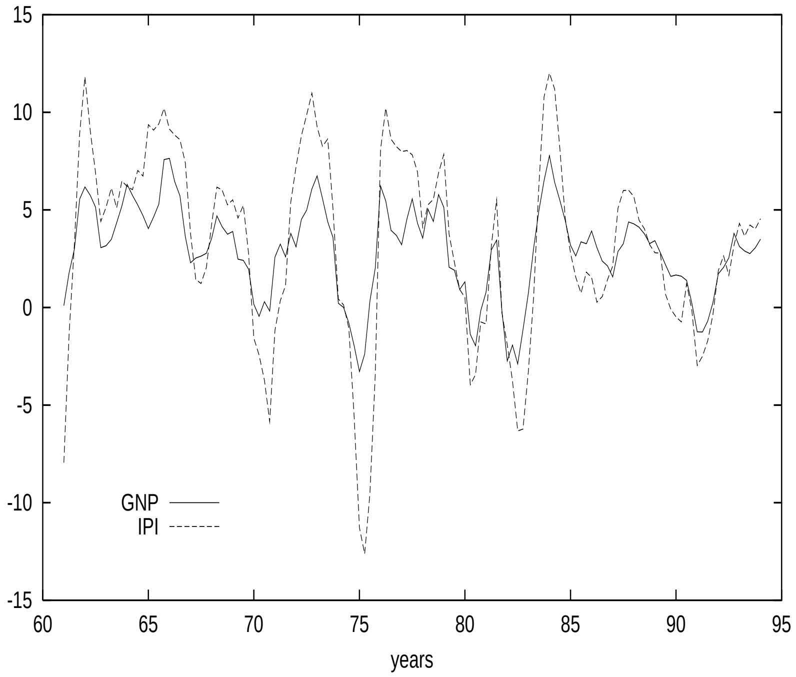
- Potter (1995), using the same data but on the period 1947:1-1990:4, adjusts a two regime step transition TAR(5) model with unequal variances where $d = 2$ and $c = 0$. Though his model seems to fit the data correctly, the linearity tests he reports, based on Hansen's (1996) theoretical results, reject linearity at 10%, but not at 5%.

- Hansen (1996) himself concludes a statistical artifact for nonlinearity in the US GNP.

- However, tests presented in Tiao and Tsay (1994) do conclude nonlinearity of the US GNP. Teräsvirta and Anderson (1992) underline that an industrial production index (IPI) has a greater variance than the GNP series and thus is more capable of displaying nonlinearity.

The US real GNP and the IPI for the period 1960:1-1994:1. Seasonally adjusted for the GNP. Monthly, not seasonally adjusted data for the IPI. Converted to quarterly by averaging. Fourth difference to stationarise and to remove possible remaining seasonality. See Figure **??**.

Figure 6: US business indicators.



Verify the presence of nonlinearity by a sequence of augmented regressions starting from an AR(1) where d varies between 1 and 5.

Table 1: Selecting linearity or nonlinearity for US GNP and US industrial production

	GNP		IPI	
d	Schwarz	P–value	Schwarz	P–value
-	-3.91	-	-2.63	-
1	-3.81	0.74	-2.52	0.89
2	-3.87	0.036	-2.62	0.004
3	-3.86	0.069	-2.66	0.001
4	-3.90	0.006	-2.55	0.25
5	-3.80	0.87	-2.53	0.68

The "Schwarz" columns give the Bayesian Schwarz criterion, and the "P–value" columns the classical P–value for the F–test of linearity. At the classical 5% level, linearity is rejected for the US GNP. The smallest P–value is obtained for $d = 4$.

However, a Schwarz criterion selects the linear model. If I consider an AR(5) model, linearity is also no longer rejected by classical P–values. This is in agreement with some of the empirical results mentioned above.

For the US IPI, linearity is rejected at the classical level of 1%. The Schwarz criterion selects the nonlinear model with $d = 3$. This is in agreement with the results of Teräsvirta and Anderson (1992) on a smaller sample size. These results are confirmed with an AR(5).

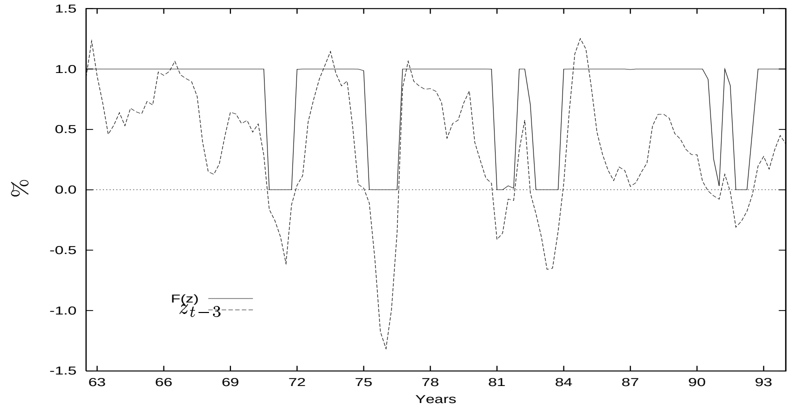
Let me now try to adjust an LSTAR model on the IPI data only, as there is not enough evidence in favour of nonlinearity in the GNP data. Let me call the transformed variable y_t. I shall use a restricted uniform prior on c, a Cauchy prior on γ, and a conditional normal prior on the second regime regression parameters as defined in (**??**).

$$
\begin{aligned}
y_t \;=\;& \underset{[0.0051]}{0.016} + \underset{[0.11]}{1.06}\, y_{t-1} - \underset{[0.90]}{0.36}\, y_{t-2} - \underset{[0.19]}{1.06}\, y_{t-4} + \underset{[0.20]}{0.59}\, y_{t-5} \\
& + (1+\exp(-\underset{[30]}{43}\; 20.34\,(y_{t-3} + \underset{[0.0020]}{0.0036})))^{-1} \\
& \times (\underset{[0.0062]}{0.020} + \underset{[0.10]}{0.25}\, y_{t-1} + \underset{[0.20]}{0.97}\, y_{t-4}) - \underset{[0.21]}{0.59}\, y_{t-5},
\end{aligned}
$$

$$\mathrm{E}(\sigma^2|y) = 0.000245,$$

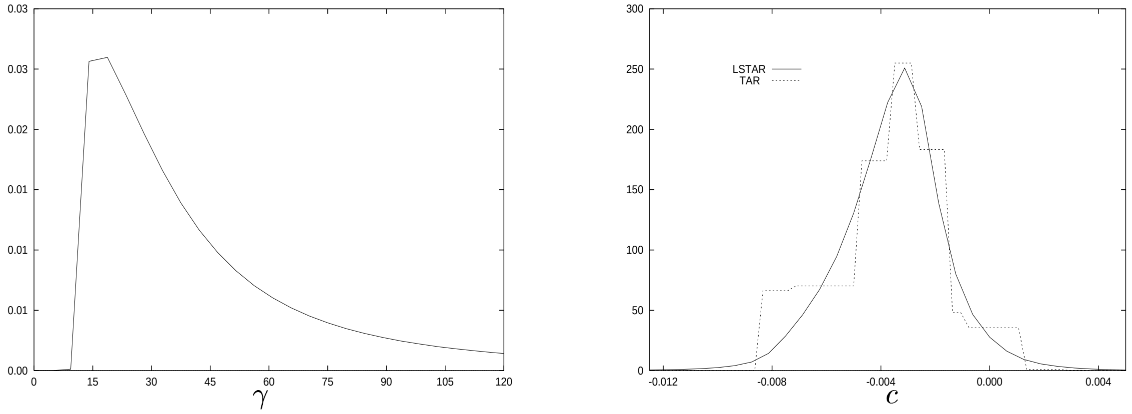
$$\text{Schwarz} = -3.164.$$

Figure 7: Transition function of the LSTAR model for US industrial production.



As the transition variable was normalised by its standard deviation, the posterior expectation of γ can be directly interpreted. With a value of 43, the transition between expansions and recessions is quite abrupt; witness the corresponding graph of $F(z)$ in Figure **??** which is shown together with the rescaled series z_{t-3}. Note that with a shorter sample (ending in 1986), $E(\gamma|y)$ was much lower.

Figure 8: Posterior densities of γ and c of the LSTAR model for US industrial production.



The posterior expectation of c is slightly negative at -0.0036, but with such a large standard deviation that it could have been taken equal to zero. The posterior density of c has not a regular shape (see Figure ??). In fact, it tends toward an irregular histogram as $\gamma \to \infty$. Note that despite the very high posterior expectation of γ, there is still a difference between the step transition and the logistic transition.

The step transition model has a Schwarz criterion of -3.165, which is only marginally better than that of the smooth transition model. There is no major change in the regression coefficients. The posterior density of the threshold is displayed in Figure ?? on the same graph as for the smooth case. It shows how the shape of this posterior density is modified by the nature of the threshold. It is less scattered, but of course presents a series of points where it is not continuous. By the way, it is still centred on zero, which suggests again that the threshold could be pinpointed at this value. In this case the model is reduced to a linear regression model with multiplicative dummy variables.

The model is characterised by very different dynamic behaviour between the two regimes. Teräsvirta and Anderson (1992) suggest computing the dominant root of the lag polynomial for each regime.

- A dominant complex root of 1.124 for the recession regime
- A dominant complex root of modulus 0.827 for the expansion regime.

Consequently, the dynamics of the expansion is stationary, while the dynamics of the recession is explosive, suggesting that recessions are short–lived and violent. Out of 127 observations, 25 are in the recession regime, while 102 are in the expansion regime.

5 Conclusion

Flexibility of Bayesian methods to analyse nonlinear time series models. Better to integrate than to maximize a likelihood function.

What about the Gibbs sampler versus Simpson integration?

- Not useful here. Simpson better.
- Useful for more complicated models: especially when data augmentation (disequilibrium models for instance)
- Hidden Markov chain models: Ghysels, McCulloch, and Tsay (1994), Albert and Chib (1993) and McCulloch and Tsay (1993).

6 Statistical artefacts and break detection

In this section, I would like to treat a problem which is recurrent in empirical work. What do we mean by detecting a break in an empirical equation. Has the behaviour of the agents really changed or is it just a convenient way of fitting better the data with an unachieved model? The question can be illustrated with break and unit roots. An I(1) series usually present several jumps. If the model allows for a break every time there is a jump, the I(1) property of the series disappears into the deterministic breaks and we have a statistical artefact.

6.1 A unit root model

The model I introduce now is a slight generalisation of the regression model model with a break as it allows for autocorrelated errors

$$\begin{cases} y_t & = & x_t\beta_1 + ID(\tau)x_t\beta_2 + u_t \\ u_t & = & \rho u_{t-1} + \epsilon_t \\ \Delta x_t & \sim & N(0, v_t) \end{cases}$$

x_t is an I(1) exogenous variable and consequently y_t is also I(1). The error term is autocorrelated. For $\rho < 1$, the non-linear combination of of y_t and x_t is stationary, which means that these two variables are cointegrated.

6.2 Detecting artefacts

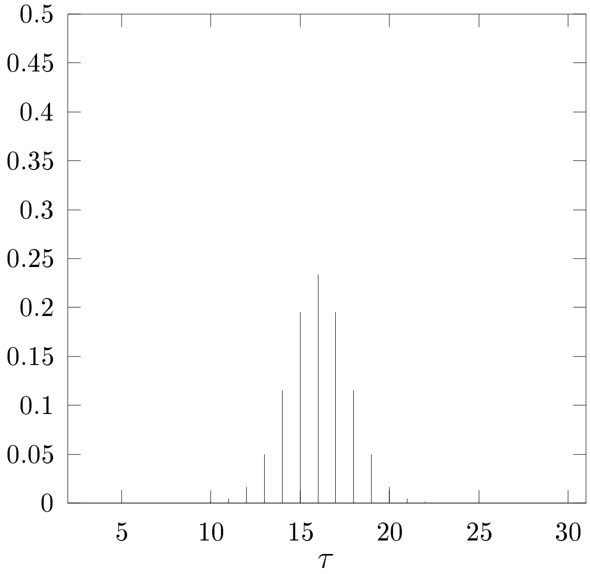
When a break is allowed for in a cointegration relationship, what is the probability of finding a spurious break in the middle of the sample as the model favours the presence of a break on a fit argument. A very simple exemple consists in regressing a trend on a constant with a break. We should find a break in the middle of the sample as in the consumption relationship. But we have diagnostic tools to show that this is in this case a real artefact.

Let us consider a trend of 32 observations and let us regress it over a constant with a break. Using a uniform prior on the coefficients, we find

$$y_t = \underset{[1.52]}{8.50} + ID(\tau)\ \underset{[1.76]}{16.00}$$

The posterior mode and expectation of τ are both equal to 16. So a break is effectively found in the middle of the sample. The first tool for detecting the misspecification consists in interpreting correctly the posterior density of ρ. It is a discrete density where each line represents the posterior probability of a break point. This it is the posterior odds of a break point compared to the whole set of all possible break points. Here the greatest mode is very small with 0.23. So there is no dominant breaking point with a posterior probability of 0.5.

Figure 9: Posterior density of τ



The second tool is provided by the posterior density of ρ. In order to have a valid cointegration relationship, we must have $\rho < 1$. The graph of the posterior density of ρ already says us that things are going badly. If we now compute the posterior probability that $\rho > 1$, we find 0.32, whih is the proof that we have a unit root. Consequently, the model is twice badly specified and we have amply detected the statistical artefact.

Figure 10: Posterior density of ρ