The econometrics of inequality and poverty
Chapter 6: Equivalence scales

Michel Lubrano

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1 Introduction

Surveys are devoted to collecting data on households. Welfare is concerned with individuals. Up to now, we have not made the link between the two. It is evident that in order to reach the same level of welfare a family with two children need more income than a family with no children. And that a couple need more income than a single person, always to reach the same level of welfare. The usual statistical practice consists in dividing the household income by a function of the household size, say $g(n)$ so as to be able to measure the welfare of an adult equivalent, supposing that welfare is equally distributed in the household. For instance, if an household with two adults has an equivalence scale of 1 and an household with two adults an one child has an equivalence scale of 1.2, this means that the second household need 1.2 times more income than the first household in order to get the same level of welfare. Or that the cost of the first child is 20%.

2 Usual scales

Various scales were proposed in the literature and used by statistical agencies. The first equivalence scale that appeared in the literature was the Oxford scale, later named the OECD scale of 1982. This scale, and all subsequent scales give a weight of 1 to the first adult, usual the head of the household, a smaller weight to the second adult (mainly the spouse) and a smaller weight to children. There is a discussion about the age of children. The weights used in the Oxford scale are presented in Table 1.

Table 1: Oxford or OECD (1982) scale

Member	coefficient
Head of household	1
Other adults	0.7
Child	0.5

As a side remark, we can note that the French fiscal system uses a particular equivalence scale for defining the tax burden of an household. The two adults have the same weight of 1, while children have a weight of 0.5, except that the third child has a weight of 1. We shall comment on this later on.

2.1 The need for changing scales

An equivalence scale reflects the fact that there are scale economies in an household. There are collective goods which are consumed by everybody and private goods which are consumed specifically by one individual. The equivalence scale depends on the proportion of collective versus private goods in the household. This proportion can vary over time and across countries, especially when there are large differences between urban and rural areas. There is a huge difference in housing costs between Shanghai and a small rural village and even between Shanghai and Nanchang. So we cannot keep the same equivalence scale over time and across countries. This explain the diversity of solutions which were adopted in the past. The OECD modified its equivalence scale in 1994 to modify the weight of the second adult and of the children. The weight of the second adult is decreased while a child is no longer a child after 14 years old and is counted as an adult. Canada Statistics has adopted a similar rule, but fixed the outing of childhood at 16

Table 2: Modified OECD (1994)

Member	coefficient
Head of household	1
Everybody else aged more than 14	0.5
Every child below 14	0.3

and gives a weight of 0.4 instead of 0.5 to this class.

The French system of taxation includes something which is similar to an equivalence scale because it introduces shares for computing taxes. Both heads of the household are counted for one share which gives a high incentive to get married. Each child receive a share of 0.5, which is again larger than the common share in OECD equivalence scales. Finally the third and subsequent children have a full share of 1, which is a strong fiscal incentive to have children. We are here in the domain of fiscal incentives and not of finding a clear statistical and economic rule for finding an adult equivalent when measuring income.

Finally, in many studies a simple parametric form is adopted for simplification instead of the linear system described up to now. If N is the size of the household, an approximation to the linear scale is simply N^α where α is a coefficient between 0 and 1. This restriction gives a concave form to the equivalence scale. It implies that the cost of the second child is lower than that of the first and so on. The Luxemburg scale takes $\alpha = 0.5$ when more common values are around 0.60.

We can summarize the effect of these different scales in Table 3. The last row of Table 3 gives the implied elasticity of consumption demand with respect to the size of the household. The per capita solution with $\alpha = 1$ means that each new member consumes the same amount as the previous members individually. There is no economy of scale. The extreme case $\alpha = 0$ means that there is a full economy of scale, which is unrealistic.

Table 3: Effect of equivalence scale

Household size	Equivalence scale				
	per capita income	Oxford Old OECD	New OECD scale	Square root scale	Household income
1 adult	1	1	1	1	1
2 adults	2	1.7	1.5	1.4	1
2 adults, 1 child	3	2.2	1.8	1.7	1
2 adults, 2 children	4	2.7	2.1	2.0	1
2 adults, 3 children	5	3.2	2.4	2.2	1
Elasticity	1	0.73	0.53	0.50	0

2.2 The influence of equivalence scales

We want to illustrate the influence of the equivalence scale on the shape of the scaled income distribution. For that, we take the data set `Ilocos` which is provided in `R` with the library `ineq`. It contains 632 observations coming from the *Family and Income and Expenditure Survey* made in one region of the Philippines called Ilocos in 1997. The data contains household income and other information such as sex of the household head, family size, urbanity and province.

We take the most simple scale N^α as we have only information about N. We then let α vary between 0 and 1. For each case we estimate the income distribution.

```
library(ineq)
data(Ilocos)
y = Ilocos$income
n = Ilocos$family.size
alpha = c(0.0,0.3,0.6,1.0)
tit = c("alpha=0.0","alpha=0.3","alpha=0.6","alpha=1.0")
split.screen(c(2,2))
for (i in 1:4){
screen(i)
yz = y/(n^alpha[i])
yz = yz/max(yz)
plot(density(yz),main=tit[i],xlab="",ylab="",xlim=c(0,0.6))
}
```

The income distribution was normalized so as to have comparable graphs despite the different equivalence scales. When $\alpha = 0$, we have a bimodal density, where a group of richer persons appear: we do not take into account family composition. The importance of this group decreases when α is increased, which means when we give more and more weight to children.

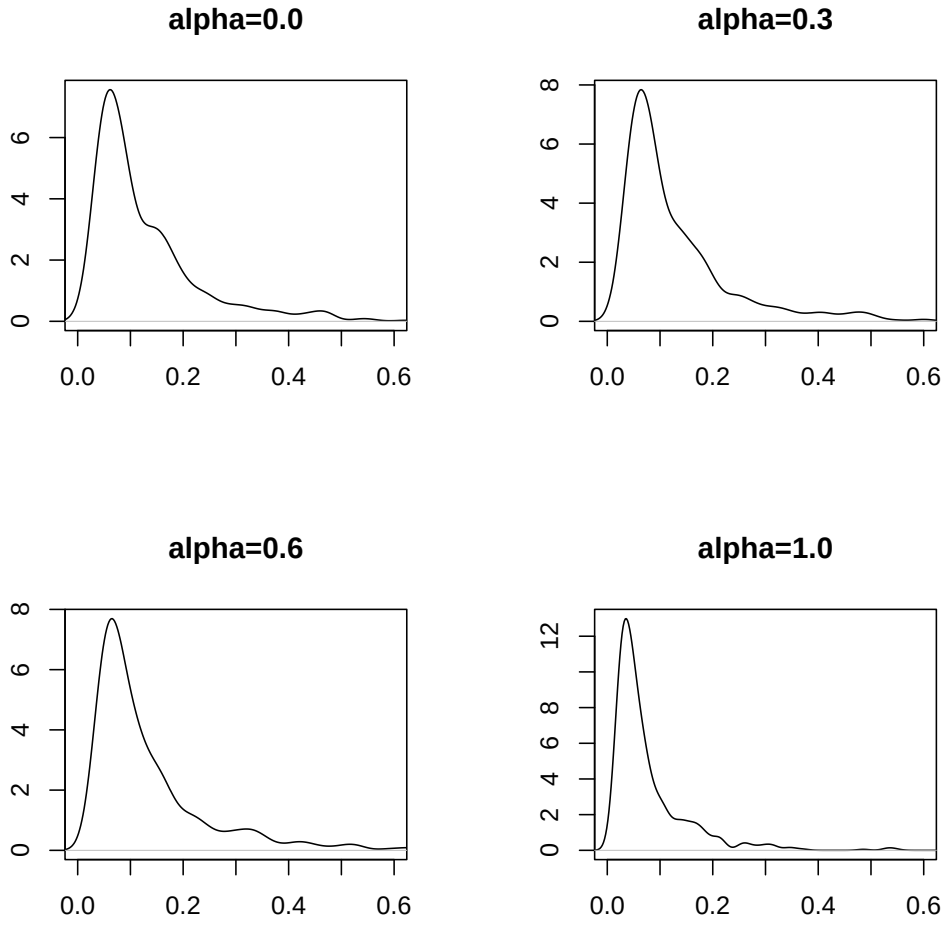


Figure 1: Influence of the equivalence scale

2.3 Family composition in France

Before applying an equivalence scale and looking at the income distribution while forgetting family composition, it is wise to have an idea of how income is distributed among the families. I have taken the example of France as it is given in the 2011 report of the CONSEIL DES PRÉLÈVEMENTS OBLIGATOIRES, page 187, Table 12. We reproduce this table now and it becomes Table 4 of our chapter.

We must first of all try to figure out the different columns are computed. Net income represents the total disposable income of the household, after payment of social contributions, but before taxes and redistribution. It corresponds to the fiscal income, which means the income that is taken into account by the fiscal authorities. The standard of living corresponds to the same notion, but taking into account family composition. We can try to deduce from the comparison

of these two columns which kind of scale has been used. For a couple it corresponds exactly to the new OECD scale, but the correspondence is slightly less precise for the children.

Table 4: Standard of living and net income as a function of family composition

	Net income	Standard of living	Poverty rate (in %)
Couple without children	41 310	27 540	9
one child	44 900	23 740	13
two children	47 990	21 550	17
three children and more	45 840	17 230	41
Head of household single	20 580	20 580	22
one child	21 420	15 160	39
two children	21 580	12 040	54
three children and more	14 470	6 320	82
Overall	34 220	22 260	20

Poverty rate is defined as the percentage of households with a standard of living lower than 60% of the median standard of living. INSEE computes the poverty rate as a function of disposable standard of living. After a continuous drop between 1996 and 2002, dropping from 14.5% to 12.9%, it is now stable at 13% in 2008.

The great advantage of this table is to shed some light on the relation between family composition and income. The number of children is not uniformly distributed with respect to income, so that an equivalence scale cannot remove the influence of the number of children on the standard of living. The rate of poverty is computed with respect to the standard of living and thus takes into account family composition. But even with this adjustment, the proportion of poor households increases with the number of children and becomes dramatic for households with three children and more. Depending on income, individuals do not have the same number of children. Fertility is a complicated function of income.

Let us now consider the case of households with a single head. The picture is here even more dramatic. Even if net income is increasing with the number of children as in the previous case (if we drop the case with three children and more), the standard of living is strongly decreasing with the number of children and we reach an incredibly high rate of poverty of 82% for this sub-population.

We can finally try to characterize the dispersion of standard of living. It is much higher among large families and single headed households. The Gini index is 50% higher in this sub-population than in the whole population. This greater dispersion is due to the presence of very low standard of living in this category. Donner les chiffres précis

2.4 Taxation and redistribution in France

What is the impact of taxation and redistribution on the standard of living of different types of households? In France, redistribution is very favourable to large families and single headed families. There is nothing abnormal in this as we saw that poverty was concentrated on these family compositions.

Households having two kids or more receive 54% of the total allowances. Single persons without any children receive 17% of the total allowances. Redistribution operates from households without kids and households with one kid toward single headed families and couples with three children or more. From Table 5 (Table 13 page 189 of the report), we see that the effect of taxation and redistribution on standard of living is maximum for single headed families with three kids or more.

Table 5: The impact of redistribution as a function of family composition

	Net standard of living	Disposable standard of living	Variation	Variation (in %)
Couple without children	27 540	25 580	-1 960	-7
with 1 child	23 740	22 870	-870	-4
with 2 children	21 550	21 640	90	0
with 3 children or more	17 230	19 420	2 190	13
Isolated without children	20 580	20 030	-550	-3
with 1 child	15 160	16 840	1 680	11
with 2 children	12 040	15 020	2 980	25
with 3 children or more	6 320	12 180	5 860	93

These families are also the poorest ones. Despite this effect of redistribution, the poverty rate of couples with three children or more remains greater than the average (19,7% against 13.0% in 2008). The report notes that this poverty rate has decreased by 8 points since 1996.

On the contrary, the poverty of single headed families is greater than that of every other type of family, whatever the number of children. It has increased since 1996, especially since 2004, passing from 26% to 30% in 2008.

3 Equivalence scales and demand systems

The various equivalence scales which were given seem to be totaly arbitrary. In fact they are not. They are the result of a precise economic theory dealing with utility and household consumption. The key question and difficulty is that we want to compare households which have not the same composition and consequently not the same utility functions. We want to find a number which says by how much the income of a household has to be multiplied if an extra member is added and if this household wants to keep the same level of utility. The individual utility theory does not know how to perform welfare comparisons between households. We must introduce specific assumptions. The theory is explained for instance in Chapter 4, section 4.3 of Deaton (1997) and also in Hourriez and Olier (1997) at a more elementary level. These are the two sources that we shall use.

3.1 The model of Prais and Houthakker

One fundamental assumption underlying equivalence scales is that there are economies of scale because consumption goods can be divided into:

1. collective goods, those which are consumed collectively by all the members of the household and

2. individual consumption goods which are consumed only by one individual.

As a collective good, we have mainly housing, as an individual good, we can quote adult clothing or tobacco. The model of Prais and Houthakker (1955) explains the household consumption of various items as a function of income and of the size of the household. Are thus explained the structure of consumption and the influence of the structure of the household from which an equivalence scale can be derived.

Consumption is divided into K different items such as lodging, food, clothing, leisure, ... The size of the household is called N which means the total number of persons which are members of the household. The size effect is introduced both as a deflator of income and as an explanation of a particular consumption item. After various computations which are not reproduced here, the model is written as:

$$\log(C_k) = A_k + \alpha_k \log(N) + \beta_k \log(R/N^{\alpha}),$$

where C_k is the consumption of good k, R household income, and N the size of the household. This model can also be expressed in term of budget shares $\omega_k = C_k/R$, obtained by subtracting $\log(R)$ from each side:

$$\log(\omega_k) = A_k + (\alpha_k - \alpha) \log(N) + (\beta_k - 1) \log(R/N^{\alpha}).$$

We have as many equations as there are consumption items, but only $K - 1$ equations are independent. This model analyzes how the structure of consumption is modified as a function of N, when we compare two households which have the same income R. We have two effects:

- A size effect. The budget share of individual goods for which $\alpha_k > \alpha$ increases. The budget share of collective goods for which $\alpha_k < \alpha$ decreases.

- An income effect. When N is increased, R/N^{α} decreases. The structure of consumption is modified. The budget share of luxury goods which have a β_k greater than 1 is decreased while the budget share of primary goods which have a β_k lower than 1 increases.

This model is not identified, as that can be easily seen by developing it.

$$\log(C_k) = A_k + (\alpha_k - \beta_k \alpha) \log(N) + \beta_k \log(R)$$

In this linear equation, we have two regressors and three parameters. As we have only $K - 1$ independent equations, the identification problem cannot be solved by considering the system as a whole. Either we fix α and then, we cannot estimate the equivalence scale we were looking for, or we impose an identification constraint on one of the α_k.

3.2 The identification assumptions of Engel and Rothbarth

The oldest method dates back to Engel (1857). It is based on the assumption that the budget share devoted to food gives a good indication of the level of welfare, independently of the composition of the household. A large and a small family are equally well-off if they devote the same budget share to food. *A poor family devotes a very large part of its budget to food. While a rich family devotes a rather small part of its budget to food.* This is the first law of Engel. Let us introduce this law in the consumption model of Prais and Houthakker. We suppose that food is the good labeled 1, so that budget share of food noted C_1/R depends only on the household welfare measured as R/N^α and not on $\log(N)$. Consequently, Engel assumption implies that

$$\alpha_1 = \alpha.$$

In full the model writes as

$$\begin{aligned}
\log(C_1) &= A_1 + \alpha \log(N) + \beta_1 \log(R/N^\alpha), \\
\log(C_2) &= A_2 + \alpha_2 \log(N) + \beta_2 \log(R/N^\alpha), \\
&\dots \\
\log(C_K) &= A_K + \alpha_K \log(N) + \beta_K \log(R/N^\alpha),
\end{aligned}$$

and using consumption shares:

$$\begin{aligned}
\log(\omega_1) &= A_1 + (\alpha - \alpha) \log(N) + (\beta_1 - 1) \log(R/N^\alpha), \\
\log(\omega_2) &= A_2 + (\alpha_2 - \alpha) \log(N) + (\beta_2 - 1) \log(R/N^\alpha), \\
&\dots \\
\log(\omega_K) &= A_K + (\alpha_K - \alpha) \log(N) + (\beta_K - 1) \log(R/N^\alpha).
\end{aligned}$$

This assumption means that food is a mid-range good, between a strictly individual and a strictly collective good in term of scale economies. This assumption is verified for a poor society, like the one of Engle at his time, but not for the rich western societies of nowadays.

An alternative identifying assumption was proposed by Rothbarth (1943). The budget share of the first adult for his clothing is a good measure of the household welfare: remember that in France the price of the suit of family head played a strong role in the presidential election. For a given welfare, the clothing consumption of the first adult does not depend on the size of the household. If C_2 represent the clothing consumption of that first adult, then, Rothbarth assumption implies that

$$\alpha_2 = 0.$$

There are some adjustments depending on the sex of the adult. For instance, we know that a women spend 1.3 times more than a man for clothing. So for a couple, we can take $C_2 = (C_{2M} + C_{2W}/1.3)/2$. An alternative form of the Rothbarth assumption considers the total consumption for clothing of the whole household and says that

$$\alpha_2 = 1.$$

There is a second problem concerning clothing consumption. From the *Enquête sur le Budget des Familles*, it is apparent that individual living alone spend much more money on clothing than individuals living in a couple. Consequently, we can restrict Rothbarth's identifying assumption to couples. This would mean that clothing consumption for adults living in couples does not depend on the number of children.

3.3 An estimation for France

Hourriez and Olier (1997) report an estimation of the model of Prais and Houthakker (1955) using the *Enquête sur le Budget des Familles* of 1985,1989, 1995 using the identifying assumption $\alpha_2 = 0$. Various extra dummy variables where introduced. Some of these variables were socio-economic variables such as localization and employment status. Other dummy variables concerned the status of the head of the household: single parent households and bachelors; because they have a very different type of consumption. So the scale of equivalence concerns in fact only married couples with or without children.

Table 6: Estimation of the Prais-Houthakker model

Consumption	Size elasticity α_k			Income elasticity β_k		
	1985	1989	1995	1985	1989	1995
Food	0.74	0.67	0.72	0.54	0.55	0.64
Clothing	1.03	0.95	0.99	1.37	1.46	1.51
Lodging	0.46	0.38	0.39	0.73	0.68	0.61
House equipment	0.60	0.55	0.71	1.41	1.43	1.60
Health	0.56	0.40	0.80	0.70	0.70	1.08
Transport-Tel.	0.73	0.49	0.57	1.43	1.40	1.22
Leisure	0.92	0.90	0.94	1.33	1.41	1.39
Misc.	0.90	0.72	0.97	1.71	1.59	1.63
Global scale α	0.69	0.58	0.65			

As the Rothbarth assumption was imposed only for adult clothing, a scale for clothing was estimated. The size elasticity is consistently equal to 1, so clothing a pure individual good. The same can be said for leisure. The good for which the scale economy is the greatest is lodging with a size elasticity of 0.40 on average. This means that a family of four spend $1.7 = 4^{0.4}$ more than a single person for lodging. Leisure on the contrary is an individual consumption because the corresponding α_i is near 1.

The main result is given for α and corresponds to an equivalence scale of N^{α}. The value of α is not constant over time. It is a function of the budget share and of the individual elasticities α_k. In the estimations, α is around 0.60. If we go back to Table 3, the Oxford scale corresponds to $\alpha = 0.73$ while the new OECD scale corresponds to $\alpha = 0.53$. The Oxford scale corresponds to a now outdated structure of consumption. At that time, food was was the first budget share and lodging had a much smaller share than nowadays. Scale economies were thus much smaller at that time.

When the size of a household is increased, the share of collective goods in total consumption decreases while the share of individual goods increases, just because of the different values of the α_i. In France, the size of households has slightly decreased over the years. Consequently, the budget share of housing has increased. relate that to the expression of the equation in budget shares and the estimated values.

This model might be too simple to describe household consumption as it does not include for instance relative price effects. A large family might decide to increase collective consumption, just because the price of individual goods has increased too much: for instance video cassettes that can be watched collectively versus individual cinema seats. For a single person, the cost can be the same between the two alternatives.

4 Subjective equivalence scales

The estimation of the previous equivalence scales relies on a particular consumption model which is rather restrictive. First of all, the literature has produced more elaborated models, such as the model of Barten, of Deaton and Muellbauer to quote only the main ones. Secondly, there is a fundamental identification problem that the econometrician has to solve by saying what is exactly welfare and how it is measured.

A second approach is possible by a direct questioning of the household, asking them what is their welfare level, most of the time using an indirect question. The task of the econometrician is then to explain the collected answers by the composition and size of the household and the level of income.

4.1 Qualitative questions

This is an important domain in the field of poverty and inequality analysis. Up to now, we have studied what can be called objective criteria which were based on quantitative data. Welfare was identified to a certain level of consumption or income. A vast field of the literature on inequality and poverty has interest on what people feel: for instance how would they define their health status. Here, a survey can ask questions on the intensity of the budget constraint of the household. *How do you manage to make the both ends meet* is a sensible question for the housekeeper. In the French *Enquête sur le Budget des Familles*, the following question was asked:

Concerning your budget, which of the following proposition fits the best your budget situation

1. *It is difficult*
2. *you are just, you have to pay attention to your spending*
3. *your budget situation is fine*
4. *You experience a good financial ease*
5. *Do not know*

The respondent has to situate on a subjective scale. Several factors can influence his position on that scale, only some of which being of interest for our purpose. We have to take account of the other factors in order to obtain unbiased results. Financial ease is a function of income, but also of wealth. Being the owner of one's apartment makes a great difference. Being in a large town like Paris induces an increase of spending of 28% for Paris for instance in order to get the same level of welfare. And other variables have their importance.

4.2 Ordered logit models

The probability of picking one of the answers is explained by an ordered logit model. There is a great liberty for selecting the explanatory variables at the difference of the Prais-Houthakker model. We observe an ordered variable Y which can have values 1,2,3,... It corresponds to the ordered answers. This observed polychotomous variable is a function of the unobserved level of welfare Y^*. The model says that the responder answers 1 if his unobserved level of welfare is below the threshold κ_1, answers 2 if his unobserved level of welfare is between the thresholds κ_1 and κ_2 and so on:

$$\begin{array}{rcll} Y_i & = & 1 & \text{if } Y_i^* \leq \kappa_1 \\ Y_i & = & 2 & \text{if } \kappa_1 \leq Y_i^* \leq \kappa_2 \\ Y_i & = & 3 & \text{if } \kappa_2 \leq Y_i^* \leq \kappa_3 \\ Y_i & = & 4 & \text{if } Y_i^* \geq \kappa_3 \end{array}$$

The latent variable Y_i^* is supposed to be determined by the following linear regression

$$Y_i^* = x_i'\beta + u_i$$

where u_i follows a $(0, \sigma^2)$ normal or logistic distribution. The variance is assumed to be 1 for identification reasons. The logistic distribution is more convenient because it has an analytical cumulative distribution. Choosing the logistic assumption, we have the following ordered logit probabilities:

$$\begin{array}{rcl} \Pr(Y_i = 1) & = & \Pr(Y_i^* \leq \kappa_1) = \dfrac{1}{1 + \exp(x_i\beta - \kappa_1)} \\ \Pr(Y_i = 2) & = & \Pr(\kappa_1 \leq Y_i^* \leq \kappa_2) = \dfrac{1}{1 + \exp(x_i\beta - \kappa_2)} - \Pr(Y_i = 1) \\ \Pr(Y_i = 3) & = & \Pr(\kappa_2 \leq Y_i^* \leq \kappa_3) = \dfrac{1}{1 + \exp(x_i\beta - \kappa_3)} - \Pr(Y_i = 2) \\ \Pr(Y_i = 4) & = & \Pr(Y_i^* \geq \kappa_3) = 1 - \dfrac{1}{1 + \exp(x_i\beta - \kappa_3)} \end{array}$$

There are thus three threshold parameters to estimate, κ_1, κ_2 and κ_3, together with the structural parameters β.

This model has identification problems due to the fact that unobserved utility has an unknown scale and unknown range. The range is identified by constraining the variance σ^2 of u_i to 1 and the scale identification is usually solved by constraining one of the κ to be zero. Usually we impose $\kappa_1 = 0$. Under this constrain, we can introduce a constant term in the regressors. Otherwise, the constant term has to be omitted. likelihood function, marginal effects, probit versus logit

4.3 MASS in R

There are several packages that can be used to run ordered logit or probit regressions. I have selected the package MASS:

```
library(MASS).
```

There is a data set in it named `housing` which can be described as follows. This is a *Frequency Table from a Copenhagen Housing Conditions Survey*. The data frame has 72 rows and 5 variables. These variables are:

1. `Sat`: Satisfaction of householders with their present housing circumstances, (High, Medium or Low, ordered factor).
2. `Infl`: Perceived degree of influence householders have on the management of the property (High, Medium, Low).
3. `Type`: Type of rental accommodation, (Tower, Atrium, Apartment, Terrace).
4. `Cont`: Contact residents are afforded with other residents, (Low, High).
5. `Freq`: Frequencies: the numbers of residents in each class.

The main command is

```
polr(formula, data, weights, method = c("logistic", "probit")).
```

The following example is build around the data set housing:

```
house.plr <- polr(Sat ~ Infl + Type + Cont,
            weights = Freq, data = housing)
             summary(house.plr, digits = 3)
```

We have the following results given in Table 7. The first part of the table gives the coefficients of the regression, how is satisfaction explained by the different variables. There are three different levels of satisfaction, and consequently two estimated values for κ. One κ is fixed for identifications reasons. The last bloc gives indications on the fit. As the explanatory variables are categorial variables, one item has to be excluded and act as a reference. For instance tower for the apartment type.

Table 7: Ordered logit model

Coefficients	Value	Std. Error	t value
Infl-Medium	0.566	0.1047	5.41
Infl-High	1.289	0.1272	10.14
Type: Apartment	-0.572	0.1192	-4.80
Type: Atrium	-0.366	0.1552	-2.36
Type: Terrace	-1.091	0.1515	-7.20
Cont: High	0.360	0.0955	3.77
Intercepts			
Low-Medium	-0.496	0.125	-3.974
Medium-High	0.691	0.125	5.505
Residual Deviance	3479.149		
AIC	3495.149		

The endogenous variable is satisfaction with lodging, high, medium or low. Reference for perceived inflation is *low*. Reference for apartment type is *tower*.

4.4 A subjective equivalence scale model for France

The basic model give the level of unobserved utility or welfare as a function of income and household size

$$Y_i^* = f(R_i, N_i).$$

The equivalence scale $m(N)$ is found by solving the equation

$$f(R_i, N_i) = f(R_i/m(N), 1).$$

Depending on the shape of the function f, the scale $m(N)$ can be concave or convex. If we decide for

$$Y_i^* = a + b\log(R_i) + c\log(N_i)$$

the implied scale is of the form N^α with $\alpha = -c/b$ and is concave. There is a decreasing cost for every extra child. Using the answers to the subjective question about financial ease, Hourriez and Olier (1997) obtain an estimated $\alpha = 0.62$ in 1995 which is very similar to the answer given by the estimation of the Prais-Houthakker model. So the subjective method is valid and it is simpler to implement. This value is in between the elasticity implied by the old OECD (Oxford) scale and the new OECD scale. In this estimation, the size of the household was taken equal to $N = N_a + 0.55N_c$ where N_a is the number of adults and N_c the number of children below 14 years. The value 0.55 will be explained below. It is related to the cost of a child and estimated in a regression.

The second possibility is to use the regression

$$Y_i^* = a + b\log(R_i) + c\,N_i.$$

In this case, the scale is A^{N-1} with $A = \exp(-c/b)$. There is an increasing cost of one extra child.

It is difficult to discriminate between the two models, because they are in fact unsatisfactory approximations of a larger model. A larger model would be

$$Y_i^* = a + b\log(R_i) + c\log(N_i) + d\,N_i. \tag{1}$$

In this case, the equivalence scale would be $N^\alpha A^{N-1}$.

4.5 The cost of a child

The cost of a child is not uniform with his age. In order to investigate the cost of a child, Hourriez and Olier (1997) opted for the following regression

$$Y^* = a + b\log(R) + c_1 N_{0-4} + c_2 N_{5-9} + c_3 N_{10-14} + c_4 N_{15-19} + c_5 N_{20-24} + c_6 N_{adults}$$

where N_{0-4} is the number of children between 0 and 4 years present in the household. The cost of an extra child of age i is given by

$$100[\exp(-c_i/b) - 1].$$

This equation was estimated by Hourriez and Olier (1997), with results given in Table 8. Ac-

Table 8: Cost of a child as a function of his age

Age group	1979	1985	1989	1995
0-4 years	21	20	18	12
5-9 years	16	15	16	11
10-14 years	22	18	20	18
15-19 years	29	34	28	28
20-24 years	45	38	49	41
Adult	43	47	45	44

Figures are in percentage.

cording to Table 8, the cost of an extra child is between 10% and 20% of the income of his household if the age of the child is below 15 years. Above 15 years, there is a jump in the cost of a child. Hourriez and Olier (1997) said that it is roughly the same as the cost of an extra adult. Consequently, we can simplify the final model and make only the difference between children under 15 years and adults. Thus the size of the household can be simplified so as to be

$$N = N_a + k\,N_c$$

The value of k can be estimated directly with the following regression

$$Y^* = a + b\log(R) + c_1 N_a + c_2 N_c$$

Then k is given by c_2/c_1, which is approximately estimated as being 0.55. The final equivalence scale, whenever we have simply N in a regression will be in fact

$$N^\alpha = (N_a + 0.55\,N_c)^\alpha,$$

for instance in the Prais-Houthakker model of Table 6.

4.6 The minimum income question

Another type of question can be asked. It concern the minimum income necessary to make the two ends meet. This question was introduced in the *Enquête sur le budget des ménages* in 1989 and 1995. And is also present in many different types of survey. This question is phrased as follows in the French survey:

> *Quel est, selon vous, le revenu mensuel minimal dont un ménage comme le votre doit absolument disposer pour pouvoir simplement subvenir à ses besoins? (réponse en clair).*

In the German Socio Economic Panel, we find a similar question as reported page 301 of van Praag and Ferrer-i-Carbonell (2008):

> *what is in your opinion the minimum amount of income that your family in your circumstances would need to be able to make ends meet? That would be DM... per month.*

Individuals are asked this question and their answer might depend on their income level and not just on family composition. This is know as preference drift. The RMI was introduced in France in 1988 and is different for a single person, a couple, a couple with one child, two children, etc. It was enlarged as the RSA in 2009. In January 2009, the RMI was determined on the following basis given in Table 9. Individuals do not have necessarily these figures in mind.

Table 9: RMI in France, January 2009

Nber of Children	Single	Couple
0	454.63	681.95
1	681.95	818.34
2	818.34	954.73
per extra child	181.85	181.85

In euros per month.

So their answers are determined by a series of factors on top of family composition and mainly depend on their level of income. Consequently, the model we have to estimate is

$$\log(R_{MINI}) = a + b\log(R) + c\log(N)$$

Parameter b measures the preference drift. One can think that only the household that have an income R near from the reported R_{MINI} have a correct perception of what is a minimum income as a function of N. We note this function $S(N)$. Consequently, we have to solve a fixed point equation, imposing $S(N) = R = R_{MINI}$:

$$\log(S(N)) = \frac{a}{1-b} + \frac{c}{1-b}\log(N).$$

Taking the exponential, the equivalence scale $m(N)$ is given by

$$S(N) = S(1)m(N) = e^{\frac{a}{1-b}} N^{\frac{c}{1-b}}.$$

The estimation produced in Hourriez and Olier (1997) gives $\alpha = 0.37$, which is a much lower value than that obtained by the question on financial ease.

The minimum income question is also used to determine a subjective poverty line. It is present in the EU-SILC (EU Statistics on Income and Living Conditions) survey on minimum income, and specifically *the lowest income to make ends meet variable* (Eurostat 2003).

Question : Using the information contained in Table 9, determine the implicit equivalence scale that was used to determine the RMI in France.

4.7 Linear or non-linear equivalence scales?

We come back here on the shape of the equivalence scale. It is an increasing function of N, but is it concave or convex? With a concave function like N^α, an extra member in the household increases smaller needs in a large family than in a small family. The cost of the third child is smaller than the cost of the first child. We have increasing economies of scale. When the function is convex as with A^{N-1}, this is just the reverse. And we saw that the data could not select one of these models against the other. So a more general form is

$$N^\alpha A^{N-1},$$

which was estimated by Hourriez and Olier (1997) and reported in Table 10. These parameters

Table 10: Estimation of α and A in equation (1)

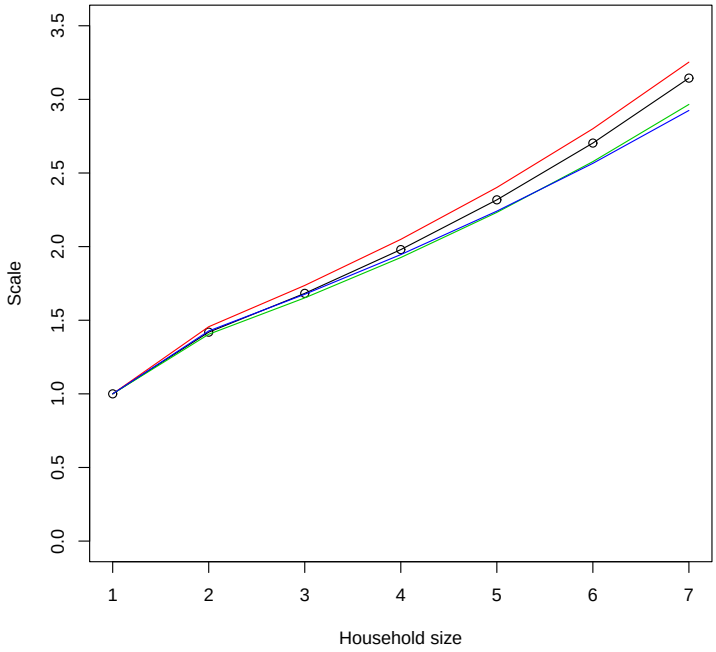
Parameter	1979	1985	1989	1995
α	0.16	0.22	0.18	0.25
A	1.27	1.25	1.24	1.20

are not directly interpretable, we need to translate them for typical household compositions. This is done in Table 11 Scale economies resulting from forming a couple remained fairly constant

Table 11: Final linear equivalence scale

	1979		1985		1989		1995	
Children age	-14	+14	-14	+14	-14	+14	-14	+14
Couple	1.43	1.43	1.46	1.46	1.42	1.42	1.42	1.42
Couple 1 child	1.70	1.92	1.74	1.99	1.67	1.87	1.66	1.86
Couple 2 children	2.01	2.56	2.05	2.65	1.96	2.45	1.92	2.38
Couple 3 children	2.37	3.37	2.41	3.48	2.29	3.16	2.21	3.00

over the years. To reach the level of welfare, a couple needs 40% more income than a single person. The cost of the first child seems to remain the same too with a value of 27% (1.70-1.43 = 0.27). On the contrary, the cost of the third child seems to be decreasing over time. Figure 2 show that there are large economies of scale for a married couple compared bachelors. The economies of scale for the children under 14 years are of course lower, they tend to be linear.



1979 is the black line with circles, 1985 the red upper line, 1989 the green lower line and 1995 the blue lower line.

Figure 2: Convex or concave scales?

5 Child poverty and single equation equivalence scales

The aim of this section is to investigate the methods which can be used in order to measure child poverty while using equivalence scales.

A usual and simple procedure is to look at the level of income of households having children. And then to count the total number of children that belong to the households which are below the poverty line. However this crude method assumes that adults and children have the same consumption needs when in fact children consume proportionally more food than adults. So instead of taking a standard poverty line based on identical value for everybody (like the one-dollar-a-day), a better alternative is to scale that official poverty line by an equivalence scale in terms of the number of equivalent adults. Thus a smaller weight is given to children. The qualification for an household to be below the poverty line changes, but we can still count the total number of children living in those households and compare this number to the total number of children in the sample.

Things might be a bit more complicated. If we look simply at total household income, poverty will be located in small households, because in general income increases with the size of the household. But if we divide the income by the household size, poverty will be concentrated in large households. How to determine the actual needs of the children? We have to solve an allocation problem inside the household. What is the consumption pattern of a child compared to a male adult? There is also the definition of what is the actual cost of a child. A child is seen as a benefit in traditional rural societies or as a high economic cost in industrial societies. We can illustrate this view by looking at the two following pictures taken in China. The left

Figure 3: What is the cost of a child in China?

picture was taken by Liu Zheng and is entitled *A Flower Boy at the Roadside*, the right one was taken in Nanchang TGV railway station. Deaton and Muellbauer (1986) emphasize the fact that there are different views concerning the definition of child cost, so the different approach in the literature do not measure the same thing. In this section, we shall examine two traditional approaches which imply estimating only one equation instead of a complete demand system.

They correspond to the two alternative identification assumptions introduced in a demand system as we have seen above: Engel and Rothbarth. They rely on different assumptions and of course lead to different results. Roughly speaking, Engel's model overestimate the cost of a child, while Rothbarth's model underestimate it.

5.1 Engel

Engel approach starts from two empirical observations:

1. for households of the same demographic composition, the food share varies inversely with income or total expenditure.
2. for households with the same income or total expenditure level, the food share is an increasing function of the number of children.

For a given household, the arrival of a new child implies some consumption recomposition. With the same income, the household consumes for the new born and thus decreases the consumption of previous items. This means that the share of food is increased (a new mouth to feed). In order to reach the same level of welfare the income of the household has to be increased by a certain level, corresponding to the cost of the child. However, there are evidences that households with the same food share, but different composition have the same level of welfare: with a new-born child, the taste of the parents changes. A child consumes mainly food, so that restoring the same food share assumes wrongly that adults and children have the same proportion between food and non-food expanses and leads to an over-compensation. This can be explained in Figure 4. The small reference household is on the left. For a given level of food share, we must increase

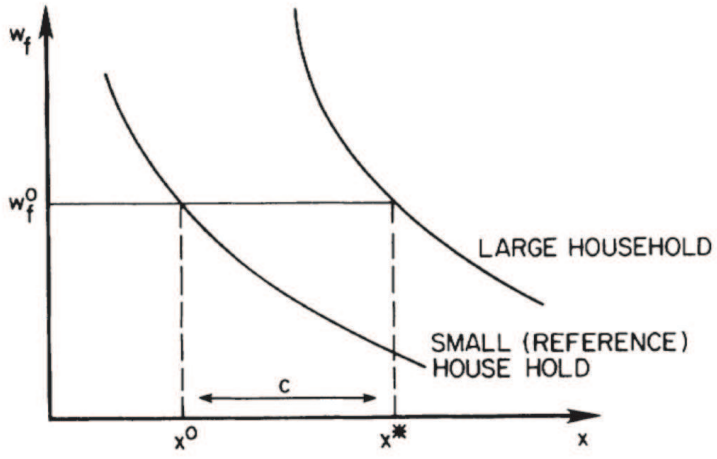


FIG. 1.—Engel's method for child costs

Figure 4: Engel method for equivalence scale

income from x^0 to x^* so that the larger household gets the same level of food share. This distance measure the cost of a child. Of course it depends on the level of income.

5.2 Rothbarth

The method proposed by Rothbarth is similar to that of Engel, but the good is different. In Engel method the share of food decreases when welfare increases, here the share of adult good increases when welfare increases. So The slope of the curves in Figure 5 is positive when it was negative in Figure 4. What we have to compute is the reduction of income that would be necessary to apply

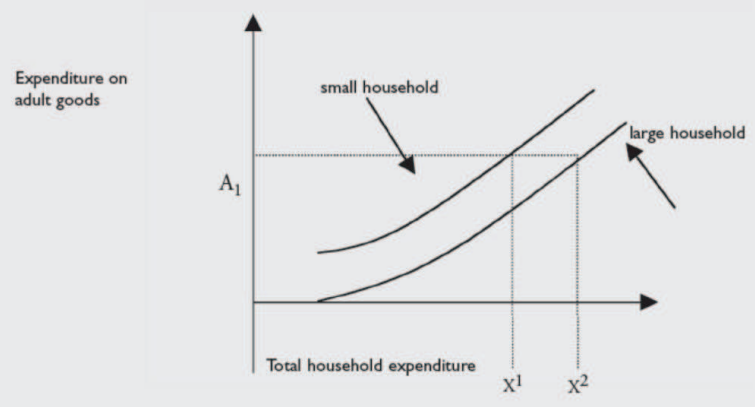


Figure 5: Rothbarth method for equivalence scale

to a childless reference household in order to reach the reduction in budget share that was cause by the arrival of a new child in the considered household. The difference between x^1 and x^2 is the Rothbarth's cost of a child. This method does not take into account substitution effects that are entailed by the arrival of a child. There is no consideration for reduction in the consumption of other items. So in fact, the cost of an extra child is underestimated. Just the contrary of the Engel's method.

Question Explain why Engel's method over-estimate the cost of a child while Rothbarth's method underestimate it.

5.3 Implementation

The equation considered in Koohi-Kamali and Liu (2013) to implement the Rothbarth's method for measuring child poverty in China is:

$$w_j^h = \alpha + \beta \ln(x_h/n_h) + \eta \ln(n_h) + \sum_{i=1}^{2} \delta_i \frac{n_{ih}}{n_h} + \delta z_h + \varepsilon_h,$$

where w_j^h is the budget share of adult good j for household h, followed by per capita expenditure, n_h household size. Using the estimated parameter values and the average total expenditure $\bar{x}$, they compute the reference budget share of adult good j:

$$w_j^0 = \hat{\alpha} + \hat{\beta} \ln(\bar{x}/2) + \hat{\eta} \ln(2) + \delta_1(2/2) + \delta_2(0/2).$$

In order to find the level of income x^* necessary to reach that reference budget share when a child is included in the household, they solve numerically in x^* the following equation:

$$w_j^0 = \hat{\alpha} + \hat{\beta}\ln(x^*/3) + \hat{\eta}\ln(3) + \delta_1(2/3) + \delta_2(1/3).$$

This is used for each type of additional member, adult or child. The adult equivalence scales (AES) for a specific family is obtained as the ratio of the total expenditure of the selected household to the base reference household.

If we do not take into account the economy of scale that is entailed by the arrival of a new member, White and Masset (2002) explain that the actual share of that new individual will be underestimated. They devote many explanation on how to take that into account, while Koohi-Kamali and Liu (2013) neglect this kind of question.

5.4 Equivalence scales and household size

White and Masset (2003) analyse how to compare households of different composition when making poverty profiles. The question they want to address is to know if there is a relation between poverty and household size or if this relation disappears when one takes into account economies of scale. The first point is to take into account difference in prices over time and space. Food, which enter for 70% of household spending in low income countries is much cheaper in rural areas as shown in Table 12.

Table 12: Rural versus Urban poverty lines

	Indonesia 1990	Bangladesh 1995	Peru 1997
Rural	13 295	349.57	1 037
Urban	20 614	455.86	1 968
Ratio	0.64	0.77	0.53

Source: World Bank and White and Masset (2003).

If now we want to have a calory-based poverty line, we see with Table 13 that calory consumption is largely different between male, females and children. We could also have added workload. We can use this information for building an adult equivalence scale, which will be valid for 70% on average of household spending. This table allows to build equivalence taking into account gender and fine age intervals. However, they would concern only for 70% of total spending and thus would assume that the remaining 30% of spending are done in a proportional way, which has not been tested.

Econometric methods derive equivalence scale, using particular assumptions, mainly those done either by Engel or Rothbarth. These methods try to evaluate the cost of a child, which means by how much the welfare of a household is lowered when a new child is added. White and Masset (2003) found that on average in developing countries, the cost of a child in term of adult equivalents ranges from a minimum of 22 per cent to a maximum of 82 per cent, with an

Table 13: Recommended Caloric Intakes by Age and Sex

Age	Male	Female
1	820	820
1-2	1150	1150
2-3	1350	1350
3-5	1550	1550
5-7	1850	1750
7-10	2100	1800
10-12	2200	1950
12-14	2400	2100
14-16	2650	2150
16-18	2850	2150

Source: WHO (1985) and White and Masset (2003).

average value of 43 per cent. If we take the data of Table 13, the calory scale gives to a child between 1 to 15 a weight of 65% of that of an adult. One has also to take into account the economy of scale, which means that there are public goods in the household. If α is the economy of scale coefficient, then the normalized expenditure E_i of household i will be:

$$es_i = \frac{E_i}{AE_i^{1-\alpha}}$$

where AE_i is the number of adult equivalents in household i. Empirical estimates of α are generally in the range [0.15-0.3].

Using Vietnamese data, White and Masset (2003) proposed to modulate the official poverty line computed as the necessary sum to buy 2100 calories per person over a year, using first household composition and second economy of scale. The assumption made were: a child consumes 65% of an adult, the economy of scale is either 0.15 or 0.30. Compared to the World Bank evaluation of poverty, this gives very different poverty profiles for Vietnam: Depending on the assumptions made (no equivalence scale with the World Bank, Equivalence scale and different economy of scale), the relation between family size and poverty can change a lot. Or the impact of having a female headed household can be significant or not. With economies of scale, poverty becomes mostly rural, and hits mostly uneducated people.

5.5 Child poverty in China

Why focussing on child poverty? In fact child poverty is at the heart of an endless cycle of poverty. Poor education, poor health will have a serious impact when entering the labour market and will lead to poor earnings in adulthood. This is the reason why there are grants, family allowances. If adults can be held responsible for their poverty status, this is certainly not the case for kids. For targeting anti-poverty measures, we have to know where child poverty is located.

Table 14: Poverty headcount using different scales

		World Bank no scale	$\alpha = 0.15$ and scale	$\alpha = 0.30$ and scale
Area				
	Rural	18.5	12.9	4.9
	Urban	2.5	1.6	0.2
Household Head				
	Male	16.2	10.7	3.8
	Female	10.6	9.1	3.8
Education				
	< 5 years	20.1	14.7	6.8
	5-10 years	12.6	8.9	2.3
	> 10 years	9.6	5.1	1.0
Children				
	None	4.0	6.9	3.0
	$< 25\%$	8.1	6.9	2.5
	$25\% - 50\%$	16.5	10.6	3.6
	$> 50\%$	33.5	18.3	7.1

Source: White and Masset (2003) using the Vietnam Living Standard Surveys of 1992 and 1997.

Koohi-Kamali and Liu (2013) applies Rothbarth's model to two household expenditure surveys led on the urban part of an east coast province of China in 2002 and 2009 in order to measure child poverty in China. The costal provinces in China are those who benefited the most from the economic reforms and the opening of the economy with an average yearly growth rate of 10% over the period between the two waves of the surveys. Once they have obtained the Adult Equivalent Scale (AES), they modify the poverty line calculated for a two adult family by multiplying it by the adequate AES. With these lines, poor households are located. From this locations, the number of poor children is cumulated.

The average per capita income in this coastal province is US$8 682 in 2012 against US$6 076 for the entire country. The question of the official line is delicate because it is much too low for that part of the country as we have already seen in the empirical application detailed in Chapter 3. The point which is adopted in Koohi-Kamali and Liu (2013) consists in taking the bottom first or second decile of the per capita expenditure distribution, and twice hat number for a two adult childless household. Even if urban areas are over-represented in the sample, this does not invalidate that kind of poverty line as underlined in Ravallion and Chen (2007). Inequality has risen a lot in urban districts in China and that might have a negative impact on poverty.

The estimated results for the Rothbarth equation parameters using OLS are not very good, probably because of data quality and might be inadequate treatment of income variables. The equivalence scale obtained for the two years are reported in Table 15. We note that the implicit cost of a child has risen over the period and is quite similar to the cost reported in the western world.

Table 15: Adult Equivalence scale for a costal province in China

	2002	2009
Two adults	1.00	1.00
Two adults + 1 adult	1.24	1.33
2 Parents + 1 child	1.16	1.22
2 Parents + 2 children	1.23	1.33

Table 16 shows that there has been a reduction in child poverty over the period. However, this drop is rather small, compared to the 10% yearly growth rate of the economy over that period. This illustrate that extreme poverty is much harder to eradicate and requires specific targeted public policies. Moreover, when we detail poverty rates with the number of children, child poverty increases with the size of the family.

Table 16: Child poverty rates

Poverty line	2002 no child	2009 no child	2002 2 children	2009 2 children
First decile	13.8%	12.4%	16.2%	25.1%
Second decile	24.9%	22.3%	31.9%	30.8%

Koohi-Kamali and Liu (2013) conclude that *reduction in urban poverty in China has proved disappointing despite China's impressive economic growth over the decade examined here*.

Question Compare the equivalence scale found in Table 15 for China by Koohi-Kamali and Liu (2013) to the OECD scale. What could you conclude?

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