

# The econometrics of inequality and poverty

## *Lecture 7: Stochastic dominance*

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Some authors like Sen prefer to use a discrete representation of income, which is based on the assumption that the population is finite. Atkinson (1970) and many following authors prefer to suppose that income is a continuous variable. It implies that the population is implicitly infinite, but the sample can be finite. The notion of stochastic dominance is in most cases based on continuous distributions. Atkinson (1970) was the first to make an extensive use of properties of continuous distributions in order to compare income distributions. He drew on the parallel with the formally similar problem of measuring risk and risk aversion in the theory of decision making under uncertainty. See also the first part of Davidson and Duclos (2000) which is an excellent survey of the topic.

The interest in comparing income distributions is to know for instance if the income distribution is more equalitarian after taxes have been paid and transfers realized. The tax and transfer system may not be efficient so that it can transform the income distribution differently at different quintiles.

## 1 Poverty indices, poverty deficit curves

It is quite illuminating to come back to the poverty indices of Foster et al. (1984), supposing now that  $X$  represents a continuous random variable. The FGT indices are based on partial moments with respect to the income distribution.

### 1.1 FGT poverty indices

If  $F(\cdot)$  is the income distribution and  $z$  the poverty line, then for a given  $\alpha$  this family of poverty indices is defined by

$$P_\alpha = \int_0^z (1 - x/z)^\alpha f(x) dx. \quad (1)$$

And of course, we recover some usual poverty indices, letting  $\alpha$  vary between 0 and 2.

**For**  $\alpha = 0$ , we have the headcount measure:

$$P_0 = \int_0^z f(x) dx = F(z). \quad (2)$$

Multiplying  $P_0$  by the population size, we get the number of poor. But we cannot make the difference between poor and very poor people.

**With**  $\alpha = 1$ , we introduce the poverty gap or the poverty deficit  $z - x_i$ :

$$P_1 = \int_0^z (1 - x/z) f(x) dx. \quad (3)$$

This index fulfill the principle of transfers, contrary to the head count measure  $P_0$ . It is continuous in  $x$ , while  $P_0$  is not. But it is not sensitive some types of transfers between the poor.

**For**  $\alpha = 2$ , we arrive at a measure which is sensitive to the income distribution among the poor:

$$P_2 = \int_0^z (1 - x/z)^2 f(x) dx, \quad (4)$$

As underlined in Atkinson (1987) and Foster and Schorrocks (1988), these indices are useful for ranking distributions, to determine for instance in which of two countries there are more poor. The result might depend on the value of  $z$ . As underlined in Foster and Schorrocks (1988), one country may have more poor than the other for a given  $z$ , but just the reverse for another value of  $z$ . We would like to have a result which is independent of the value of  $z$ , or at least valid for a given range of values of  $z$ . This will be the notions of stochastic dominance and restricted stochastic dominance, for which we give an empirical example at the end of this chapter.

## 2 Incidence curves, poverty deficit curves

$P_0(z)$  is a measure of the impact of poverty. It indicates the proportion of poor, the persons below the poverty line  $z$ . This is the headcount measure. If we now let  $z$  varying in the domain of definition of  $x$ , we get the poverty incidence curve, using a terminology due to Ravallion (1996).

Poverty can be measured by counting the poor using  $F(z)$ , the poverty incidence curve. We might like to measure the severity of poverty by measuring the surface under the poverty incidence curve:

$$\int_0^z F(x)dx.$$

We can decompose this surface, using integration by parts with  $u = dx$ ,  $v = F(x)$  and  $z = \int_0^z dx$ :

$$\int_0^z F(x)dx = z \int_0^z f(x)dx - \int_0^z xf(x)dx = z \int_0^z (1 - x/z)f(x)dx.$$

The surface below the incidence curve is thus equal to the poverty line times the truncated mean of the relative poverty gap, the latter being defined by

$$1 - \frac{x}{z}.$$

If we divide on both sides by  $z$ , we get a second poverty index noted  $P_1$ . It is called the normalized poverty deficit by Atkinson

$$\frac{1}{z} \int_0^z F(x)dx = \int_0^z (1 - x/z)f(x)dx = P_1.$$

If we now let  $z$  vary over the domain of  $x$ , we get the poverty deficit curve. If we now call  $\mu_p$  the average standard of living of the poor and using some integral calculus we get

$$P_1 = F(z) \left[ 1 - \frac{\mu_p}{z} \right] = P_0 \left[ 1 - \frac{\mu_p}{z} \right].$$

## 3 Stochastic dominance

### 3.1 The veil of ignorance

The paradigm of the veil of ignorance is a good starting point. It is a method of determining the morality of a certain issue concerning social choice. It prevents the participants from knowing

about who they will be in that society for which a decision has to be made. When they are selecting the principles for the distribution of rights, for positions and resources in the society they will live in, the veil of ignorance prevents them from knowing about who they will be in that society. Considering the paradigm of an immigrant having to choose a society where to go is a good illustration of this paradigm.

We consider an individual decision problem under uncertainty where an immigrant has to choose between two countries  $A$  and  $B$ , each having an income distribution  $X_A$  and  $X_B$ . Once he has chosen a country, his social position will be drawn at random within the income distribution. If he chooses the country where dispersion is lower, he will minimize his risk. If he chooses the country with more inequality, he might get more opportunities of getting richer, but also the risk of being poorer. The immigrant would choose the country for which his expected utility is maximum

$$\max_i \int U(x) f_i(x) dx.$$

The aversion for inequality in the distribution of income  $f_i(x)$  corresponds to the concavity of the utility function  $U(x)$  which means risk aversion. This is similar to an individual decision problem under uncertainty.

Atkinson (1970) took advantage of the story of the veil of ignorance in order to use stochastic dominance to compare income distributions. The individual utility function is transformed into a social welfare function in his paper. Just because whenever we want to compare income distributions, there is a form of social welfare to consider. Under which assumptions over the the social welfare function can we form judgements or choose between different income distributions. When usual indices give conflicting results, how can we restrict the shape of the social welfare function so that, conditional on these supplementary assumptions we can rank different income distributions?

## 3.2 Some definitions and results

Individual decision problems under uncertainty were studied in a series of papers:

- Quirk and Saposnik (1962) Review of Economic Studies
- Hadar and Russell (1969) American Economic Review
- Hanoch and Levy (1969) Review of Economic Studies

The main result of these papers is that:

- if the individual utility function is increasing, he will choose option  $i$  instead of option  $j$  if and only if  $F_i$  stochastically dominates  $F_j$  at the first order.
- If the utility is increasing and concave (risk aversion), then the individual will select option  $i$  instead of option  $j$  if and only if  $F_i$  stochastically dominates  $F_j$  at the second order.

The particular case of Markowitz (1952) based on means and variances is valid only for quadratic utility functions and for Gaussian distribution of the outcomes.

Stochastic dominance is a mathematical notion that allows to compare distributions. It comes from the theory of probabilities (Blackwell (1953)), was used to solve decision problems under uncertainty (Hanoch and Levy (1969)), then in finance to characterize portfolio choices (Fishburn (1977)). Finally, it was used by Atkinson (1970) to compare income distributions.

### 3.3 Mathematical characterization

The usual (simplified) definition of stochastic dominance at the order one (or first degree stochastic dominance) is (see e.g. Hadar and Russell (1969)):

**Definition 1** *The probability distribution  $F$  stochastically dominates the probability distribution  $G$  at the order one if and only if*

$$F(x) < G(x) \quad \forall x \in [0, +\infty[. \quad (5)$$

This definition means that the probability of getting  $x$  or less is not larger with  $F$  than it is with  $G$ , whatever the value of  $x$ . The usual definition make use of loose inequality, but add the restriction that there are at least one point where the inequality is strict.

This definition allows to compare two distributions only when they do not intersect. If they intersect, we cannot conclude. In this case, it might be useful to use a second notion, which is stochastic dominance at the second order. Second order (or second degree) stochastic dominance is based on the comparison of the surface under the cumulative distribution functions and may remove this indeterminacy. We have:

**Definition 2** *The probability distribution  $F$  stochastically dominates the probability distribution  $G$  at the order two if and only if*

$$\int_0^x [F(t) - G(t)]dt < 0 \quad \forall x \in [0, +\infty[. \quad (6)$$

We can define stochastic dominance for any order because there is a strict relation between each order. It is useful to consider a sequence of integrals for a density  $f$  that we define as follows:

$$\begin{aligned} F_0(x) &= f(x) \\ F_1(x) &= \int_0^x F_0(t)dt \\ F_2(x) &= \int_0^x F_1(t)dt \\ &\dots \end{aligned} \quad (7)$$

that we can generalize in the following recurrence relation

$$F_s(x) = \int_0^x F_{s-1}(t)dt = \frac{1}{(s-1)!} \int_0^x (x-t)^{s-1} f(t)dt. \quad (8)$$

The same for density  $g$ . Because distributions are positive and increasing functions of  $x$ , stochastic dominance at the order  $s$ , which can be written as

$$F_s(x) \leq G_s(x) \quad \forall x \in [0, +\infty[$$

implies stochastic dominance at any higher order. In particular, stochastic dominance at the order two

$$F_2(x) \leq G_2(x), \quad \forall x$$

implies

$$F_{2+j}(x) \leq G_{2+j}(x), \quad \forall j \geq 1$$

but does not rely on stochastic dominance at the order 1

$$F_1(x) \leq G_1(x), \quad \forall x.$$

## 4 Ordering income distributions and poverty indices

Let us start from the general recurrence relation:

$$F_s(x) = \int_0^x F_{s-1}(t)dt = \frac{1}{(s-1)!} \int_0^x (x-t)^{s-1} f(t)dt. \quad (9)$$

This last writing is particularly interesting as it directly links the Foster et al. (1984) poverty indices to the notion of stochastic dominance. As a matter of fact, if we set  $x$  equal to the poverty line  $z$ , we discover that the dominance function  $F_s(z)$  is identical to the poverty incidence curve  $P_{s-1}(z)$  modulo a proportional factor that depends only on  $s$ . Stochastic dominance thus correspond to the generalization of these indices when we let the poverty line  $z$  vary over the whole segment  $[0, +\infty[$ . This is the point of view developed in Atkinson (1987) and in Foster and Schorrocks (1988). Let us note that the notion of poverty deficit curve is obtained when we let  $zP_1(z)$  be a function of  $z$ .

The link with poverty indices is even more direct if we consider a notion of restricted dominance instead of a notion of full dominance. We no longer consider inequalities for all  $x$ , but inequalities for a restricted interval  $[z_*, z^*]$ . We thus consider

$$F_s(z) = \frac{1}{(s-1)!} \int_0^z (z-t)^{s-1} f(t)dt \quad \forall z \in [z_*, z^*]$$

This writing allows to compare two income distributions when the poverty line varies between two boundaries. This leads to a robust comparison which will no longer be strictly depend on the definition chosen for the poverty line. If we call  $z - x$  the poverty gap, that is to say the difference between the observed income  $x$  and the poverty line  $z$ , then for  $s = 1$ , we count the number of poor, for  $s = 2$ , we measure the mean of the poverty gap and for  $s = 3$ , we focus our attention on the dispersion of income within a given interval.

## 4.1 Partial moments and poverty indices

Because it may be time consuming to check for stochastic dominance when selecting among a large set of distributions as in portfolio selection, a branch of the literature has devoted efforts to finding necessary (but not sufficient) conditions which enable one to eliminate irrelevant alternatives. In that spirit,

- Bawa (1975) Journal of Financial Economics
- Fishburn (1977) American Economic Review
- Jean (1984) The Journal of Finance

among others introduced *lower partial moments* (LPM) of order  $s$  ( $s \geq 1$ ) for a distribution  $F$  with a reference value  $z$

$$LPM_F^s(z) = \int_0^z (z - t)^s dF(t). \quad (10)$$

The semi-variance corresponds to  $s = 2$ . Fishburn (1977) uses it as a measure of risk in portfolio selection. For a given  $z$ , this measure is asymmetric because it does not treat upper and lower deviations from the mean or from the target symmetrically as the variance does. It concentrates on the left tail of the distribution.

Using integration by parts, it is easy to show by recurrence, the link between the sequence of integrals (9) and the LPM definition (10):

$$F_s(z) = \frac{1}{(s-1)!} LPM_F^{s-1}(z) \quad s \geq 1. \quad (11)$$

Stochastic dominance at the order  $s$  implies the ordering of partial moments starting from order  $s - 1$ . For instance, stochastic dominance at the order two implies the ordering of all partial moments; but the ordering of semi-variances is not a necessary condition for stochastic dominance at the order four. See Jean (1984) for more results on partial moments.

Of course, we see the immediate relation between partial moments and the FGT poverty indices.

## 4.2 Necessary and sufficient condition

LPM can be transformed into a function of  $x \in [0, +\infty[$  as follows:

$$LPM_F^s(x) = \int_0^x (x - t)^s dF(t). \quad (12)$$

Because we have now

$$F_s(x) = \frac{1}{(s-1)!} LPM_F^{s-1}(x), \quad (13)$$

the ordering of LPM functions of order  $s - 1$  for distributions  $F$  and  $G$  corresponding to

$$LPM_F^{s-1}(x) \leq LPM_G^{s-1}(x) \quad \forall x \in [0, +\infty[ \quad (14)$$

is strictly equivalent to the condition for stochastic dominance of  $F$  over  $G$  at the order  $s$ .

## 5 Lorenz curves and stochastic dominance

### 5.1 General equivalence

In a previous chapter, we have studied the Lorenz curve and defined the notion of Lorenz ordering. We are here particularly interested by the notion of generalized Lorenz curve which is defined as:

$$GLC(p) = \int_0^p F^{-1}(q) dq, \text{ for } p \in [0, 1].$$

We say that a distribution  $F$  is preferred to a distribution  $G$  in the Lorenz sense if and only if:

$$GLC_F(p) \geq GLC_G(p) \text{ for all } p \in [0, 1].$$

In other words,  $F \succeq_{GL} G$  if the mean income of the  $100p$  per cent of the population in  $F$  is no smaller than that in  $G$  and for some  $p$ , this mean income is greater (full definition). This notion is strictly equivalent to that of second order stochastic dominance.

**Theorem 1** *Let us consider two income distributions  $F$  and  $G$ . It is strictly to say that  $F$  stochastically dominates  $G$  at the second order or to say that  $F$  Lorenz dominates  $G$  in the Generalized Lorenz sense.*

To prove this theorem, we have to show that

$$\int_0^z [G(t) - F(t)] dt \geq 0 \text{ for all } z \Leftrightarrow \int_0^p [F^{-1}(q) - G^{-1}(q)] dq \geq 0$$

These two conditions are equivalent. First note that we have

$$\int_0^z [G(t) - F(t)] dt = \int_0^z [(1 - F(t)) - (1 - G(t))] dt$$

Then using integration by parts with presumably  $u = t$  and  $v = -(1 - F(t))$ , we have

$$\int_0^z (1 - F(t)) dt = \int_0^z t f(t) dt$$

Let us introduce the change of variable  $t = F^{-1}(q)$  with  $dq = f(t) dt$ , then

$$\int_0^z (1 - F(t)) dt = \int_0^z t f(t) dt = \int_0^p F^{-1}(q) dq$$

which completes the proof.

### 5.2 What to do when Lorenz curves intersect?

When Lorenz curves intersect or when dominance curves intersect, this means that there is no unambiguous ordering of income distributions whatever the class of social welfare functions. In this case, we have to make assumptions about the shape of the welfare function, which means



operating under more restrictive assumptions than those made above. Atkinson (1970) promotes the use of a particular welfare function  $U(y)$  which is symmetric and additively separable in individual income:

$$U(y) = A + B \frac{y^{1-\epsilon}}{1-\epsilon}.$$

Inequality is measured as:

$$I_A = 1 - \left[ \int \left( \frac{y}{\mu} \right)^{1-\epsilon} f(y) dy \right]^{1/(1-\epsilon)}.$$

The sole question is then to select the value of the parameter of aversion to inequality. Atkinson (1970) in his reported empirical application shows that the Gini is coherent with rankings produced when  $\epsilon < 1$  in the above Atkinson index. The Gini index is more sensitive to changes for the middle class. With  $\epsilon > 1$ , a greater concern for the poor is incorporated in the welfare function.

## 6 Stochastic dominance for usual parametric distributions

### 6.1 Stochastic dominance for Pareto distributions

The cumulative distribution for the Pareto process is:

$$F(x) = 1 - h^\alpha x^{-\alpha} \quad \text{if } x \geq h, \quad 0 \text{ otherwise.}$$

For two Pareto processes with respective distributions  $F_1(x|h_1, \alpha_1)$  and  $F_2(x|h_2, \alpha_2)$ , we have stochastic dominance at the order one of  $F_1$  over  $F_2$  if and only if:

$$F_1(x|x_{m1}, \alpha_1) < F_2(x|x_{m2}, \alpha_2), \quad \forall x \in [0, \infty[.$$

This condition is verified if we have first:

$$h_1 \geq h_2.$$

The condition on  $\alpha_1$  and  $\alpha_2$  is more difficult to find.

The second order dominance curve can be found either by integration (using Maple):

$$F_2(x) = \frac{1}{\alpha - 1} (h^\alpha x^{1-\alpha} + (\alpha - 1)x).$$

Parametric conditions for stochastic dominance at the order two are difficult to find. A condition like

$$\alpha_1 \geq \alpha_2, \quad \frac{h_1}{\alpha_1 + 1} \leq \frac{h_2}{\alpha_2 + 1},$$

seems to work for stochastic dominance at the second order of  $F_1$  over  $F_2$ . But a proof is needed.

## 6.2 Ordering log normal distributions

Stochastic dominance for the log-normal process was first analysed in Levy (1973). More precisely, his theorem 4 states:

**Theorem 2** *Let  $F_1$  and  $F_2$  be two log-normal distributions with parameters  $\mu_j$  and  $\sigma_j$ ,  $j = 1, 2$ .  $F_2$  dominates  $F_1$  at the order one if  $\mu_2 > \mu_1$  and  $\sigma_1 = \sigma_2$ .*

Stochastic dominance at the order one requires that the two processes have the same log variance. Otherwise we cannot conclude. Note that this result is different than for Lorenz dominance where the condition implied the comparison of  $\sigma_1$  and  $\sigma^2$ , whatever the value of  $\mu_j$ .

This criterion is very restrictive, since the probability that the variance of two distributions are identical is very small. Then, Levy and Kroll (1976) have introduced a new criterion, which is less restrictive, but includes the previous result as a special case. This criterion is as follows:

**Theorem 3** *Let  $F_1$  and  $F_2$  be two log-normal distributions with parameters  $\mu_j$  and  $\sigma_j$ ,  $j = 1, 2$ .  $F_2$  stochastically dominates  $F_1$  at the first order if and only if the following two conditions simultaneously hold:*

$$\exp\left(\mu_2 + \sigma_2 \left(\frac{\mu_2 - \mu_1 + \ln(\sigma_2/\sigma_1)}{\sigma_1 - \sigma_2}\right)\right) > \exp\left(\mu_1 + \sigma_1 \left(\frac{\mu_2 - \mu_1 + \ln(\sigma_2/\sigma_1)}{\sigma_1 - \sigma_2}\right)\right).$$

*If there is equality in the first condition, the second condition reduces to  $\mu_2 > \mu_1$ .*

However, when we try to apply this criterion, it does not seem to work, and more work and verification is certainly needed. Using a Monte Carlo experiment where the parameters of two lognormal processes are drawn at random, a necessary condition seems to be that  $\sigma_2 \leq \sigma_1$ , contrary to what is stated in the paper. In the same experiment, the second condition seems to be implied by stochastic dominance at the first order, but not the reverse. Certainly more verification is needed.

Finding a criterion for stochastic dominance at the order two is certainly more easy to find. We have the first original result in Levy (1973). But we can consult also Levy and Kroll (1976) or Yitzhaki (1982). Theorem 5 in Levy (1973) states that:

**Theorem 4** *If  $F_2$  and  $F_1$  are log-normal distributions, we can say that  $F_2$  stochastically dominates  $F_1$  at the second order if and only if the following three conditions simultaneously hold:*

$$\begin{aligned} \mu_2 &\geq \mu_1, \\ \sigma_2 &\leq \sigma_1, \\ \mu_2 + \sigma_2^2/2 &\geq \mu_1 + \sigma_1^2/2. \end{aligned}$$

This criterion is more easy to apply than those given for first order stochastic dominance. Essentially,  $\sigma_2 \leq \sigma_1$  correspond to Lorenz ordering while the two other conditions insure that the mean of the random variable is greater in  $F_2$  than in  $F_1$ .

### 6.3 Stochastic dominance for Weibull distributions

For the Weibull distribution, the cumulative distribution is given by:

$$F(x) = 1 - \exp(-(kx)^\alpha),$$

while the mean is:

$$E(x) = \frac{1}{k} \Gamma(1 + 1/\alpha).$$

Stochastic dominance between two Weibull distributions  $F_1$  and  $F_2$  of parameters  $\alpha_1, k_1$  and  $\alpha_2, k_2$  was characterized in Lubrano and Protopopescu (2004).

For stochastic dominance at the order one, we have:

**Theorem 5** *If  $F_1$  and  $F_2$  are two Weibull distributions with parameters  $\alpha_i$  and  $k_i$ ,  $i = 1, 2$ , we can say that  $F_1$  stochastically dominates  $F_2$  at the first order if and only if the following two conditions are met:*

$$\begin{aligned} \alpha_1 &= \alpha_2, \\ k_1 &\leq k_2. \end{aligned}$$

**Proof 1** *Find the condition for which the analytical cumulative distributions can be compared for all  $x \in R^+$ .*

We can now characterize generalized Lorenz dominance and then stochastic dominance at the order two.

**Theorem 6** *If  $F_1$  and  $F_2$  are two Weibull distributions with parameters  $\alpha_i$  and  $k_i$ , we can say that  $F_1$  stochastically dominates  $F_2$  at the second order if and only if the following conditions are met:*

$$\begin{aligned} \alpha_1 &\geq \alpha_2, \\ \frac{1}{k_1} \Gamma\left(1 + \frac{1}{\alpha_1}\right) &\geq \frac{1}{k_2} \Gamma\left(1 + \frac{1}{\alpha_2}\right), \end{aligned}$$

where  $\Gamma(\cdot)$  is the Gamma function.

**Proof 2** *Find the conditions for which the generalized Lorenz curves can be compared for all  $p$ .*

## 7 Empirical application

We consider again the FES data in order to compare various notions of dominance. In a routine written in  $R$ , we first read the data and scale them using the consumption price index as was done in the previous chapters.

```

data1=read.table("fes79.csv",header=F,sep=";")
data2=read.table("fes88.csv",header=F,sep=";")
data3=read.table("fes92.csv",header=F,sep=";")
data4=read.table("fes96.csv",header=F,sep=";")

y79 = data1[,1]/223.5*223.5
y88 = data2[,1]/421.7*223.5
y92 = data3[,1]/546.4*223.5
y96 = data4[,1]/602.4*223.5

```

## 7.1 Lorenz curves

We then use the library *ineq* to compute and plot a non-parametric estimate of the Lorenz curves for three years: 1979, 1992 and 1996. 1988 is not indicated to simplify the graph. It is just in between 1992 and 1996.

```

library(ineq)
plot(Lc(y79))
text(0.21,0.15,"1979")
lines(Lc(y92),col="blue")
text(0.70,0.40,"1992")
lines(Lc(y96),col="green")
text(0.50,0.30,"1996")

```

Figure 1 clearly indicates a perfect Lorenz ordering. None of the curves are intersecting. The year 1979 presents a rather low inequality in the income distribution. There is a great increase in income inequality in 1992. With 1996, there is a marked return to less inequality, but without reaching the lower level of 1979.

## 7.2 Dominance curves

We must now remember that Lorenz ordering is not equivalent to stochastic dominance. The only equivalence we can find is between generalised Lorenz ordering and stochastic dominance at the order two. We know also that the FGT indices are strictly equivalent (up to a scale factor) to the dominance curves when we let the poverty line  $z$  vary.

```

Dc1a = 1:100
Dc1b = 1:100
Dc1c = 1:100
xmin = 8
xmax = 200
a = 1
dx = (xmax-xmin)/99
z = seq(from = xmin, to = xmax, by = dx)

```

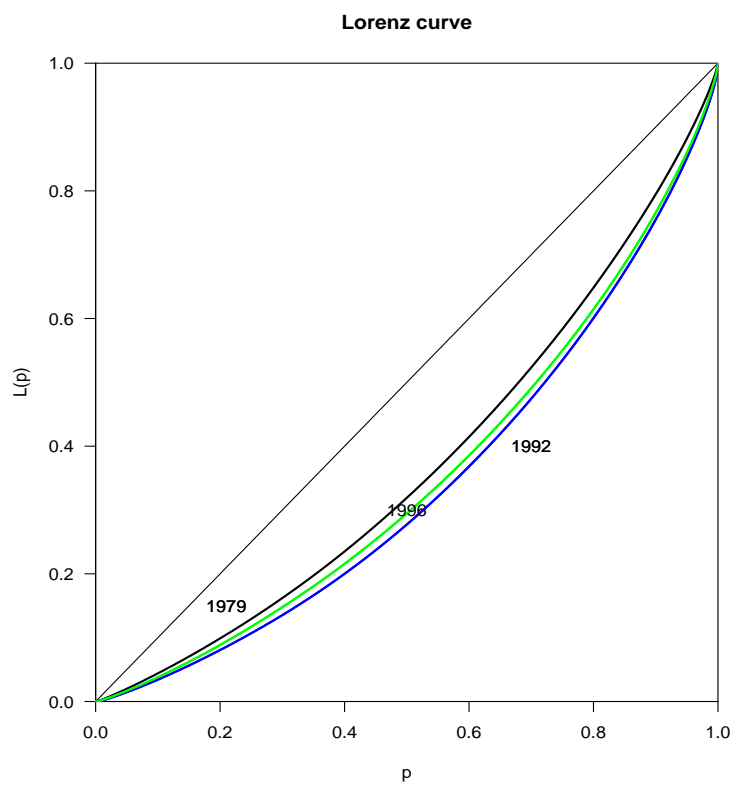


Figure 1: Lorenz curves for the FES in the UK

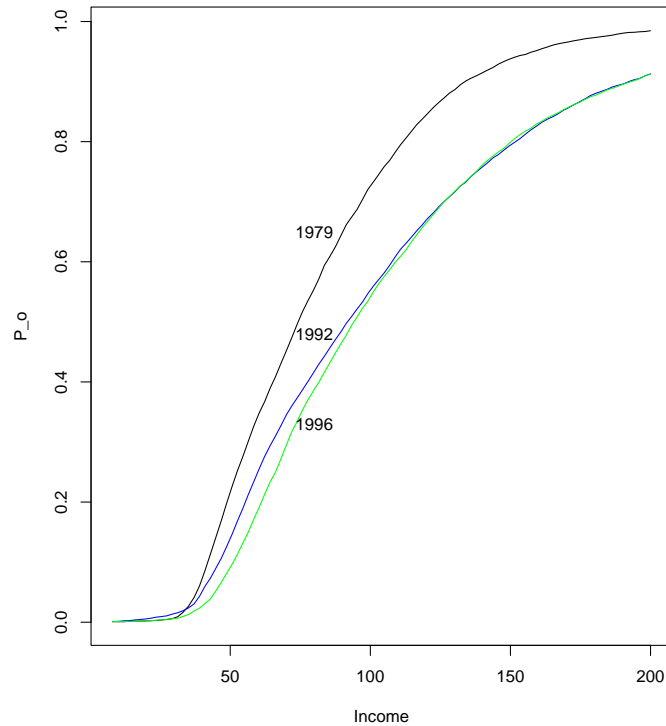


Figure 2: Dominance curves at the first order for the FES in the UK

```

for(i in 1:100)
{
Dc1a[i] = Foster(y79,z[i],parameter=a)
Dc1b[i] = Foster(y92,z[i],parameter=a)
Dc1c[i] = Foster(y96,z[i],parameter=a)
}
plot( z,Dc1a,type="l",xlab="Income",ylab="P_o")
text(80,0.65,"1979")
lines(z,Dc1b,col="blue")
text(80,0.48,"1992")
lines(z,Dc1c,col="green")
text(80,0.33,"1996")

```

Let us recall that the poverty line is defined as half the mean of income which gives 41£ for 1979, 55£ for 1992 and 56£ for 1996. However, the dominance curves are computed for a whole range of values, so that these values are only indicative. Figure 2 represents the dominance curves at the order one. They give the proportion of poor for a given level of  $z$ . They are graphed for  $z \in [8, 200]$  which represents a very large interval. In Table 1, we give the extreme quantiles of

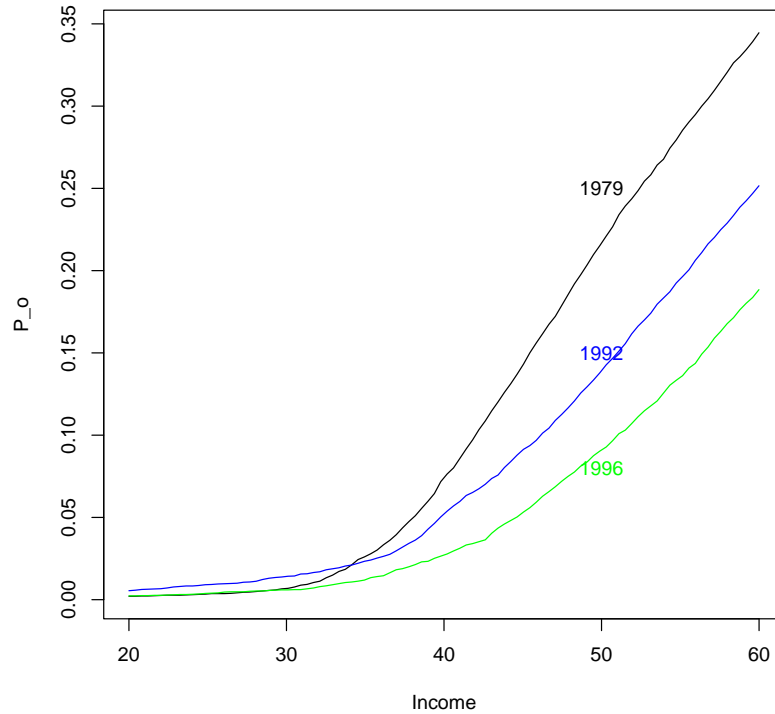


Figure 3: Close up of dominance curves

the income distribution: Consequently, this means that the dominance curve covers on average

Table 1: Quantiles of the UK income distribution  
in 1979 pounds per week

Year	1%	50%	99%
1979	31.63	74.23	217.66
1992	26.86	91.91	350.15
1996	33.51	94.20	346.49

90% of the income values, but with a slight translation toward the poorer part of the distribution.

The situation in 1992 dominates the situation in 1979, despite the increase of inequality. However, as shown in Figure 3, the two curves intersect for very low incomes below 35£. The situation of the very poor was worse in 1992. The whole situation became significantly better in 1996. For people over 100£ of income, there is no change. But the change is important for people under that value as shown clearly in Figure 3. For a given level of income, the proportion

of poor is lower in 1996 than both in 1992 and in 1979.

### 7.3 Atkinson welfare function

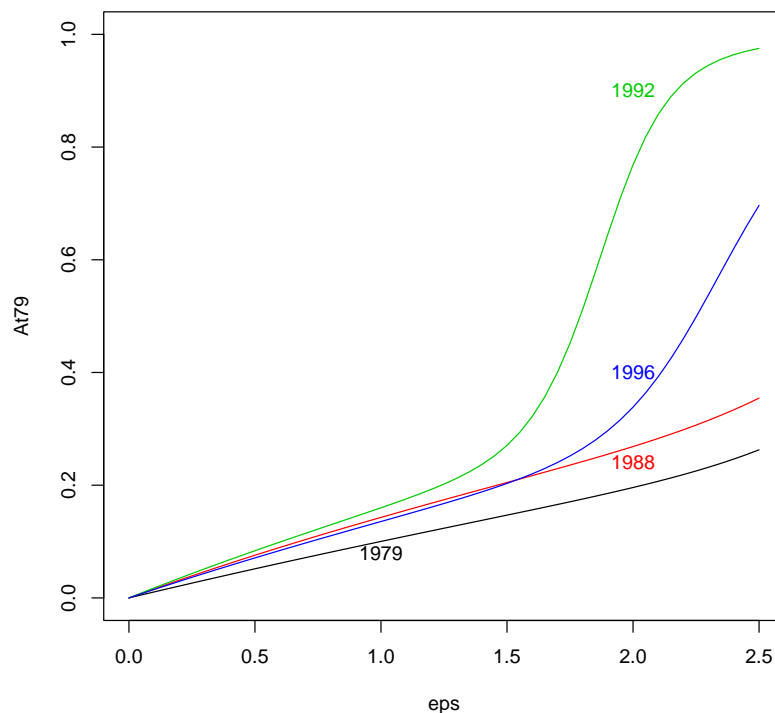


Figure 4: Atkinson indices as a function of  $\epsilon$

When dominance curves intersect, we cannot conclude the comparison between different income distribution, whatever the social welfare function. We have to choose a specific social welfare function and decide for a level of inequality aversion. In Figure 4, we give Atkinson indices as a function of  $\epsilon$ , the degree of aversion to inequality. We will thus be able to compare income distribution when focussing on a particular part of it. We notice first of all that the ranking valid for small  $\epsilon$  is no longer valid for larger values. Lower inequality in 1996 than in 1988 is a valid result when  $\epsilon < 1.5$ . The Gini coefficients are 0.256, 0.307, 0.321, 0.297 for the four samples. Gini coefficient favour changes for middle classes. For a larger concern to the poor ( $\epsilon = 2$ ), Inequality was reduced in 1996 compared to 1992, but is still much higher than in 1988 and surely 1979.



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