BAYESIAN TESTS FOR CO-INTEGRATION IN THE CASE OF STRUCTURAL BREAKS: AN APPLICATION TO THE ANALYSIS OF WAGE MODERATION IN FRANCE *

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Abstract

This paper considers a special non-linear time series problem, that of testing for co-integration in a Bayesian framework when there is a break in the co-integrating relationship. It is shown that a partial linearization of the likelihood function solves many puzzling questions, in particular identification and common factor restrictions which are originally imbedded in the model. A generalization of the Jeffreys' prior is derived for the dynamic parameter which monitors co-integration. The procedure is applied to a one time much debated question in France which concerns the wage regulation policy implemented at the beginning of the eighties.

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Résumé

Ce papier considère un problème particulier de séries temporelles non linéaires qui consiste à tester la cointégration dans un cadre Bayésien quand il y a une rupture dans la relation de long terme. On y montre qu'une linéarisation partielle de la fonction de vraisemblance résoud bon nombre de questions embarassantes, en particulier d'identification et de restriction de facteur commun qui sont originellement contenues dans le modèle. Une généralisation de l'a priori de Jeffreys est calculée pour le paramètre dynamique qui commande la cointégration. La procédure est appliquée à un problème empirique qui a été longuement débattu en France et qui concerne la politique de régulation salariale mise en place au début des années quatre vingt.

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INTRODUCTION

Most economic time series present a break around the year 1974. The possible presence of this break in the trend of the series may have drastic consequences for the investigation of various questions such as unit roots testing (see e.g. the paper by Perron [1989]), the measure of the importance of the unit root in the series usually called the measure of persistence, and finally in the investigation of the presence of co-integration relationships in multivariate time series. The aim of this paper is precisely to investigate this last question. For this purpose, I use a Bayesian framework and this decision is grounded on the following reasons. First the date of the break is usually unknown even if 1974 is a good candidate. The break can appear around this date, or may be rather vague. It has to be determined endogenously. Besides, the linearity of the model (no break) has to be tested. The Bayesian approach is especially convenient for that purpose. Finally, the model of interest is fundamentally non-linear; the Bayesian approach, with its full use of the likelihood function (integration versus maximization) is the most capable of extracting all the information contained in the sample, especially when the latter is rather small. Classical methods, on the contrary, are not well equipped for this kind of problems, as I shall detail in the paper.

Beside its theoretical interest for the development of new Bayesian tools, the paper is motivated by an economic question which concerns the apparent wage de-indexation that was implemented in France (and also in Belgium) at the beginning of the eighties. Using a Phillips' curve model and classical methods, Ralle and Toujas-Bernatte [1990] concluded that wages and consumption prices ceased to move at the same pace after 1982 due to a wage regulation policy. I shall propose a different model which tells a slightly different story.

The paper is organized as follows. In a first section, I detail the non-linear time series model I use in the paper for testing for co-integration and the statistical problems that are induced by its use in a Bayesian Framework. In a second section, I propose new solutions for a Bayesian analysis and derive the Phillips-Jeffreys' prior for the co-integration case. In the last section, I propose a reappraisal of the wage regulation policy that was implemented in France at the beginning of the eighties.

1 A CO-INTEGRATION MODEL WITH AN ENDOGENOUS BREAK

In a Bayesian framework, a model is composed of a sample space Y, a parameter space Θ and a joint probability measure Π on these two spaces. The purpose of inference is a dual decomposition of this joint probability into:

$$\begin{aligned}\Pi(y,\theta) &= D'(\theta)\, l(y|\theta) \\ &= D''(\theta|y)\, p(y)\end{aligned}$$

$D'(\theta)$ is the prior density and $l(y|\theta)$ the likelihood function. After the sample is observed, the alternative decomposition of $\Pi(y,\theta)$ gives the predictive density $p(y)$ and the posterior density $D''(\theta|y)$. The questions I shall ask to the data in the empirical part of the paper will find answers in the inspection of the marginal posterior density of various parameters of my model.

1.1 The basic non-linear model

The model I shall consider is a non-linear regression model with autocorrelated errors and integrated regressors. More precisely:

$$\left\{\begin{array}{rcll} y_t & = & x_t'\beta_1 + ID(\tau)\, x_t'(\beta_2 - \beta_1) + u_t & \\ u_t & = & \rho\, u_{t-1} + \epsilon_t & \epsilon_t \sim IN(0, \sigma^2) \\ \Delta x_t' & = & V_t & V_t \sim N(0, \Omega) \end{array}\right. \tag{1}$$

$ID(\tau)$ is an indicator function which is zero if the parameter τ is lower or equal than an unknown date t_0 and equal to one otherwise. The first equation in (1) represents typically a long run co-integration relationship between y_t and x_t. This relationship may have a break at time $\tau = t_0$ which induces a change in the regression coefficient β which is thus equal to β_1 in the first period and to β_2 in the second. The second equation in (1) determines if there is co-integration or not. If ρ is equal to one, this means that the error term u_t contains a unit root. So there is no linear (non-linear here) combination of y_t and x_t which is stationary. The last equation in (1) says that x_t is I(1) and y_t also by consequence. It says also that x_t is exogenous for inference on β. V_t is independent of ϵ_t.

If ρ is lower than one, the first two equations in (1) can be combined into an Error-Correction-like model with:

$$\begin{aligned} \Delta y_t = \Delta x_t'\beta_1 + \Delta(ID(\tau)\, x_t')(\beta_2 - \beta_1) \\ + (\rho - 1)\,(y_{t-1} - x_{t-1}'\beta_1 - ID(\tau)\, x_{t-1}'(\beta_2 - \beta_1)) + \epsilon_t \end{aligned} \tag{2}$$

Three posterior densities are interesting for my purpose:

- the posterior density of ρ will serve as a basis for the test of co-integration.
- the posterior density of τ will give indications on the presence of a break and its place in the sample. It may eventually give indications on the presence of several possible breaks.
- the posterior density of β will finally produce information on the co-integrating vector. It may eventually serve to test for the presence of a break just by testing if the difference $\beta_2 - \beta_1$) is zero.

1.2 Statistical problems

The analysis of this type of model in a Bayesian framework has long history, but the latter is split in two different branches of the literature. Non-linear models with an endogenous break point based on a time index have been studied for instance in Bacon and Watts [1971] and Ferreira [1975]. The interest was focused on the posterior density of the break indicator (sharp or smooth). But no autocorrelation was allowed for. Conversely, regression models with autocorrelated errors were investigated by many authors such as O'Brien [1970] or Zellner [1971], but in a linear context. Many problems are introduced by the autocorrelation in the error process.

- as shown in O'Brien [1970], there are numerical problems as soon as ρ approaches 1 when there is a constant term. The marginal likelihood function in ρ has a singularity at $\rho = 1$ as shown in Lubrano [1995]. If x_t is reduced to a constant term μ (and maybe a trend), the writing of the model includes the term $(\rho - 1)\,\mu$ which vanishes at $\rho = 1$, creating thus an identification problem for μ. O'Brien [1970] recommends to use a prior on ρ which cancels out a factor of the likelihood function. This solution seems too radical and introduces a prior information on ρ which favours very much stationarity[1] ($\rho < 1$).

- the model contains a common factor restriction as it is apparent in (2). The long term and short term coefficients are constrained to be the same. This restriction is most of the time not validated empirically. This creates a distortion for testing for co-integration as shown in Kremers *et al* [1992] in a classical framework.

- if the common factor restriction is removed by another parameterization such as:

$$\begin{aligned}\Delta y_t = \Delta x_t'\gamma_1 + \Delta(ID(\tau)\,x't)(\gamma_2 - \gamma_1) \\ + (\rho - 1)\,(y_{t-1} - x_{t-1}'\beta_1 - ID(\tau + 1)\,x_{t-1}'(\beta_2 - \beta_1)) + \epsilon_t\end{aligned} \tag{3}$$

  the long term parameters β are no longer identified when ρ approaches 1. This is natural as no co-integration implies no long term solution, but the numerical problem induced by this lack of local identification at $\rho = 1$ begins well below this value. Together with the last equation in (1), (3) is similar to the model used in Banerjee *et al* [1986], except of course for the non-linear term $ID(\tau)$.

- The question of the prior density on ρ, at least when x_t is reduced to a constant term and a trend has been much debated. In a classical framework, the OLS estimates of ρ is downward biased, favouring stationarity. In a Bayesian framework, a flat prior on ρ favours stationarity. Phillips [1991] has proposed a Jeffreys' prior which tends to infinity for explosive values. This prior has been very much criticized (see the October 1991 special issue of the Journal of Applied Econometrics) as it is hardly compatible with any subjective assessment. Schotman [1994], following Schotman and van Dijk [1991], has proposed a joint prior on μ (the unconditional mean of the series) and ρ. The posterior density of ρ can vary very much according to the degree of prior dependence between μ and ρ. Berger and Yang [1994] compares the use of various non-informative priors, including the Jeffreys' prior. They recommend the use of a symmetrized proper version of the stationary reference prior. De la Croix and Lubrano [1995] have extended this prior to the case of models with co-integration.

[1] See Lubrano [1995] for more comments.

- Kleiberger and van Dijk [1994] have a funny way to solve the identification problem. They take advantage of the very special shape of their Jeffreys' prior to skip the problem. When the long term regression (1) contains a constant term, the Jeffreys' prior is zero for $\rho = 1$. So a null weight is put on the point causing trouble. However, this prior implicitly exclude the case when there is no co-integration. If I am interested in testing for co-integration, this prior is thus not very suitable. On the contrary, a Jeffreys' prior derived without the assumption that x_t contains a constant term does not present this drawbacks, but of course does not solve the identification problem.

1.3 A classical test for co-integration

Gregory and Hansen [1992] have proposed a co-integration test[2] along the lines of Engle and Granger [1987] which takes into account the possibility of a structural break. The main question in a classical framework is to derive and tabulate the distribution of a test statistics under the null and eventually give an indication of its power.

The starting point is a static regression as defined in (1) which gives for each different value of τ an estimate of $\hat{u}_t$. Then for each τ an AEG test[3] is computed. The final value of τ is chosen so as to give the minimum value to the AEG test.

The above described classical approach for testing co-integration in the presence of a structural break has two drawbacks:

- as it is well-known (Kremers *et al* [1992]), the AEG test imposes a common factor restriction which distorts the size of the test and reduces its power if there is no common factor restriction in the data generating process.

- the procedure of Gregory and Hansen [1992] violates the likelihood principle as $\hat{\tau}$ is not determined by the likelihood function of the model, but by the value of the test statistics. Moreover the value of the AEG test is dependent on the chosen number of lags p which should be adjusted for each value of τ during the grid search, in pure logic, but which is not in practice.

These reasons give ample justifications for the Bayesian approach to this problem.

2 BAYESIAN ANALYSIS OF NON-LINEAR ERROR CORRECTION MODELS

A Bayesian analysis of model (3) is not an easy task, witness the discussion conducted above. There are essentially two problems. The identification problem when ρ approaches the value 1 and the choice of an adequate reference prior for ρ which does not favour too much the hypothesis of co-integration.

[2]See also Bai *et al* [1991].

[3]Augmented Engle Granger test, the co-integration test proposed by Engle and Granger [1987] with p additional lags introduced to cope with remaining autocorrelation.

The identification problem comes from the simultaneous inference made on ρ and β. If I manage to separate the inference on these two parameters, the identification problem disappears. In the classical literature, there is a method which uses a similar approach: the two step estimation method of Engle and Granger [1987]. I shall show below how a simple linearization of (3) helps to build a Bayesian equivalent to this estimation method.

In the simple model $\Delta y_t = \Delta x_t \gamma + (\rho - 1)\,(y_{t-1} - x_{t-1}\beta) + \epsilon_t$ where x_t is scalar, let me consider the product $(\rho - 1)\,\beta$ and its first order Taylor expansion around the, for the while unspecified, values $\hat{\rho}$ and $\hat{\beta}$:

$$(\rho - 1)\beta \simeq (\hat{\rho} - 1)\hat{\beta} + (\rho - \hat{\rho})\hat{\beta} + (\beta - \hat{\beta})(\hat{\rho} - 1)$$

For $\hat{\rho} = 1$ (hypothesis of no co-integration) the linearization reduces to $(\rho - 1)\beta \simeq (\rho - 1)\hat{\beta}$ and the model becomes $\Delta y_t = \gamma \Delta x_t + (\rho - 1)\,(y_{t-1} - x_{t-1}\hat{\beta}) + \epsilon_t$. This suggests a "long term" model $y_t = x_t\beta + u_t$ to make inference on β and a "short term" model from which inference on ρ (and also on γ) can be drawn. This last model will be mainly used for testing for co-integration and it does not impose any common factor restriction. Our case is a little more complex due to the presence of the endogenous break-point.

2.1 Inference in the long term model

The "long term" model comes from (1) when considering that:

$$u_t \sim IN(0, \omega^2) \tag{4}$$

It is non-linear in τ and will be used for inference both on β and τ. It is convenient to define:

$$x'_t(\tau) = [x'_t, ID(\tau)\, x'_t] \tag{5}$$

and write the corresponding likelihood function:

$$L(y; \beta, \tau, \omega^2) \propto \omega^{-T} \exp -\frac{1}{2\,\omega^2} \sum_{t=1}^{T} (y_t - x'_t(\tau)\beta)^2 \tag{6}$$

Under a diffuse prior for β and ω^2, the conditional posterior density of β is Student[4]:

$$D''(\beta|\tau, y) = f_t(\beta|\beta_*(\tau), M_*(\tau), s_*(\tau), T) \tag{7}$$

together with:

$$\left\{ \begin{array}{lcl} M_*(\tau) & = & \sum x_t(\tau)\, x'_t(\tau) \\ \beta_*(\tau) & = & M_*^{-1}(\tau) \sum x_t(\tau)\, y_t \\ s_*(\tau) & = & \sum y_t^2 - \beta'_*(\tau)\, M_*(\tau)\, \beta_*(\tau) \end{array} \right. \tag{8}$$

[4] The notation of the Student density is as follows:

$$f_t(\theta|\bar{\theta}, M, s, \nu) \propto |M|^{1/2} s^{\nu/2} [s + (\theta - \bar{\theta})' M (\theta - \bar{\theta})]^{-(\nu+k)/2}$$

The posterior density of τ obtains as one over the integrating constant of the above Student density times the prior density of τ:

$$D''(\tau|y) \propto |s_*(\tau)|^{-T/2} \, |M_*(\tau)|^{-1/2} \, D'(\tau) \tag{9}$$

The posterior moments τ and a graph of its posterior density are obtained by numerical integration of this posterior density. The posterior moments of β are obtained by numerical integration using the traditional formulae for conditional expectations and variances. Starting from:

$$\begin{aligned} \mathrm{E}(\beta|\tau, y) &= \beta_*(\tau) && (10) \\ \mathrm{Var}(\beta|\tau, y) &= s_*(\tau)\frac{M_*^{-1}(\tau)}{T-2} && (11) \end{aligned}$$

the marginal moments are computed with the following formulae:

$$\mathrm{E}(\beta|y) = \int \mathrm{E}(\beta|\tau, y) \, D''(\tau|y) \, d\tau \tag{12}$$

for the marginal expectation. The marginal variance of β is more complex to compute as:

$$\mathrm{Var}(\beta|y) = \mathrm{E}_\tau[\mathrm{Var}(\beta|\tau, y)] + \mathrm{Var}_\tau[\mathrm{E}(\beta|\tau, y)] \tag{13}$$

These computations are decomposed into:

$$\begin{aligned} \mathrm{E}_\tau[\mathrm{Var}(\beta|\tau, y)] &= \int \mathrm{Var}(\beta|\tau, y) \, D''(\tau|y) \, d\tau && (14) \\ \mathrm{Var}_\tau[\mathrm{E}(\beta|\tau, y)] &= \int \mathrm{E}(\beta|\tau, y) \, \mathrm{E}(\beta|\tau, y)' \, D''(\tau|y) \, d\tau - \mathrm{E}(\beta|y) \, \mathrm{E}(\beta|y)' && (15) \end{aligned}$$

2.2 Inference in the short term model

The short term model comes from (1) when the first two equations are combined together, the common factor restriction relaxed as in (3) and β and τ replaced by their posterior expectation obtained from the "long term" model. Let me define:

$$\hat{u}_t = y_t - x_t'\hat{\beta}_1 - ID(\hat{\tau}) \, x_t'(\hat{\beta}_2 - \hat{\beta}_1) \tag{16}$$

where $\hat{\beta}$ and $\hat{\tau}$ come from the previous step. Introducing this expression into (3), I get:

$$\Delta y_t = \Delta x_t'\gamma_1 + \Delta(ID(\hat{\tau}) \, x_t')(\gamma_2 - \gamma_1) + (\rho - 1) \, \hat{u}_{t-1} + \epsilon_t \tag{17}$$

which is linear in ρ. However I want to be free to use any type of prior density for ρ. The use of a general prior implies that I get out of the natural conjugate framework and may have to numerically integrate out ρ. Consequently it is convenient to consider that (17) is non-linear in ρ. Let me define the following notations:

$$\left\{ \begin{array}{lcl} y_t(\rho) & = & \Delta y_t - (\rho - 1) \, \hat{u}_{t-1} \\ \tilde{x}_t' & = & [\Delta x_t, \Delta(ID(\hat{\tau}) \, x_t)] \\ \theta' & = & [\gamma_1, \gamma_2 - \gamma_1] \end{array} \right. \tag{18}$$

The likelihood function of (17) is:

$$L(\Delta y; \hat{u}_{t-1}, \theta, \rho, \sigma^2) \propto \sigma^{-T} \exp \frac{-1}{2\,\sigma^2} \sum_{t=2}^{T} (y_t(\rho) - \tilde{x}_t'\theta)^2 \tag{19}$$

The conditional posterior density of θ has a Student form and is:

$$D''(\theta \mid y, \rho) = f_t(\theta \mid \theta_*(\rho), N_*, ss_*(\rho), T-1) \tag{20}$$

with:

$$\begin{cases} N_* &=& \sum \tilde{x}_t\, \tilde{x}_t' \\ \theta_*(\rho) &=& N_*^{-1} \sum \tilde{x}_t y_t(\rho) \\ ss_*(\rho) &=& \sum y_t^2(\rho) - \theta_*'(\rho) N_* \theta_*(\rho) \end{cases} \tag{21}$$

The posterior density of ρ obtains as one over the integrating constant of (20):

$$D''(\rho \mid y) \propto [ss_*(\rho)]^{-(T+p-1)/2}\, D'(\rho) \tag{22}$$

and has to be analyzed numerically. The posterior moments of θ are obtained using the same formulae as the one used to computed the posterior moments of β.

2.3 Remaining autocorrelation

I started with model (1) where the autocorrelation process is of order $p = 1$. The general case implies:

$$A(L)\, u_t = \epsilon_t \tag{23}$$

where $A(L)$ is a lag polynomial of order p with $A(L) = 1 - a_1 L - a_2 L^2 - \ldots - a_p L^p$. It is always possible to factorise this lag polynomial into:

$$A(L) = (1 - \rho L) - (1 - L) A^*(L) \tag{24}$$

with $\rho = 1 - A(1)$. The coefficients of $A^*(L)$ are defined by $a_j^* L^j = -(\sum_{i=j}^{p-1} a_{i+1}) L^j$, $j = 1, \ldots, p-1$. Factorising $A(L)$ in (23) gives after some manipulations:

$$\Delta u_t = (\rho - 1)\, u_{t-1} + a_1^* \Delta u_{t-1} + \ldots + a_{p-1}^* \Delta u_{t-p+1} + \epsilon_t \tag{25}$$

Let me replace Δu_t by its analytical expression and the remaining u_t by $\hat{u}_t$ as defined in (16). I get:

$$\begin{aligned} \Delta y_t = \Delta x_t' \gamma_1 + \Delta(ID(\hat{\tau})\, x_t')(\gamma_2 - \gamma_1) + (\rho - 1)\, \hat{u}_{t-1} \\ + a_1^* \Delta \hat{u}_{t-1} + \ldots + a_{p-1}^* \Delta \hat{u}_{t-p+1} + \epsilon_t \end{aligned} \tag{26}$$

Thus the question of remaining autocorrelation for inference and test on ρ can be solved by simply adding extra lags of $\Delta \hat{u}_t$ to the initial short term model. The computations are the same as before; I just have to alter the definitions of $\tilde{x}_t'$ and θ into:

$$\begin{cases} \tilde{x}_t' &=& [\Delta x_t, \Delta(ID(\hat{\tau})\, x_t), \Delta \hat{u}_{t-1}, \cdots, \Delta \hat{u}_{t-p+1}] \\ \theta' &=& [\gamma_1, \gamma_2 - \gamma_1, a_1^*, \cdots, a_{p-1}^*] \end{cases} \tag{27}$$

The number of degrees of freedom in (20) has to be changed from $T - 1$ to $T - p$.

2.4 The choice of a prior density

The choice of a prior density for the AR(1) model has a long story. The uniform prior, advocated by Sims [1988] or by DeJong and Whiteman [1989] has been widely criticized as it does not take into account the specificity of the model which behaves very differently in the stationary and nonstationary case. Considering $y_t = \mu + \rho y_{t-1} + \epsilon_t$, Thornberg [1967] (without a constant term), then Zellner ([1971] p.216-220) propose a Jeffreys' prior which, based on the information matrix, should reflect the properties of the data generation process. Their prior is of the form:

$$D'_J(\rho) \propto \left[\frac{1}{1-\rho^2}\right]^{1/2} \tag{28}$$

but is confined to the stationary case. It is equivalent to a limiting Beta prior. Phillips [1991] computed the Jeffreys' prior for the nonstationary case:

$$D'_{PJ}(\rho) \propto \left[\frac{1}{1-\rho^2}\left(T - \frac{1-\rho^{2T}}{1-\rho^2}\right)\right]^{1/2} \tag{29}$$

This prior has some very nasty properties as discussed in the 1991 special issue of the Journal of Applied Econometrics. It gives an infinite prior weight to the explosive case, because of the leading term ρ^{2T}. Berger and Yang [1994] advocate the use of a reference prior. The general theory of reference prior was initiated by Bernardo [1979]. The asymptotic reference prior minimizes the expected Kullback-Liebler divergence between the prior $D'(\theta)$ and the posterior $D''(\theta|y)$ when $T \to \infty$. For the AR(1) model, they show that the asymptotic reference prior is equal to (28). The method is however sensible only for the stationary case. So they propose a symmetrized asymptotic reference prior which is obtained by considering the transformation $\tilde{\rho} = 1/\rho$ with Jacobian $|1/\rho^2|$. The result is:

$$D'_{SR}(\rho) = \begin{cases} \dfrac{1}{2\pi}\left[\dfrac{1}{1-\rho^2}\right]^{1/2} & \text{if } |\rho| < 1 \\[2ex] \dfrac{1}{2\pi}\dfrac{1}{|\rho|}\left[\dfrac{1}{\rho^2-1}\right]^{1/2} & \text{if } |\rho| > 1 \end{cases}$$

This prior is a proper prior integrating to one. It gives the same weight to stationary and nonstationary regions. Figure 1 show that the reference and the Phillips' prior have the same left tail, but of course differ for their right tail.

The reference prior seems very suitable for testing for unit roots. It has been extended to the co-integration case in De la Croix and Lubrano [1995]. So in this paper I shall perform another exercise and extend the Phillips-Jeffreys' prior to the co-integration case. Despite its drawbacks, this prior can be interesting. The linearization may have put too much weight on the stationary hypothesis. This opinion find grounds on Lubrano [1995] where it is shown that true non-linearity may be crucial in testing for unit roots. A prior with an explosive tail may thus act as a compensation. The application to the Phillips-Jeffreys' principle to the simple regression model with autocorrelated errors and I(1) regressors leads to the following theorem:

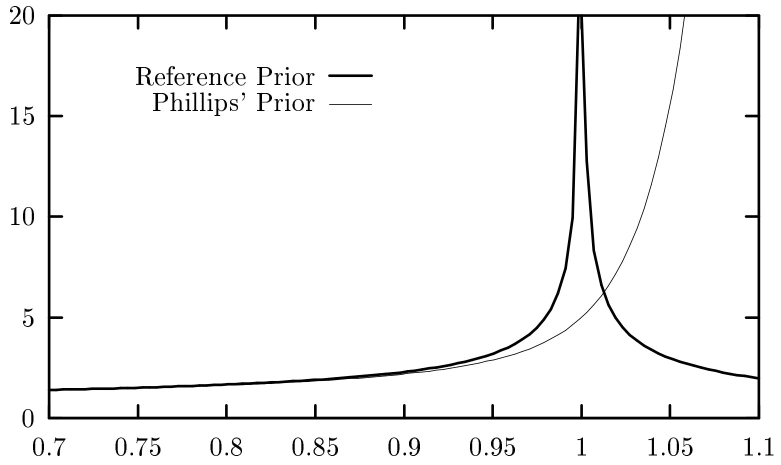

Figure 1: Two prior densities for the AR(1) model

Theorem 1 *Let us consider the regression model* $y_t = x_t'\beta + u_t$ *with autocorrelated error term,* $u_t = \rho u_{t-1} + \epsilon_t$ *and integrated regressors* $x_t \sim$*I(1). The Phillips-Jeffreys' prior for this model is:*

$$\begin{aligned}
D'(\sigma^2) &\propto \frac{1}{\sigma^2} \\
D'(\beta|\sigma^2) &\propto \frac{1}{\sigma^k} \\
D'(\rho|\beta,\sigma^2) &\propto \left[\frac{1}{1-\rho^2}[T - \frac{1-\rho^{2T}}{1-\rho^2}][1 + \tfrac{1}{2}(T-1)(1-\rho)^2]^k\right]^{1/2}
\end{aligned}$$

where k *is the dimension of* x_t*.*

Proof: Conditionally on the first observation ($t = 0$), the log likelihood function of the model is:

$$L(y;\beta,\rho,\sigma^2) \propto -\frac{T}{2}\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}((y_t - \rho y_{t-1}) - (x_t' - \rho x_{t-1}')\beta)^2$$

The Jeffreys' prior is obtained by taking the square root of the product of the diagonal elements of the information matrix. The second order derivatives of the likelihood function are:

$$\begin{aligned}
\frac{\partial^2 L}{\partial\sigma^4} &= \frac{T}{2\sigma^4} - \frac{1}{\sigma^6}\textstyle\sum_{t=1}^{T}(y_t - \rho y_{t-1} - (x_t' - \rho x_{t-1}')\beta)^2 \\
\frac{\partial^2 L}{\partial\rho^2} &= -\frac{1}{\sigma^2}\textstyle\sum_{t=1}^{T}(y_{t-1} - x_{t-1}'\beta)^2 \\
\frac{\partial^2 L}{\partial\beta\partial\beta'} &= -\frac{1}{2\sigma^2}\textstyle\sum_{t=1}^{T}(x_t - \rho x_{t-1})(x_t - \rho x_{t-1})'
\end{aligned}$$

The expectation of the first block:

$$-\mathrm{E}\frac{\partial^2 L}{\partial \sigma^4} = \frac{T}{2\sigma^4}$$

For the second block, as I cannot assume $\rho < 1$, I have to use a decomposition of $y_t - x_t'\beta = \epsilon/(1-\rho L)$ which is conditional on the initial condition:

$$y_t - x_t'\beta = \sum_{i=0}^{t-1} \rho^i \epsilon_{t-i} + \rho^t (y_0 - x_0'\beta)$$

Assuming zero initial conditions leads to the result:

$$\begin{aligned} -\mathrm{E}\frac{\partial^2 L}{\partial \rho^2} &= \frac{1}{\sigma^2} \textstyle\sum_{t=0}^{T-1} \sigma^2 \displaystyle\frac{1-\rho^{2t}}{1-\rho^2} \\ &= \frac{1}{1-\rho^2}\left[T - \frac{1-\rho^{2T}}{1-\rho^2}\right] \end{aligned}$$

The last block concerns the I(1) regressors. Each regressor is defined as:

$$x_{it} = \sum_{i=0}^{t-1} v_{t-i} \qquad v_t \sim N(0,1)$$

Then:

$$-\mathrm{E}\frac{\partial^2 L}{\partial \beta_i^2} = \frac{1}{2\sigma^2}\sum_{t=1}^{T} \mathrm{E}(x_{i,t} - \rho x_{i,t-1})^2 = \frac{T}{2\sigma^2}(1 + \tfrac{1}{2}(T-1)(1-\rho)^2)$$

Considering the k elements of x_t and taking the adequate product gives the desired prior. □

As it is apparent in figure 2, the symmetrized reference prior for co-integration used in De la Croix and Lubrano [1995] and the Phillips-Jeffreys' prior are similar in a subinterval of the stationary region, but diverge outside this region. Both have higher left tails as k and T grow.

The prior on τ is chosen to be a discrete uniform probability distribution over a finite interval of dates:

$$D'(\tau) \propto \begin{cases} \dfrac{1}{\mathrm{Card}[a,b]} & \text{if } \tau \in [a,b] \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

where $\mathrm{Card}[a,b]$ means the number of elements in the discrete interval $[a,b]$.

2.5 Testing with posterior densities

Two types of tests are interesting to conduct. A test on ρ to get an answer to the question "is there co-integration between the I(1) variables?" and a test on τ which is a test of linearity

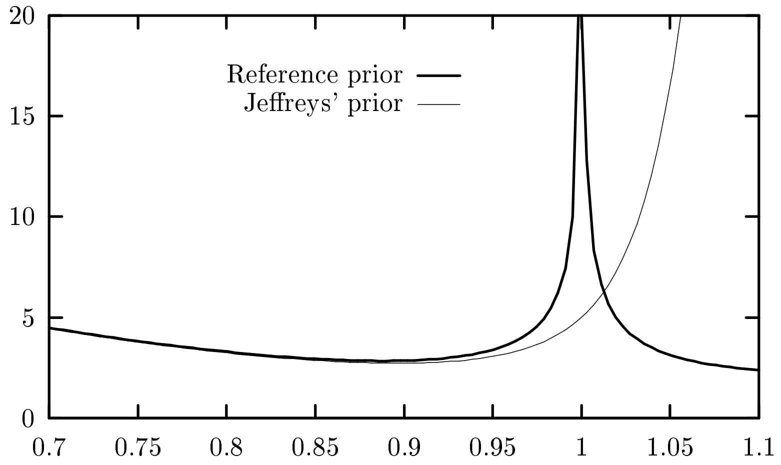

Figure 2: Two prior densities for the co-integration model

to known if it is necessary to consider a model with a break or a simple linear model. In each case, I am going to use the corresponding marginal posterior density which contains all the relevant information.

The marginal posterior density of ρ is continuous and defined on the whole real line. To decide if there is co-integration or not, I shall compute a posterior confidence interval for ρ and say that there is no co-integration if the value 1 belongs to this interval. I can alternatively compute

$$\Pr(\rho \geq 1 \mid y) = \int_1^\infty D''(\rho|y)\, d\rho \tag{31}$$

and compare this value to a pre-assigned probability level like 5%.

The marginal posterior in τ has a different structure. It is a discrete density. Moreover, the model is not identified at the end points of the support of τ, as then there are less observations than parameters for one of the two regimes. So I cannot decide that the model is linear by simply testing if τ belongs to one of the end points of the sample dates. However, each point of this marginal discrete density can be interpreted as the probability that τ takes a particular value. A date will dominate all the other dates if a particular value of τ has a probability greater than 0.5. This is a posterior odds approach for deciding if the model is non-linear with a single break. Alternative models are the linear model with no break at all or a smooth transition model if the graph of the posterior density of τ is rather well concentrated around a particular date.

Another approach is possible to decide if the model is linear or not. It is not based on a test, but on a model selection approach. I simply have to compute a Schwarz criterion:

$$Sc = \log(\mathrm{E}(\sigma^2|y)) + \left(\frac{k}{T} + 1\right) \log(T) \tag{32}$$

for the estimated model and compare it to the same criterion computed for the corresponding linear model. A last approach would be to use the posterior distribution of β and test for $\beta_2 - \beta_1 = 0$.

3 WAGE REGULATION IN FRANCE, A NEW APPRAISAL

In European countries wages are de facto more or less indexed on prices. There is a tradition of unit indexation in Belgium and Italy. In France no official indexation exists, but there was an informal agreement about price indexation in wage negotiations. The same case applies to England. The first oil shock resulted in a tremendous change in relative prices and a drop of labour productivity. Wages did not fully adjust to this new setting. A wage moderation policy was inforced by governments at the beginning of the eighties. This led to an apparent de-indexation of wages on prices. Ralle and Toujas-Bernatte [1990] discusses the case for France, Bogaert *et al* [1991] for Belgium. The two cases just mentioned are different one from the other as in France there was just a pressure from the Government to reduce wage growth when in Belgium this policy was enforced by law, resulting in a modification of the automatic indexation procedure. Ralle and Toujas-Bernatte [1990], using an augmented Phillips' curve model and quarterly data covering the period 1971:1-1988:4 show the effectiveness of the wage moderation policy: their coefficient for the rate of inflation is equal to one in the first period of the sample and lower than one in the second. The aim of this section is to verify if a break really appeared in a co-integration relationship based on the Phillips' curve and if not what really happened [5].

3.1 Testing with the original Phillips' curve model

The relation which is considered by Ralle and Toujas-Bernatte [1990] relates the rate of growth of wages to the rate of growth of consumption prices and the logarithm of the unemployment rate. They adjust an error correction model on these data. A different coefficient is allowed for prices before 1982:1 and after 1983:1. The difference is found to be significant. A unit price indexation is accepted before 1982. No break on other coefficients was significant. Any attempt to include productivity failed. Ralle and Toujas-Bernatte treat the breaking point of course as exogenous. Moreover, they exclude observations around the chosen date, thinking that the wage equation was not stable in the transition period. An error correction model with endogenous breaking point seems to be the right tool to use in order to analyze these data. If the policy of wage regulation enforced at the end of 1982 was really effective, a Phillips' curve model would need a change in the consumption price coefficient in order to display co-integration. Without a break, the model should indicate an absence of co-integration.

Let me first analyze the behavior of the individual series. Various Bayesian tests of unit root were developed in the literature. I shall use Lubrano [1995], Phillips [1991] and Schotman and van Dijk [1991]. The first two tests are based on a posterior confidence interval

[5] The data used in this empirical application come a data bank designed by Laroque *et al* [1990] which initialy covered the period 1946:1-1989:4 and which was extended till 1991:4.

so the unit root is chosen if $\Pr(\rho \geq 1|y) > 0.05$. The last one is based on posterior odds and the unit root is preferred if $\Pr(\rho = 1|y) > 0.5$. I shall compute these tests for various orders of differentiation and various configurations of the trend, including an exogenous break at the appropriate date which was chosen to be 82.2 on graphical inspection of the series. The results are regrouped in table 1. I can conclude from the inspection of this table that, on average, the log of the wage rate and the log of the consumption price index are I(2) and the log of unemployment rate is I(1).

series	p	trend	break	Lubrano $\Pr(\rho \geq 1\|)$	Phillips $\Pr(\rho \geq 1\|)$	Schotman van Dijk $\Pr(\rho = 1\|y)$
lw	2	yes	82.2	0.94	0.11	0.77
Δ lw	1	no	-	0.04	0.18	0.33
$\Delta\Delta$ lw	1	no	-	0.00	0.00	0.00
lpc	2	yes	82.2	0.71	0.004	0.74
Δ lpc	1	no	-	0.06	0.58	0.43
$\Delta\Delta$ lpc	1	no	-	0.00	0.00	0.00
lur	2	yes	82.2	0.11	0.00	0.48
Δ lur	1	no	-	0.00	0.00	0.00

Table 1: Unit root tests for the Phillips' curve model

Wage indexation was not automatic quarter by quarter, but based on annual wage bargaining. Thus it seems more reasonable to work with annual log differences than quarterly log differences. The obtained series are much smoother and thus more reliable to make inference on long term relationships. A first analysis of the sample with Johansen's [1991] approach is quite fruitful. It indicates that on the period 1971:1-1988:4 there is one co-integrating vector between the variables $\Delta 4 \log w$, $\Delta 4 \log pc$ and $\log ur$. This vector is 1, 0.92, -0.026. The unit price indexation is not rejected with a P value of 0.25. Extending the sample to the period 1963:1-1991:1 does not change the result. For the sample 1971:1-1991:4, there is still one co-integration vector, but the unit price indexation can be nearly rejected as this hypothesis receives a P value of 0.043. Conclusions: first, as there is only one co-integrating vector, a one equation analysis as that developed above is valid; second, the equation seems quite stable over the period with a unit price elasticity (this last hypothesis could be questioned at the very end of the period); third, a break does not seem very necessary for the equation to hold.

I shall first present inference results with no break allowed for on the period 1971:1-1988:4. With a Jeffreys' diffuse prior on all the parameters, I get:

$$\hat{z}_t = \Delta 4 \log w_t - (\underset{[0.0052]}{0.079} + \underset{[0.037]}{0.85}\, \Delta 4 \log pc_t - \underset{[0.0021]}{0.025}\, \log ur_t)$$

$$\sqrt{\mathrm{E}(\omega^2|y)} = 0.0101$$

$$\Delta\Delta 4 \log w_t = \underset{[0.078]}{0.57}\, \Delta\Delta 4 \log pc_t - \underset{[0.012]}{0.026}\, \Delta \log ur_t + (\underset{[0.069]}{0.58} - 1)\hat{u}_{t-1} + \underset{[0.10]}{0.48}\, \Delta\hat{u}_{t-1}$$

$$\sqrt{\mathrm{E}(\sigma^2|y)} = 0.00533 \quad \text{Schwarz} = -6.038 \quad \Pr(\rho \geq 1|y) = 0.0$$

So, in a Bayesian framework, there is co-integration without the need of a break. If I now try to impose the unit price indexation:

$$\hat{z}_t = \Delta 4 \log(w/pc)_t - (\underset{[0.0040]}{0.064} - \underset{[0.0022]}{0.023} \log ur)$$

$$\sqrt{\mathrm{E}(\omega^2|y)} = 0.0111$$

$$\Delta\Delta 4 \log(w/pc)_t = -\underset{[0.015]}{0.033}\, \Delta \log ur_t + (\underset{[0.077]}{0.63} - 1)\hat{u}_{t-1} + \underset{[0.11]}{0.49}\, \Delta\hat{u}_{t-1}$$

$$\sqrt{\mathrm{E}(\sigma^2|y)} = 0.00636 \quad \text{Schwarz} = -5.747 \quad \Pr(\rho \geq 1|y) = 0.0$$

there is still co-integration. With this type of inference model, it is rather difficult to judge if the unit indexation constraint is a valid one. But first remember the results obtained in a classical framework with the Johansen's method, and second imposing the unit price indexation restriction does not remove co-integration. These results do not change very much if I extend the sample till 1991:4.

Let me now introduce the possibility of a break in the value of the price indexation coefficient by applying the full statistical model developed above. With a Jeffreys' diffuse prior on $(\beta,\, \rho,\, \sigma^2)$ and a uniform prior on τ on the range 1972:4-1987:1, inference results are as follows:

$$\hat{z}_t = \Delta 4 \log w_t - (\underset{[0.010]}{0.079} + \underset{[0.091]}{0.79}\, \Delta 4 \log pc_t - \underset{[0.0058]}{0.021} \log ur_t - II^{nd} \underset{[0.16]}{0.15}\, \Delta 4 \log pc_t)$$

$$\sqrt{\mathrm{E}(\omega^2|y)} = 0.00875 \quad \mathrm{Mode}(\tau|y) = 82.3 \quad \Pr(Mode = 82.3|y) = 0.497$$

$$\begin{aligned}\Delta\Delta 4 \log w_t = \underset{[0.079]}{0.46}\, \Delta\Delta 4 \log pc_t - \underset{[0.012]}{0.016}\, \Delta \log ur_t + II^{nd} \underset{[0.041]}{0.031}\, \Delta\Delta 4 \log pc_t \\ + (\underset{[0.061]}{0.61} - 1)\hat{u}_{t-1} + \underset{[0.090]}{0.50}\, \Delta\hat{u}_{t-1}\end{aligned}$$

$$\sqrt{\mathrm{E}(\sigma^2|y)} = 0.00511 \quad \text{Schwarz} = -6.063 \quad \Pr(\rho \geq 1|y) = 0.0$$

II^{nd} indicates the second regime. It appears that the difference between the parameters of the first and the second regimes is near zero as the posterior standard deviation of $(\beta_2 - \beta_1)$ is rather high (0.16 to be compared to 0.15). There seems to be a break in 1982.3, but the mode of τ is just below 0.5. Figure 3 displays the corresponding posterior density of τ. If I use a more informative prior on τ, restricting the range to 1976:1-1985:4, the equation changes a bit. The coefficient of $II^{nd}\Delta 4 \log pc_t$ becomes -0.21 with a standard deviation of

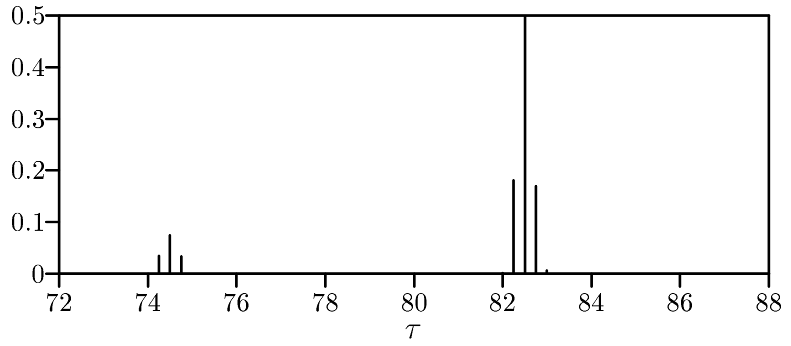

Figure 3: Posterior density of τ for the Phillips' curve

0.045. The posterior mode of τ does not change, but its posterior probability is now above 0.5 with 0.58. There is co-integration in any case.

Let me now impose the unit price indexation using the same prior as before. The equation becomes:

$$\hat{z}_t = \Delta 4 \log(w/pc)_t - (\underset{[0.0056]}{0.063} - \underset{[0.0042]}{0.020} \log ur_t - II^{nd} \underset{[0.20]}{0.028} \Delta 4 \log pc_t)$$

$$\sqrt{\mathrm{E}(\omega^2|y)} = 0.0103 \quad \mathrm{Mode}(\tau|y) = 77.1 \quad \Pr(Mode = 77.1|y) = 0.131$$

$$\Delta\Delta 4 \log(w/pc)_t = -\underset{[0.015]}{0.039} \Delta \log ur_t - II^{nd} \underset{[0.058]}{0.074} \Delta\Delta 4 \log pc_t + (\underset{[0.077]}{0.63} - 1)\hat{u}_{t-1} + \underset{[0.11]}{0.49} \Delta \hat{u}_{t-1}$$

$$\sqrt{\mathrm{E}(\sigma^2|y)} = 0.00628 \quad \mathrm{Schwarz} = -5.710 \quad \Pr(\rho \geq 1|y) = 0.0$$

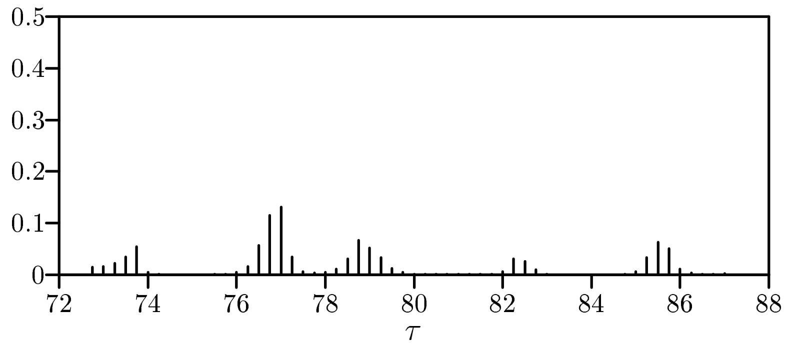

Figure 4: Posterior density of τ for the Phillips' curve with unit indexation

With a unit price indexation, the break really disappears, witness figure 4. There is no serious candidate for a break point. Moreover, the coefficient of $\Delta \log pc_t$ in the second regime is roughly zero (0.028, with a standard error of 0.20). Finally, if I compare the Schwarz criterion attached to the model with break (-5.710), to that attached to the model without break (-5.747), clearly the simpler model without break dominates.

In sum, if I do not impose the unit price indexation for the first regime, it is possible to detect a break in the coefficient of price indexation at the correct date which is the middle of 1982, provided I accept to allow for a mild prior information on the date of occurrence of the break. But with a model where unit price indexation is already built in, it is not possible to relax this hypothesis by allowing a second regime with no unit price indexation. Moreover, a break is never necessary to get co-integration. These empirical results cast some doubts on the effectiveness of the wage regulation policy enforced in France at the beginning of the eighties and on the results presented in Ralle and Toujas-Bernatte [1990]. However something really happened if it was not a price de-indexation, and I have to find what.

3.2 Testing with a wage bargaining equation

series	p	trend	break	Lubrano $\Pr(\rho \geq 1\|)$	Phillips $\Pr(\rho \geq 1\|)$	Schotman van Dijk $\Pr(\rho = 1\|y)$
lwpcse	0	yes	76.2	0.49	0.03	0.70
Δ lwpcse	0	yes	-	0.00	0.00	0.00
lppc	0	no	-	0.21	0.14	0.63
Δ lppc	0	no	-	0.00	0.00	0.00
lq	0	yes	74.4	0.15	0.33	0.59
Δ lq	0	no	-	0.00	0.00	0.00
ur	1	yes	-	0.05	0.02	0.34
Δ ur	0	no	-	0.00	0.00	0.00
ur2	1	yes	-	0.08	0.01	0.45
Δ ur2	0	no	-	0.00	0.00	0.00

Table 2: Unit root tests for the wage bargaining model

The second type of model which is used to explain wage formation is the model of wage bargaining. The dependant variable is no longer the growth rate of nominal wages, but the level of real wages which is the quantity under bargaining between the firm and the representative union. A very simplified wage equation inspired from De la Croix and Lubrano [1992] could be as follows:

$$\log(w\,(1+cse)/p)_t = \alpha_0 + \alpha_1 \log(p/pc)_t + \alpha_2 \log q_t + \alpha_3 ur_t + \alpha_4 ur_t^2 + u_t$$

where w represents the gross wage rate paid to workers, pc the consumption price index, p the output price index, cse the rate of social contribution paid by firms, q the labour productivity and ur the rate of unemployment. As I no longer have the equations of Ralle and Toujas-Bernatte as a stick yard, I can extend my sample size and consider the period[6] 1971:1-1991:4. The variables can be shown to be I(1) in table 2. The unemployment rate may be trend stationary with a break at 1974:4 in the trend; but the break is not significant. With a linear trend, the variable is I(1).

Let me first present inference for the wage bargaining equation on the whole period 1971:1-1991:4 without any break. With a Jeffreys' diffuse prior on all the parameters, I get:

$$\begin{aligned}\hat{z}_t = {} & \log(w\,(1+cse)/p)_t \\ & - (\underset{[0.29]}{4.80} - \underset{[0.13]}{0.71}\log(p/pc)_t + \underset{[0.038]}{0.70}\log q_t + \underset{[0.0040]}{0.069}\,ur_t - \underset{[0.00024]}{0.0042}\,ur_t^2)\end{aligned}$$

$$\sqrt{\mathrm{E}(\omega^2|y)} = 0.0141$$

$$\begin{aligned}\Delta\log(w\,(1+cse)/p)_t = {} & -\underset{[0.12]}{0.43}\,\Delta\log(p/pc)_t + \underset{[0.088]}{0.49}\,\Delta\log q_t \\ & + \underset{[0.0092]}{0.035}\,\Delta ur_t - \underset{[0.00059]}{0.0018}\,\Delta ur_t^2 + (\underset{[0.14]}{0.93} - 1)\hat{u}_{t-1} \\ & \sqrt{\mathrm{E}(\sigma^2|y)} = 0.00689 \quad \text{Schwarz} = -5.324 \quad \Pr(\rho \geq 1|y) = 0.22\end{aligned}$$

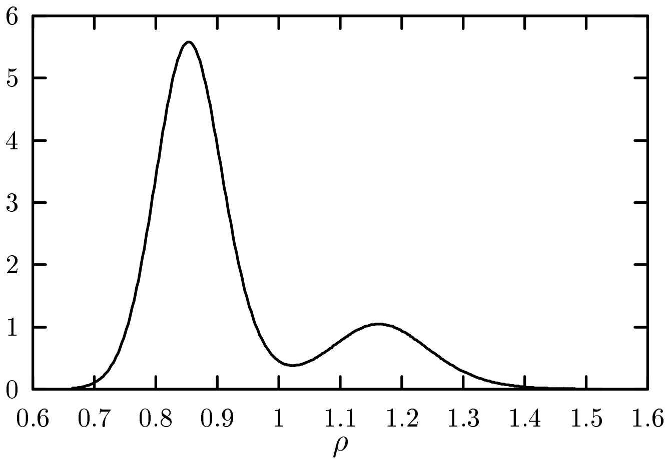

Figure 5: Posterior density of ρ for the wage bargaining equation without break

There is no co-integration. The graph (5) of the posterior density of ρ is quite illuminating, showing the hide and seek party between the Jeffreys' prior and the sample. Had I used the reference prior of De la Croix and Lubrano [1995], the right bump would have been replaced by a peak at $\rho = 1$ and the right tail would have gone to zero much faster. Without a break this model cannot give an account of wage negotiations in France on the period considered.

[6] Extending the period back to 1963:1 is perhaps not very wise. The basis of National Accounts changed in 1971. In 1968:3, wage formation was very peculiar due to the May events which were followed by huge wage negotiations and agreements on very high wage rise.

Figure 6: Wage share in France

In a wage bargaining model, firms and trade unions bargain for a wage share (or a profit share) in the total value added. The wage share is defined as $ws = w/(p \times q)$. Figure 6 shows that ws has experienced a drop after 1982, but that this drop started around 1975. This suggest a change in the coefficient of productivity in the wage equation. One of the aims of governments after 1974 was to restore firms' profits. Let me thus introduce a break in the constant term and the productivity coefficient of the previous equation. With a Jeffreys' prior for (β, ρ, σ^2) and a uniform prior for τ on the range 1974:1-1987:1, I get:

$$\hat{z}_t = \log(w\,(1+cse)/p)_t - (\underset{[0.24]}{6.06} - \underset{[0.087]}{0.63}\log(p/pc)_t + \underset{[0.042]}{1.06}\log q_t$$
$$+ \underset{[0.0041]}{0.047}\,ur_t - \underset{[0.00027]}{0.0023}\,ur_t^2 + II^{nd}(\underset{[0.16]}{1.75} - \underset{[0.052]}{0.55}\log q_t))$$

$$\sqrt{\mathrm{E}(\omega^2|y)} = 0.00864 \quad \mathrm{Mode}(\tau|y) = 76.2 \quad \Pr(Mode = 76.2|y) = 0.23$$

$$\Delta\log(w\,(1+cse)/p)_t = -\underset{[0.10]}{0.49}\,\Delta\log(p/pc)_t + \underset{[0.11]}{0.76}\,\Delta\log q_t + \underset{[0.0072]}{0.036}\,\Delta ur_t - \underset{[0.00048]}{0.0015}\,\Delta ur_t^2$$
$$+ II^{nd}(\underset{[0.47]}{1.44} - \underset{[0.15]}{0.45}\,\Delta\log q_t) + (\underset{[0.076]}{0.56} - 1)\hat{u}_{t-1}$$

$$\sqrt{\mathrm{E}(\sigma^2|y)} = 0.00582 \quad \mathrm{Schwarz} = -5.553 \quad \Pr(\rho \geq 1|y) = 0.00$$

With a break in 1976:2, this equation is co-integrated when it was not without a break. The posterior density of ρ has the usual bell shape and is concentrated away from 1. The break point is located around around 1976:2 with a standard deviation of 3 quarters as illustrated in figure 7. This suggests a smooth transition between the two regimes and not a single break point as the mode has a probability of only 0.23. The coefficient of productivity is one in the first regime and experiences a drop in the second regime with an indexation on productivity of only $1.06 - 0.55 = 0.51$. The standard deviation of lq is very small in the second regime.

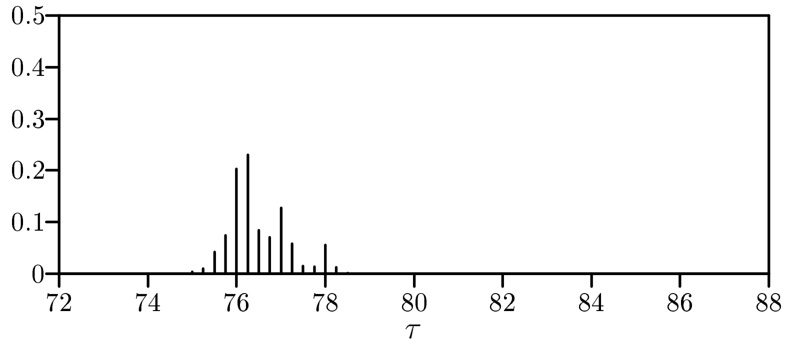

Figure 7: Posterior density of τ for wage bargaining equation

3.3 Partial conclusions

These empirical results are quite interesting. They show that the 1982 policy was mainly concentrated on keeping wages and prices in strict agreement. No suppression of wage indexation can be detected. With this respect the case of France is different from the Belgian experience. The most interesting part of the play append elsewhere and started in fact before 1982, around 1976 according to the data. Before 1976 productivity gains were fully integrated to wages. After this date they are only very partially integrated. This policy was decided to recover from the first oil shock. It started with right wing governments and did not change with the Socialist governments, except for a transitory period in 1981 when the minimum wage was strongly risen, just after the elections.

4 CONCLUSION

One of the most often worded criticism to Bayesian methods is the difficulty of implementation and the lack of empirical applications. In this paper, I have tried to show that Bayesian methods are both feasible and useful for empirical investigations. Under a diffuse prior, classical and Bayesian methods usually give similar answers. The recent literature has shown that this is no longer the case for time series. The most provocative illustration of this fact is contained in Sims and Uhlig [1991]. The shape of the non-informative prior on the dynamic parameter plays an essential role in order to reconcile the likelihood function viewed as a function of the sample (classical approach) and the likelihood function viewed as a function of the parameters conditionally on the observed sample (Bayesian approach). The difference of results I obtained when treating the empirical question raised by Ralle and Toujas-Bernatte [1990] does not seem to be due to the difference of statistical approach as the sample sample analyzed with a co-integration method à la Johansen does not produce a radically different message. The Jeffreys' prior I used is certainly not innocent in this respect.

Finding an endogenous breaking point in a relationship is very exiting, witness the large amount of recent literature on non-linear time series. When testing for a unit root, it is quite easy to guess if it is useful or not to introduce a break in the deterministic trend, just by visual inspection of the series. It is then also easy, for the same reason, to judge if the break point found by the program is realistic or not. When a complete regression is concerned, thing are not that easy. Individual series may each present a break, and the co-integration relationship may be stable over the period if all the variables have their break at the same point and if the break is of the same amplitude. But this is rarely the case. The break in the relationship may appear at different points of time, depending on the specification of the equation. This is what I have found for a wage equation. Adding some observations at one of the ends of the sample may also change a bit the date of the break. So the posterior density of τ may provide a message which seems too precise and must be taken with a grain of salt. This drawback is not specific to the Bayesian approach, but general to non-linear time series.

APPENDIX: THE DATA

date	w	pc	p	ur	q
1963.1	2.40860	29.6523	31.0655	1.4736	13.3887
1963.2	2.48120	30.1237	31.9478	1.3563	13.9828
1963.3	2.54350	30.6997	32.3265	1.2303	14.4369
1963.4	2.60580	30.9578	32.5321	1.0554	14.4225
1964.1	2.66810	31.2132	32.8338	0.9950	14.6894
1964.2	2.72000	31.2888	32.8800	1.0622	14.8168
1964.3	2.76150	31.4587	33.3979	1.0892	14.9860
1964.4	2.80310	31.5827	33.6677	1.1740	15.1095
1965.1	2.84460	31.7943	33.7888	1.2940	15.1303
1965.2	2.88610	32.0283	34.0026	1.3432	15.5454
1965.3	2.92760	32.3095	34.1607	1.3855	15.7832
1965.4	2.97960	32.4834	34.1699	1.3610	16.0126
1966.1	3.03150	32.7933	34.4260	1.3101	16.1143
1966.2	3.07300	33.0773	34.7946	1.3340	16.3235
1966.3	3.11450	33.2839	35.0476	1.3800	16.4736
1966.4	3.16640	33.5076	35.3193	1.4736	16.5294
1967.1	3.21830	33.7537	35.6149	1.5980	16.8101
1967.2	3.25990	33.9612	35.7973	1.7177	16.9945
1967.3	3.30140	34.3348	36.1176	1.8312	17.1738
1967.4	3.33250	34.6306	36.2444	2.0393	17.3524
1968.1	3.38440	35.1345	36.6604	2.1019	17.8256
1968.2	3.57130	35.6647	37.1385	2.2582	18.0680
1968.3	3.75820	36.1965	37.6167	2.0584	18.2839
1968.4	3.84120	36.8678	38.3143	1.8481	18.4254
1969.1	3.90350	37.6414	39.2365	1.7411	18.3899
1969.2	3.97620	38.1817	39.3240	1.6016	18.7824
1969.3	4.03850	38.6301	39.6674	1.4862	18.8904
1969.4	4.16310	39.0642	40.1131	1.4780	18.9517
1970.1	4.28770	39.5921	40.8581	1.5828	19.3161
1970.2	4.40190	40.0367	41.3995	1.6994	19.5926
1970.3	4.50570	40.4788	41.9826	1.7670	19.6875
1970.4	4.65100	40.9632	42.3574	1.8722	19.9652
1971.1	4.78600	41.5213	43.1071	1.9762	20.1302
1971.2	4.90020	42.1751	43.5236	2.0960	20.4243
1971.3	5.01440	42.7705	44.3149	2.1750	20.8286
1971.4	5.14930	43.4348	44.6898	2.2976	21.2080
1972.1	5.27390	43.8960	45.4811	2.3637	21.4612
1972.2	5.39850	44.5033	46.0642	2.3924	21.6430
1972.3	5.54380	45.3325	47.0638	2.3709	21.9121
1972.4	5.69960	46.1010	47.4803	2.2046	22.1893
1973.1	5.88640	46.4359	48.3549	2.0392	22.4102
1973.2	6.07330	47.2834	49.1879	1.9868	22.5697
1973.3	6.27060	48.4702	50.2708	2.1030	22.8127
1973.4	6.49900	49.7040	52.0201	2.0293	22.9464
1974.1	6.72890	51.8337	52.3222	1.9644	23.3248
1974.2	7.06110	53.7515	54.2188	1.9395	23.3616
1974.3	7.39520	55.6299	56.4376	2.3683	23.6670
1974.4	7.69320	57.3972	58.2568	2.9759	23.3841
1975.1	7.99140	59.0681	59.8851	3.3442	23.2559
1975.2	8.27130	60.2249	61.4965	3.7611	23.3413
1975.3	8.56230	61.5991	62.7479	4.0796	23.4601
1975.4	8.83190	62.9002	63.9443	4.1195	23.7655
1976.1	9.15740	64.4919	65.8646	4.1345	24.0291
1976.2	9.49580	66.1195	67.6825	4.0958	24.2623
1976.3	9.80800	67.5106	69.5907	4.0700	24.3098
1976.4	10.0861	69.2896	70.9852	4.1735	24.5224

date	w	pc	p	ur	q
1977.1	10.3214	70.5309	71.9920	4.4501	24.7257
1977.2	10.5592	72.4010	73.6007	4.6535	24.7865
1977.3	10.8565	74.0665	75.4239	4.8390	24.9894
1977.4	11.2090	75.4547	77.0969	4.8118	25.0540
1978.1	11.5497	76.5050	78.3532	4.7214	25.2007
1978.2	11.9146	78.7045	80.7181	4.9668	25.5862
1978.3	12.3167	81.0527	83.1023	5.3356	25.7345
1978.4	12.6691	82.6268	85.2998	5.4740	26.0371
1979.1	13.0145	84.4352	87.1334	5.6538	26.0863
1979.2	13.3784	86.7626	88.8185	5.8057	26.3177
1979.3	13.8040	89.7149	90.9453	5.8884	26.6969
1979.4	14.3075	92.2191	93.5234	5.9988	26.7614
1980.1	14.8512	95.5575	95.7131	6.0340	27.0011
1980.2	15.4228	98.4721	98.6471	6.0566	26.7482
1980.3	15.9835	101.538	101.758	6.2672	26.8596
1980.4	16.4999	104.388	103.921	6.6346	26.8053
1981.1	16.9733	107.355	106.219	7.0384	26.8169
1981.2	17.5507	110.750	108.531	7.4822	27.2639
1981.3	18.2542	115.009	112.241	7.8732	27.5155
1981.4	19.0054	118.673	116.581	8.1083	27.7136
1982.1	19.7896	121.717	120.047	8.2642	27.8621
1982.2	20.2540	125.352	123.122	8.4873	28.0704
1982.3	20.3654	127.091	124.057	8.5978	28.0194
1982.4	20.8289	129.428	126.257	8.5307	28.1445
1983.1	21.4957	133.034	130.706	8.4563	28.1033
1983.2	22.0078	136.947	134.234	8.5222	28.1834
1983.3	22.4805	139.826	137.080	8.5468	28.0361
1983.4	22.9478	142.389	139.755	8.9036	28.2797
1984.1	23.3908	144.889	141.741	9.3948	28.7234
1984.2	23.7685	147.533	144.560	9.6864	28.6024
1984.3	24.1607	150.076	146.876	9.8860	28.8175
1984.4	24.5804	152.179	148.659	10.055	28.9462
1985.1	25.0330	154.367	150.530	10.073	28.9390
1985.2	25.4241	156.764	153.243	9.9853	29.1565
1985.3	25.8524	158.416	155.761	9.9549	29.4533
1985.4	26.2275	159.408	157.691	9.6746	29.5098
1986.1	26.5691	159.675	159.931	9.8916	29.5311
1986.2	26.7930	160.981	162.324	10.198	30.0940
1986.3	27.0498	161.945	163.667	10.604	30.2740
1986.4	27.2643	162.938	164.153	10.751	30.4946
1987.1	27.5012	164.713	166.029	11.038	30.4391
1987.2	27.7830	166.281	166.969	10.872	30.8190
1987.3	28.1053	167.179	168.089	10.821	31.0682
1987.4	28.5381	168.021	169.383	10.715	31.4229
1988.1	28.9177	168.724	170.560	10.507	31.8343
1988.2	29.2623	170.394	171.938	10.604	32.0325
1988.3	29.5720	171.842	173.589	10.621	32.3628
1988.4	29.8659	173.047	174.890	10.551	32.7117
1989.1	30.1377	174.524	175.934	10.436	33.2214
1989.2	30.4315	176.569	176.704	10.409	33.4996
1989.3	30.7558	177.577	178.173	10.400	33.6053
1989.4	31.2493	178.853	179.826	10.291	33.7320
1990.1	31.6618	179.921	181.049	10.262	34.0901
1990.2	32.1371	181.133	183.074	10.230	34.0205
1990.3	32.5249	182.970	183.960	10.223	34.1347
1990.4	32.9060	184.531	184.143	10.348	33.9923
1991.1	33.2324	185.728	186.362	10.615	34.0349
1991.2	33.5736	186.894	187.747	11.039	34.3189
1991.3	33.9406	188.509	188.562	11.279	34.7817
1991.4	34.2671	189.888	189.988	11.507	34.8817

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