About the Right Weight of the Social Welfare Function when Needs Differ.

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Abstract

When equivalence scales are used to compute the well-being of individuals, two possible weighting methods of the different household types have been proposed, the first one resorts to the family size and the second to the equivalence scale itself. The latter is criticized on the ground that it does not respect an anonymity axiom. We show that this criticism vanishes in the standard microeconomic setting.

JEL classification: D63, I3, D13

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1 Introduction[1]

When one has to compare inequality or welfare of income distributions across household of different sizes, there is still some disagreement about the method to be used. More specifically, once it has been decided to use equivalent scales to compute comparable living standards, the choice of the weights attributed to each family size remains a bit controversial. There are basically two methods, the first one favoured for instance by Shorrocks (1995) where the weight is just given by the family size and the second one proposed by Ebert (1997) where the equivalence scale is used as a weighting function as well. Moreover, the latter author showed that each method is supported by an ethical principle. Favouring anonymity leads to the first method while endorsing an extension of the Pigou-Dalton principle to different household types results in the second one. Thus a dilemma seems inescapable between the two principles. Later on, Ebert (1999) advocated the merits of using equivalence scale for weighting the different household types. Nevertheless he added p. 251 that "a possible

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objection against this methodology is the fact that at first sight not all persons have the same weight and significance". We claim that this objection may be dismissed in a rigorous way.

More precisely, when one plugs the dilemma in the framework of the standard microeconomic theory applied to allocations within families, like in Bourguignon (1989), one cannot assimilate the function whose argument is the equivalent income as "an individual indirect utility function". By way of consequence, one cannot define an anonymity condition on this function and the dilemma disappears from itself. All reasons militate for choosing the equivalence scale as the appropriate weight.

2 A microeconomic setting

It is assumed that each individual is endowed with the same continuous, increasing and concave utility function U defined on two attributes, a private good x and a good g which is public within the family. Assume that each family of size n (treated as a real variable for convenience) behaves like a utilitarian society[2]. It allocates the household budget y such

$$\max_{x,g} nU(x,g)\ /\ nx+g=y. \tag{1}$$

Call $F(x,g)$ the c.d.f of the joint distribution of the private good and of the public good among individuals. The support of the associated p.d.f is denoted $A_x \times A_g$. With N^i the number of individuals and N^h the number of households in the society, a utilitarian social planner will express the social welfare as

$$W_F = N^i \int\limits_{A_x \times A_g} U(x,g)dF(x,g). \tag{2}$$

This expression can be reformulated with the introduction of the demand functions $x(y,n)$ and $g(y,n)$ associated to the maximization program (1). Let $G(y,n)$ be the c.d.f of the joint distribution of family income and family size on the support $A_y \times A_n$ among individuals.[3] Applying a general change-of-variable formula, (e.g., formula (16.18) in Billingsley (1986) Theorem 16.12, p.219) expression (2) is still equivalent to

$$W_F = W_G = N^i \int\limits_{A_y \times A_n} U(x(y,n),g(y,n))dG(y,n). \tag{3}$$

Defining $G^n(y \mid n)$ the conditional c.d.f of family income given family size among individuals and $G(n)$ the marginal c.d.f of family size among individuals,

[2] The same reasoning holds if we only assume that the household allocate goods in an efficient way. For the problem in touch, it is easier to consider that individuals (with the same utility function) are treated in a symmetric way.

[3] The link between F and G is given by an expression like (16.16) in Billingsley (1986) p.218.

(3) may be rewritten

$$W_G = N^i \int\limits_{A_n} \left[\int\limits_{A_y} U(x(y,n), g(y,n)) dG^n(y \mid n) \right] dG(n). \quad (4)$$

Introducing $V^h(y,n)$ the indirect household utility function associated to the program (1), i-e, $V^h(y,n) = nU(x(y,n), g(y,n))$ and $H(n) = \frac{N^i}{N^h} \int\limits_{[0,n]} \frac{1}{t} dG(t)$ which gives the cumulated proportion of households of size smaller than or equal to n, we obtain either

$$W_G = N^h \int\limits_{A_n} \left[\int\limits_{A_y} V^h(y,n) dG^n(y \mid n) \right] dH(n) \quad (5)$$

as final expression for the social welfare, or

$$W_G = N^i \int\limits_{A_n} \left[\int\limits_{A_y} V^i(y,n) dG^n(y \mid n) \right] dG(n) \quad (6)$$

with

$$V^i(y,n) = \frac{V^h(y,n)}{n} \quad (7)$$

the indirect individual utility function, if we are interested in expressing the social welfare as a sum of individual well-beings.

It must be clear that either (5) or (6) satisfies anonymity by construction, with respect to indirect household utility in the former case and with respect to indirect individual utility in the latter case.

Bourguignon (1989) has investigated what are the properties of the indirect household utility function and he showed that the condition $V^h_{yn}(y,n) > 0$ (where the subscript denotes a partial derivative) is not ensured and depends on the elasticity of substitution between private and public consumption (e.g., note 2 p.71). We will not pursue in this direction here and the difficulty of obtaining clear-cut properties of the indirect household utility function in that case might vindicate to call on some ad-hoc forms.

Now suppose that a valid reduced form for the indirect household utility function is given by the expression pioneered by Ebert

$$V^h(y,n) = m(n) v(\frac{y}{m(n)}), \quad (8)$$

where $m(n)$ is the equivalence scale associated to a family of size n and $v(.)$ is any real-valued continuous strictly increasing function.

Substituting this expression for the indirect household utility function in (5) gives the exact expression of social welfare which has been axiomatized by Ebert (1999).

Comparing (7) and (8), makes transparent that $v(.)$ cannot be assimilated to the indirect individual utility function. Indeed starting from (8) the true expression for the indirect individual utility function is

$$V^i(y,n) = \frac{m(n)}{n} v(\frac{y}{m(n)}). \tag{9}$$

The anonymity condition requires that if the profile of individual utility levels is permuted, then the social welfare remains unchanged. If we apply the anonymity to the wrong profile, the profile given by v,[4] anonymity is not satisfied by the social welfare given by (6). Then the paradox comes from the fact that v serves as an ingredient of an anonymity property, while it should not since v is not related to the well-being of an individual in the grand microeconomic tradition.

3 Conclusion

Social choice theory is so full of impossibility results that it may be viewed as good news that a dilemma pointed out by several authors vanishes, when it is formulated in the right setting, the one provided by the standard microeconomic theory. Since the alternative weighting solution, the family size, has been showed to be incompatible with an extension of the Pigou-Dalton principle of transfers in some circumstances (see Glewwe (1991)), there is only one weighting solution on the battle ground. One of the weaknesses of that solution on a purely empirical point of view still deserves to be mentioned. Many equivalence scales have been proposed in practice and any uncertainty about the "true" equivalence scale would weaken any ranking obtained with a specific equivalence scale. To solve this problem, Fleurbaey *et al.* (2001) proposed to consider all equivalence scales belonging to some given interval. The intersection of the rankings obtained for each admissible equivalence scale is a quasi-ordering which might be difficult to compute. They exhibit an algorithm which does so and they showed that the criterion corresponding to it is also equivalent to dominance for families of social welfare functions inspired by Atkinson and Bourguignon (1987), in which household utility is a general function of income and needs like in (5).

References

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