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Consistency between tastes and values: A universalization approach

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Abstract. The object of this paper is to propose a *consistency test* for an individual involved in collective choice process. Collective choice processes considered in the paper are those that transform individuals '*tastes*' – which reflect the self-interested view point of the individuals – into (social) ranking of alternatives. In addition to her tastes, an individual has *values* about the way by which collective decision should be made. We distinguish two categories of such values. First, there are *end-values* that restrict the class of social rankings that the individual considers ethically acceptable. Second there are *aggregation-values* that specify the way by which the social ranking should depend upon the individuals tastes. The consistency test stands on an hypothetical operation of *universalization* of the individual tastes to everyone. Five illustrations of the potential usefulness of our approach for interpreting social choice theory and welfare economics are proposed. These illustrations deal with utilitarian aggregation in the presence of income inequality aversion, the so-called 'ethics of responsibility' and the aggregation of individual ranking of opportunity sets based on their freedom of choice. A discussion of the relevance of the consistency test for addressing the problem of 'laundering' individual preferences is also provided.

'The principle of right, and so of justice, puts limit on which satisfactions have value; they impose restrictions on what are reasonable conceptions of one's

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good. In drawing up plans and in deciding on aspirations, men are to take these constraints into account. (...) the interests requiring the violation of justice have no value.' (Rawls [57], p. 31)

1 Introduction

As conventionally understood in economics, the word *consistency* refers to a property of the behavior of an agent in an *isolated context*. A behavior is considered consistent if it satisfies various properties – like contraction and/or expansion consistency in abstract choice theory,[1] revealed preferences and/or profitability axioms in standard microeconomic theory,[2] and consequentialism and other path independence axioms in the case of sequential choices[3] – that enables a rationalization of this behavior as resulting from the maximization of some objective. In conventional economics therefore, "consistent" means "can be thought of as resulting from some maximization".[4]

The aim of this paper is to inquire into the behavior of an individual who is involved in an explicit *collective choice context* and to investigate another notion of consistency for such a setting. In this broader view of the behavior of an individual, the scope for a consistency requirement is very much increased. Yet the meaning we shall give to the word consistency in this context is somewhat different than the usual one and is more related to the common sense notion. Loosely speaking, an individual is consistent if the different pieces of opinions that she expresses in the course of the public debate are free from contradictions. More specifically, this paper proposes a *consistency text* for an individual involved in a collective choice process and illustrates the usefulness of such a test for addressing a few problems related to the standard application of social choice theory.

An example can usefully illustrate what we have in mind. Suppose that Bob belongs to a parliament and is known to be very active in promoting laws against the illegal arrival of immigrants. At the same time, newspapers reveal that Bob employs Anita, an illegal immigrant, as a baby sitter. Bob's behavior is likely to be portrayed as "inconsistent' by a wide range of accounts of the word. And it will be considered as inconsistent by the test proposed herein. Let us see why this is so.

As it shall be interpreted herein, the inconsistency comes from a conflict between Bob's *tastes* and Bob's *values*. The distinction between tastes and

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1 See for instance Sen ([63], [65]).
2 See e.g. Samuelson [61], Houthakker [48], Richter [58] and Varian ([76], [75]).
3 See e.g. Hammond [42] and Bandyopadhyai [7].
4 If an objective pursued by the agent is to give rise to a rational – or *consistent* – behavior, the objective must itself produces a transitivite or, at least, an acyclical ranking (see Sen ([63], ch. 1*) for a discussion of this point). For this reason, we also often think of properties such as transitivity, quasi-transitivity and acyclicity as to *consistency* properties.

values has been made famous by Arrow [2] and Sen [66] (who phrases it in terms of 'interests' (instead of 'tastes') and 'judgements' (instead of values'). It has also been considered by Rawls ([57], p. 31) who prefers opposing 'desires and aspirations' – on the 'tastes' side – to 'principles of justice' – on the 'values' side. As interpreted herein, a 'taste' is a criterion for comparing alternative states of affair from the self-interested viewpoint of the individual. In the example, it is not so difficult to understand why Bob employs Anita rather than a legal baby sitter. It is less expensive and Anita does not complain if he and his wife come back two hours late from some party. Anita is so kind to their children who learn some words of a foreign language by the way! By any standards we can say that Bob family's well-being is higher when they employ Anita rather than a legal baby sitter. Bob's 'tastes' clearly favors a liberal immigration law.

But in addition to his tastes, Bob has 'values' which can be defined broadly as opinions on the result and the process of social choice beside his own individualistic viewpoint. In this paper, we find useful to distinguish between *two* categories of such values.

The first category consists of *end-values* which take the form of a restriction in the set of rankings of social states which are acceptable from a normative standpoint. A example of such end-values is the requirement that the social ranking be an ordering typically imposed in social choice theory. Another example, applicable to the case where social states are distributions of incomes, is a requirement of aversion for income inequality. In the example, the opinion defended by Bob in the parliament is a typical end-value. Bob defends the view that the first settlers in a country have a right to object to newcomers. According to such an end-value, social states allowing unrestricted immigration must be ranked below social states which restrict it.

In the second category are *aggregation-values* that restrict the shape of the *social aggregation function* which transforms data on individuals into rankings of social states. An example of such a restriction in social choice theory is of course the Pareto-requirement that the function be 'monotonically increasing' with respect to each individual taste. Another well-known example, that makes sense when the individual tastes are cardinally measurable by a utility function and interpersonally fully comparable, is the requirement that the social aggregation function takes the specific form of the *sum* of the individual utilities. This latter aggregation value is usually referred to as *utilitarianism*. In the example, Bob's aggregation values have not been made explicit. We could assume that, as a member of a parliament, Bob's aggregation values take the form of a Paretian requirement: If a social decision improves the self-interest of everyone (including his own), it must be adopted.

Why does the triple consisting of Bob's taste (hiring Anita), Bob end-values (promoting restrictive immigration laws) and Bob's aggregation-values (respecting everyone's tastes) seem so inconsistent?

The consistency test that we propose to answer this question is based on a *principle of universalization*. It is in effect well-known that most ethical systems have in common the property of appraising the morality of individual acts by resorting to an operation of universalization. By such an operation, the *actor* evaluates the consequences of her act by putting herself in the hypothetical situation in which everyone else acts in an equivalent way as she does. The Bible's Golden Rule 'Do unto others as ye would that others would do to you' and Kant's ([51], p. 66) categorical imperative ('Act always on such a maxim as thou canst at the same time will to be a universal law') are classical examples of the use of such a operation. Other use of universalization within the realm of public and normative economics can be found in Bilodeau and Gravel [9], Bordignon [14], Laffont [53] and Roemer [60] (ch. 6) among others. The operation of universalization is usually defined over individual actions. In this paper, we interpret the individual's 'actions' as the tastes that are transformed in a collective judgement by the social aggregation function. With such an interpretation, we define an individual taste to be consistent with a set of values when, *if adopted by everyone*, and if aggregated by a social aggregation function compatible with the aggregation-values held by the individual, it would result in a ranking of social states which is acceptable for the individual's end-values.

Applied to the example, the universalization test makes straightforward the contradiction between Bob's tastes and values. If we universalize the taste of Bob – that is if we assume that everyone in the country choose to employ an illegal immigrant as a baby-sitter – the social choice which would result from a social aggregation function that he considers legitimate would contradict the end-values he promotes in the parliament. To become consistent, he could ''launder' his tastes (using the word coined by Goodin [37]) by sacking Anita. But Bob could also resolve the inconsistency by ''laundering'' his values. As shall be discussed herein, our consistency test does not provide any specific advice to this respect.

The resort to an operation of universalization of the kind we propose seems plausible as a test for checking the consistency of a set of normative views held by an individual. Consider in effect an individual who accepts in advance a set of norms that restrict a priori the set of admissible rankings of social states and a method for aggregating individual tastes into a collective ranking. The problem of this individual is to formulate a taste which, when aggregated with the criteria of the other individuals by the method with which she identifies, produces a ranking of social states that she considers ethically defensible. Yet this individual does not have control over the tastes chosen by the other members of the society. For this reason, an assessment of the consistency of the individual's tastes with the individual's values should be *independent* from the tastes chosen by others. *A priori* there are two ways by which such an independence could be posited. One would be to consider an individual taste to be consistent with a set of values if it produces an ethically acceptable social ranking *no matter what the configuration of other individual criteria is*. The other is the one we choose to adopt. The first notion of independence would be very demanding since it amounts to requiring the individual to formulate a taste which is a 'dominant strategy' – as far as consistency is concerned – against any choice of tastes by the others. The notion of independence on which our test is based seems *a priori* less demanding since it only requires the individual taste to be consistent with the individual values when the taste is universalized to everyone. Of course, in the 'real' course of social choice, the other individuals will typically not formulate the same tastes as that of the individual. Yet, if the other individuals formulate different tastes, then at least the ethical unacceptability of the resulting social ranking could not be said to result from her own inconsistency. It would be the result of a bad coordination or of the ethical inconsistency of others.

Even if the plausibility of this test is acknowledged, the relevance of such an exercise can be questioned. The interest of our consistency test seems two-fold. First it allows one to provide some *endogenous* domain restrictions on the space of admissible preferences or on the set of utilities functions considered in aggregation procedures. Restrictions of domains have been thought as one way to escape the gloom of Arrow's or Sen's [64] theorems and have been at the origin of a large literature whose a thorough account can be found in Gaertner [36]. For instance, it is often assumed that the individual utility function are concave. Yet the status played by such an assumption is rarely made explicit. Is concavity a positive assumption about the likely relation which exists between, say, happiness and income? If so, on what kind of empirical evidence is such an assumption based? In this paper (see in particular Sect. 3.1), we propose instead to understand concavity as a normative requirement imposed by the consistency of an explicit set of individual tastes and values.

Second, the test aims at providing a kind of bridge between welfarist social aggregation, which underlies much of contemporary social choice theory and welfare economics, and various non-welfarist principles. Recall that *welfarism* is the claim that social decision should result only from the aggregation of individual subjective utility by a Pareto inclusive function.[5] As an approach to social choice, welfarism has been severely criticized from various perspectives in the last two decades. By using examples of sadistic preferences (as in Sen [68]), of 'expansive tastes' (as in Dworkin [30]) or of individual preferences that adjust to adverse conditions by a process of cognitive dissonance (the famous 'tamed wife problem' commented at length by Roemer [60]), various author have challenged the ability of welfarism to generate an ethically acceptable ranking of social states. These criticisms have led some writers (like Rawls [57], Dworkin [30] and, to a less extent, Sen ([69], [70]) to abandoning welfarism. An example of a non-welfarist method to normative issues in economics is provided by the so-called 'ethics of responsibility' examined by Roemer ([59], [60]), Bossert [15], Bossert and Fleurbaey [16] and Fleurbaey [33], [34]) among others. It has led others, like Goodin [37] (and also Arneson [1], Hare ([44], [45]), Harsanyi ([46], [47]), Broome [19] and Griffin [41]) to argue that individual utilities be 'laundered' before entering as arguments in

[5] See d'Aspremont and Gevers [23], d'Aspremont [22], Blackorby et al. [11] and Sen ([67], [68]) for classic discussions of welfarism.

the social aggregation function.[6] Yet no systematic procedure for 'laundering' individual utilities has been proposed.

Welfarism is an example of an aggregation-value while non-welfarist authors base their attack against welfarism on various end-values (such as a concern for resource inequality (in Dworkin [30]) or for responsibility (in Roemer ([59], [60]). Interpreted in this light, the issue of consistency between tastes and values can be stated as follows: For which kind of tastes are welfarist and non-welfarist view points compatible?

In this paper, we illustrate the usefulness of our consistency test to five situations that all share the following features. First the set of social states is the Cartesian product of as many sets as there are individuals. Each set of this product is therefore interpreted as the dimension of the social state that is specific to the individual to which it corresponds. An allocation of private goods among individuals is an example of this kind of situation. Second, all individual sets are identical. Problems of allocations of private goods are also consistent with this kind of situation (at least if individual consumption sets are identical or if the allocation problem is interested in their common intersection). Third, the criterion used by an individual to compare the worth of alternative social states depends only upon the dimension of the social states that is specific to this individual. Using the allocation of goods example, this would be the case if individuals cared only about what they receive in the various allocations. This rules out benevolence and malevolence as well as other kind of externalities. Within this class of problem, each individual criterion is assumed to be defined on the *dimension* of social states that is specific to the individual. It is to this 'self-regarding' criterion that we apply the notion of normative consistency examined herein. This framework is of course motivated by our interpretation of such a criterion as a 'taste'.

In the first three examples, the consistency test is applied to individual tastes that are cardinally measurable and interpersonally comparable and to an aggregation-value that is utilitarian or, at least, welfarist. We first examine the problem of ranking income distributions in the case where income recipients are identical in every respects other than their income and, possibly, the utility they derive from it. In addition to her utility (interpreted as a taste), the individual is assumed to hold a end-value that takes the form of the aversion to income inequality contained in the standard Pigou-Dalton principle of transfers and an aggregation-value that is utilitarian. Consistency in this context requires the individual utility function to be concave with respect to income.

The second illustration is taken from a similar problem in the case where income recipients are allowed to differ by another characteristic (say their health) reflecting their 'need'. Noticeable contributions on this topic are Atkinson and Bourguignon ([4], [5]), Bourguignon [18] and Ebert ([32], [31]).

[6] See also the discussion of the issue of 'laundering preferences' in the recent survey by Mongin and d'Aspremont [24].

In this context, a generalization of the Pigou-Dalton principle of transfers can be proposed. The generalization says that income inequality is unambiguously reduced when a finite sequence of transfers of income from richer *and* less needy households to poorer *and* needier ones that do not modify the ranking of household with respect to income is performed. Assuming, that each income recipients derive cardinally and interpersonally meaningful (indirect) utility from both income and need, we show that the consistency of such a taste with a utilitarian aggregation-value and with the generalized Pigou-Dalton principle of transfers as an end-value requires individual's marginal utility of income to be decreasing with respect to income and increasing with respect to the need.

Closely related to the problem of ranking income distributions when individuals differ in other dimension than income is the 'responsibility' approach mentioned above. As discussed in Fleurbaey [35], this approach proposes two principles that a rule for redistributing income should follow: A principle of *compensation* and a principle of *natural reward*. The principle of compensation says, roughly, that income should be redistributed so as to equalize (or maximin) the individuals 'achievements' – when individuals have exerted the same level of 'responsibility'. The principle of natural reward says roughly that the social ranking of income distributions should always respect the 'natural ranking' of income distributions that would result from different exercises of individual's responsibility. In the third application, the consistency test is applied to a utilitarian individual who adopts two rather weak formulations of each of the two principles as end-values. It is shown that the consistency test implies significant restrictions on the individuals' utility function.

The last two illustrations concern situations in which individuals tastes are only ordinally defined. The fourth application is devoted to a situation where individuals have (ordinal and inter-individually non-comparable) pref-erences over bundles of goods and where these preferences are used to rank alternative combinations of prices and incomes on the basis of the summation of the money metric representations (see Samuelson [62]) of the individual preferences (as an aggregation-value). We assume as in the first illustration that the end-value of such an individual is the income inequality aversion notion which underlies the compatibility with the Pigou-Dalton principle of transfers. Using a result due to Blackorby and Donaldson [10], we show in this context that the consistency of the individual system of normative views requires the individual preferences to be homothetic.

Finally, as a fifth application, we consider the problem of ranking vectors of opportunity sets in connection with a recent literature that aims at making precise the notion of individual *freedom of choice*. Using results developed in another contribution (see [40]), we show in this setting that applying the consistency test to *a priori* plausible aggregation-values and end-values that an individual can hold about the aggregation of individual rankings of opportunity sets (as the individual tastes) implies a quite definite property of the ranking of opportunity sets: That of having an *additive* numerical representation.

The organization of the rest of the paper is as follows. In the second section we define formally the general notion of normative consistency. In the third section, we apply this notion to utilitarian aggregation-values by examining various set of possible end-values. The fourth section is devoted to the aggregation of ordinally non-comparable tastes. The fifth section provides a brief discussion of the usefulness of our approach to address the problem of laundering individual preferences. The sixth section concludes.

2 A general definition of normative consistency

We consider an individual who has *normative conceptions* on the ranking of social alternatives in the class of *social contexts* in which she might evolve. For the purpose of this paper, a social context consists of the following elements. First there is a (possibly variable) finite number n of individuals. Each individual is concerned about a set X of (individual) alternatives that is taken to be the same for all individuals. The fact that individual i $(i = 1, \dots, n)$ is 'concerned' about the alternatives in X means that individual i has a *criterion* C_i for comparing the worth of the alternatives from the view point of her tastes. Every individual criterion is taken from a class $\mathscr{C}$ of admissible criteria which, in the applications considered below, will be utility functions from X to the real line, preference orderings on X or even weaker binary relations (as in Sect. 4.2 below).[7] The fourth element of a social context is a set $Y_n \subseteq X^n$ of social alternatives. In this framework, a social alternative is interpreted as a combination of individual alternatives which indicates precisely who gets what. In short, a social context is a quadruplet $(n, X, \{C_i\}_{i=1}^n, Y_n) \in \bigcup_{n \in N^+} (\{n\} \times \{X\} \times \mathscr{C}^n \times \{Y_n\})$. We denote by S the set of admissible social contexts $\left(S \subseteq \bigcup_{n \in N^+} \{n\} \times \{X\} \times \mathscr{C}^n \times \{Y_n\} \right)$.

We now turn to the *values* that an individual can have as to how alternatives in Y_n can be compared. As mentioned in the introduction, we find

[7] Our terminology, definitions and notation for binary relations is as follows. By a *binary relation* $\succeq$ on a set Ω, we mean a subset of $\Omega \times \Omega$. Following the convention used in economics, we write $x \succeq y$ instead of $(x, y) \in \succeq$. Given a set Ω, the set of all binary relations on Ω is denoted by $\beta(\Omega)$. Given a binary relation $\succeq$, we define its *symmetric factor* by $x \sim y \Leftrightarrow (x \succeq y) \wedge (y \succeq x)$ and its *asymmetric factor* $\succ$ by $x \succ y \Leftrightarrow (x \succeq y) \wedge \neg(y \succeq x)$. A binary relation $\succeq$ on Ω is *complete* if for any (possibly non-distinct) $x, y \in \Omega$, either (or both) of the statements $(x \succeq y)$ or $(y \succeq x)$ hold, is *reflexive* if the statement $x \succeq x$ holds for every x in Ω, is *transitive* if $x \succeq z$ follows $x \succeq y$ and $y \succeq z$ for any $x, y, z \in \Omega$ and is *anti-symmetric* if $x \sim y \Rightarrow x = y$. A complete (and therefore reflexive) and transitive binary relation is called an *ordering* and a reflexive and transitive binary relation is called a *quasi-ordering*. A binary relation $\succeq$ on Ω is *weakly compatible* with a binary relation $\hat{\succeq}$ on Ω if and only if $x \hat{\succeq} y \Rightarrow x \succeq y$ and is *strongly compatible* with it if it is weakly compatible and satisfies $x \hat{\succ} y \Rightarrow x \succ y$.

useful to distinguish between two categories of such values: Aggregation-values and end-values. The key element of first category of value is a *social aggregation functions* $A_n : \mathscr{C}^n \to \beta(Y_n)$ which associates a unique binary relation on Y_n to any profile of individual criteria. This aggregation function is taken to belong to a certain class $\mathscr{A}_n$ which is interpreted as the largest class of functions which satisfies ethical properties that are considered desirable by the individual. For example, $\mathscr{A}_n$ could consist in all functions from $\mathscr{C}^n$ to $\beta(Y_n)$ that are (weakly or strongly) Pareto-inclusive or that are non-dictatorial with respect to the individual criteria. It could also consist of a single element (like the sum of individual utilities if utilitarianism is adopted as aggregation value).

The other category of values concern the *range* of the social aggregation function, that is the set of (ethically) admissible rankings of social alternatives. Formally, given a number n of individuals, these end-values take the form of a subset $\mathscr{E}_n$ of the set of all binary relations on Y_n. The interpretation given to $\mathscr{E}_n$ is that it contains all rankings of social alternatives which are compatible with a particular notion of the 'social good'.

Consider an individual who evolves in a social environment belonging to S and who has aggregation-values embodied in the class $\mathscr{A}_n$ of social aggregation functions and end-values defined by the class $\mathscr{E}_n$ of admissible social rankings. Suppose now that, given the normative conception embodied in $\mathscr{A}_n$ and $\mathscr{E}_n$, a particular individual criterion, *if adopted by all members of the society*, would result in a ranking of social states that does *not* belong to $\mathscr{E}_n$. Quite clearly an individual who would adopt such a criterion as a taste could be qualified as inconsistent.

A *contrario* we define individual normative consistency as follows.

Definition 1. *Given a class S of social contexts and a class of values embodied in $\mathscr{A}_n$ and $\mathscr{E}_n$, an individual criterion C is said to be consistent with $\mathscr{A}_n$ and $\mathscr{E}_n$ if and only if $A_n(C, \dots, C) \in \mathscr{E}_n$ for every $A_n \in \mathscr{A}_n$.*

We now turn to the illustrations of this notion of normative consistency.

3 Consistency in a welfarist-utilitarian setting

3.1 Consistency with income inequality aversion: The homogenous case

We consider here a case where n is fixed, where individual alternatives are incomes ($X = \Re_+$) and where the set of social alternatives consists in all possible income distributions ($Y_n = \Re_+^n$). The individual is assumed to derive utility from income by a continuous and weakly increasingly monotonic utility functions $U : \Re_+ \to \Re_+$. The class $\mathscr{C}^{\mathrm{UI}}$ of individual criteria relevant for this social context is the set of all such utility functions from income.

In the context of income distributions, a widely accepted normative principle is an aversion for income inequality. Although this aversion may take various forms, a weak one is the consistency with the Lorenz domination criterion. Recall that income distribution $(y_1, \dots, y_n)$ (weakly) Lorenz dominates income distribution $(y'_1, \ldots, y'_n)$ (denoted $(y_1, \ldots, y_n)\,L\,(y'_1, \ldots, y'_n)$) if $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} y'_i$ and if, for all $j \in \{1, \ldots, n\}$, $\sum_{i=1}^{j} y_{(i)} \geq \sum_{i=1}^{j} y'_{(i)}$ [8] where $(y_{(1)}, \ldots, y_{(n)})$ and $(y'_{(1)}, \ldots, y'_{(n)})$ are permutations of $(y_1, \ldots, y_n)$ and $(y'_1, \ldots, y'_n)$ respectively satisfying $y_{(i-1)} \leq y_{(i)}$ and $y'_{(i-1)} \leq y'_{(i)}$ for $i = 2, \ldots, n$. It is well known that L is a (quite incomplete) quasi-ordering of $\Re^n_+$. A standard normative justification for using this criterion is the so-called Pigou-Dalton principle of transfers. This principle asserts that any money transfer from a wealthier individual to a poorer one that do not modify their rank in the income distribution is, every other things considered, a good thing. Such a kind of transfer is usually called a *progressive transfer*. As it happens, for any two distinct income distributions $(y'_1, \ldots, y'_n)$ and $(y_1, \ldots, y_n)$, $(y_1, \ldots, y_n)$ strictly Lorenz dominates $(y'_1, \ldots, y'_n)$ if and only if $(y_{(1)}, \ldots, y_{(n)})$ has been obtained from $(y'_{(1)}, \ldots, y'_{(n)})$ by a finite sequence of such progressive transfers.[9]

Suppose that the individual has the aversion to income inequality that underlies the notion of compatibility with the Lorenz quasi-ordering (or with finite sequence of progressive transfers). In terms of the formal model of the previous section, this end-value induces the following definition of the set $\mathscr{E}_n^{\mathrm{PT}}$.

Definition 2. *$\mathscr{E}_n^{PT}$ is the set of all binary relations on $\Re^n_+$ which are strongly compatible with the asymmetric factor of L.*[10]

3.1.1 Utilitarianism as aggregation value. We assume first that the individual adopts utilitarianism as its aggregation value. This aggregation-value defines the singleton $\mathscr{A}_n^{\mathrm{U}}$ which contains the utilitarian functional $A_n^{\mathrm{U}} : [\mathscr{C}^{\mathrm{UI}}]^n \to \beta(\Re^n_+)$ as its unique element. Recall for latter use that this functional is defined as follows.

Definition 3.

$$\forall (U_1, \ldots, U_n) \in [\mathscr{C}^{UI}]^n, \quad \forall (y_1, \ldots, y_n), (y'_1, \ldots, y'_n) \in \Re^n_+$$

$$(y_1, \ldots, y_n) A_n^U (U_1, \ldots, U_n)(y'_1, \ldots, y'_n) \quad \Leftrightarrow \quad \sum_{i=1}^{n} U_i(y_i) \geq \sum_{i=1}^{n} U_i(y'_i).$$

What does our definition of consistency implies in this context? What properties (in addition to those listed above) must an individual utility func-

[8] At least one of these inequalities must be strict in order to have strict Lorenz domination

[9] See Berge [8] for an elegant proof of this result due to Hardy et al. [43]. It is easily seen that the binary relation induced by the statement 'is obtained from by a finite sequence of progressive transfers' is an anti-symmetric quasi-ordering. On the other hand, the binary relation L is not antisymmetric since it considers equivalent all permutations of a given income vector.

[10] The asymmetric factor of L corresponds to the notion of strict Lorenz domination.

tion satisfy in order to be consistent with utilitarianism and the income inequality aversion underlying compatibility with the Pigou-Dalton principle of transfers? The following proposition gives the answer to that question.

Proposition 1. *$U \in \mathscr{C}^{UI}$ is consistent with the utilitarian aggregation-value $\mathscr{A}_n^U$ and the end-value underlying the respect for progressive transfers $\mathscr{E}_n^{PT}$ if and only if U is strictly concave.*

Proof: It is well-known that if U is strictly concave, then the function $\Phi : \Re_+^n \to \Re_+$ defined by $\Phi(y_1, \dots, y_n) = \sum_{i=1}^n U(y_i)$ is also strictly concave and, for this reason (see [43]), the ordering R defined by $(y_1, \dots, y_n)\ R\ (y_1', \dots, y_n') \Leftrightarrow \sum_{i=1}^n U(y_i) \geq \sum_{i=1}^n U(y_i')$ is strongly compatible with the asymmetric factor of L. Suppose now that U is not concave in income. That is, assume that for some $x, z \in R_+$ such that $x > z$ and for some $\lambda \in]0, 1[$, $U(\lambda x + (1-\lambda)z) \leq \lambda U(x) + (1-\lambda)U(z)$. Since $U(\cdot)$ is continuous, there is no loss of generality in assuming $\lambda = \frac{1}{2}$. Consider now income distributions $(y_1', \dots, y_n')$ and $(y_1, \dots, y_n) \in \Re_+^n$ such that, $y_j' = x > \frac{x+z}{2} = y_j = y_i > z = y_i'$ for two individuals i, j and $y_k' = y_k$ for all other individuals k. The two income distributions are clearly distinct. Since $(y_1, \dots, y_n)$ has been obtained from $(y_1', \dots, y_n')$ by a progressive transfer of $\frac{x-z}{2}$ from individual j to individual i, $(y_1, \dots, y_n)$ strictly Lorenz-dominates $(y_1', \dots, y_n')$. Note however that, by assumption, $\sum_{i=1}^n U(y_i) - \sum_{i=1}^n U(y_i') = 2U\left(\frac{x+z}{2}\right) - U(x) - U(z) \leq 0$. Hence the ordering R defined by $(y_1, \dots, y_n) R (y_1', \dots, y_n') \Leftrightarrow \sum_{i=1}^n U(y_i) \geq \sum_{i=1}^n U(y_i')$ is not strongly compatible with L. **QED**

Hence if John claims to derive continuously happiness from income, then his acceptance of the ethical norm contained in the Lorenz domination criterion and the utilitarian method for aggregating individual utilities forces him either to have marginal utility of income that decreases with his income or to be morally inconsistent. The usual property of decreasing marginal utility of income is, when interpreted in our setting, a normative requirement rather than a positive assumption.

3.1.2 Consistency for a mean of order-r aggregation function. Considering utilitarianism as an aggregation value of an individual who is averse to income inequality may seem a bit contradictory. After all why would an individual concerned with *income inequality* identifies herself with an ethics like utilitarianism who exhibits very little concern for *welfare inequality*? Yet utilitarianism is only a particular case of the mean-of-order r family of social aggregation function $A_n^{m(r)} : [\mathscr{C}^{\mathrm{UI}}]^n \to \beta(\Re_+^n)$ defined by

$$\forall(U_1,\ldots,U_n)\in[\mathscr{C}^{\mathrm{UI}}]^n, \forall(y_1,\ldots,y_n),(y'_1,\ldots,y'_n)\in\Re^n_+$$

$$(y_1,\ldots,y_n)A_n^{m(r)}(U_1,\ldots,U_n)(y'_1,\ldots,y'_n)$$

$$\Leftrightarrow \quad \left[\sum_{i=1}^{n}(U_i(y_i))^r\right]^{1/r} \geq \left[\sum_{i=1}^{n}(U_i(y'_i))^r\right]^{1/r}\Bigg\}$$

if $r\in\Re\backslash\{0\}$ and by

$$(y_1,\ldots,y_n)A_n^{m(r)}(U_1,\ldots,U_n)(y'_1,\ldots,y'_n)$$

$$\Leftrightarrow \quad \sum_{i=1}^{n}\ln U_i(y_i) \geq \left[\sum_{i=1}^{n}\ln U_i(y'_i)\right\}$$

otherwise. Utilitarianism corresponds to the case where $r=1$. Lower values of r correspond to higher degrees of inequality aversion, the highest of which – the maximin criterion – being the limit of the functional as r approaches $-\infty$.

If one applies the test to any social aggregation function in this class, one obtains as a corollary of proposition1 that the restriction imposed on the individual utility function is that of r-concavity introduced by Avriel [6] and discussed in Caplin and Nalebuff [20].

Recall that a function f with convex domain $B\subseteq\Re^n$ is r-concave if for any $\lambda\in[0,1]$ and any x, $y\in B$, it satisfies

$$f(\lambda x+(1-\lambda)y)\geq[(1-\lambda)f(x)^r+\lambda f(x)^r]^{1/r} \quad \text{if } r\neq 0$$

$$\ln f(\lambda x+(1-\lambda)y)\geq(1-\lambda)\ln f(x)+\lambda\ln f(x) \quad \text{otherwise.}$$

Hence, for $r=0$ (corresponding to the social aggregation function associated to the symmetric Nash bargaining solution), the restriction implied by r-concavity is that of log-concavity. As r approaches $-\infty$, r-concavity reduces to quasi-concavity. For this reason, an individual who identifies with a maximin aggregation function, who is averse to income inequality and who has a utility function that is increasingly monotonic with respect to income (and therefore quasi-concave in this unidimensional setting) will always be consistent according to definition.

3.2 *Consistency with income inequality aversion and utilitarianism: The heterogeneous case*

In the preceding example, individuals were assumed to be identical in every respect other than income (and possibly utility). Yet in many circumstances where comparisons of income distributions are performed, such an assumption can not be sustained. Utility notwithstanding, individuals or households do differ by other dimensions, like their health or handicap, than income. These dimensions typically affect the ability of income recipients to convert income into utility.

Suppose for simplicity that there is only one of these dimensions, say the health status, represented by a non-negative real variable θ, the larger value of which being associated with better health. In terms of the formal framework of the preceding section, individual alternatives are income and health status combinations (that is $X = \Re_+ \times \Re_+$) and social alternatives are all income and health status allocations ($Y_n = \Re_+^{2n}$). The class of individual criteria is the set $\mathscr{C}^{\mathrm{UHI}}$ of all utility functions for health and income that are increasing with respect to both variables and differentiable with respect to income. Here again, we assume that the individual adopts utilitarianism as her aggregation value (defined formally just as before using $\mathscr{C}^{\mathrm{UHI}}$ instead of $\mathscr{C}^{\mathrm{UI}}$ as the domain of definition of the utilitarian functional in definition 3).

In this context, a natural extension of the Pigou-Dalton principle of transfers would be to consider that, *ceteris paribus*, transferring income from a richer *and* healthier household to a poorer *and* less healthy one is a good thing when this transfer does not modify their relative ranking with respect to both income and health. Formally, we define such a *restricted progressive transfer (RPT)* as follows:

Definition 4. *Given any two distributions of incomes y and* $y' \in \Re_+^n$ *and a distribution of health status* $\theta \in \Re_+^n$, *we say that* (y, θ) *is obtained from* (y', θ) *by a RPT if and only if there are two individuals i and j such that* 1) $\theta_i \geq \theta_j$, 2) $y'_j + \delta = y_j \leq y_i = y'_i - \delta$ for some $\delta \geq 0$ *and* 3) $y'_h = y_h$ for all $h \neq i, j$.

As in the case of the Pigou-Dalton principle of transfer, we can extend this idea by saying that the fact of going from one distribution of income to another (for a given distribution of health status) by a finite sequence of *RPT* so defined is also, *ceteris paribus*, a good thing. We define the binary relation *"is obtained from by a finite sequence of restricted progressive transfers"* ($\widehat{RPT}$) on $\Re_+^n \times \Re_+^n$ (formally the transitive closure of *RPT*) as follows.

Definition 5. $\forall (y, \theta),\ (y', \theta') \in \Re_+^n \times \Re_+^n,\ (y, \theta)\widehat{RPT}(y', \theta')$ *if and only if* 1) $\theta' = \theta$ *and* 2) *there exists a sequence* $\{y^t\}_{t=0}^T (T \in N_+, y^t \in \Re_+^n$ *for all t) such that* $y^0 = y,\ y^T = y'$ *and* (y^t, θ) *is obtained from* (y^{t+1}, θ) *by a RPT for* $t = 0, \ldots, T-1$.

Suppose that we assume that end-value of this individual is the set $\mathscr{E}_n^{\mathrm{RPT}}$ of all binary relations on $\Re_+^n \times \Re_+^n$ that are compatible with $\widehat{RPT}$. Then applying our consistency test leads to the following property of the individual utility function.

Proposition 2. $U \in \mathscr{C}^{UHI}$ *is consistent with the utilitarian aggregation value* $\mathscr{A}_n^U$ *and the end-value* $\mathscr{E}_n^{RPT}$ *associated with the respect for restricted progressive transfer if and only if it is strictly concave in income and satisfies* $\theta \geq \theta' \Rightarrow U_y(y, \theta) \leq U_y(y, \theta')$ *for every* $y \in \Re_+$.

Proof: For the sufficiency part, suppose that U has the assumed property and let $\hat{y}$ and $\tilde{y} \in \Re_+^n$ be any two distinct distributions of income such that $(\hat{y}, \bar{\theta})$ is obtained from $(\tilde{y}, \bar{\theta})$ by a *RPT* for a distribution of health status $\bar{\theta} \in \Re_+^n$. We need to show that $\sum_{h=1}^{n} U(\hat{y}_h, \bar{\theta}_h) - \sum_{h=1}^{n} U(\tilde{y}_h, \bar{\theta}_h) > 0$. Clearly, $\sum_{h=1}^{n} U(\hat{y}_h, \bar{\theta}_h) - \sum_{h=1}^{n} U(\tilde{y}_h, \bar{\theta}_h) = U(\tilde{y}_i - \delta, \bar{\theta}_i) - U(\tilde{y}_i, \bar{\theta}_i) + U(\tilde{y}_j + \delta, \bar{\theta}_j) - U(\tilde{y}_j, \bar{\theta}_j)$ with $\bar{\theta}_j \leq \bar{\theta}_i$ and $\tilde{y}_j + \delta = \hat{y}_j \leq \hat{y}_i = \tilde{y}_i - \delta$ for some $\delta > 0$. If $\bar{\theta}_j = \bar{\theta}_i$, the continuity and strict concavity of U with respect to income ensures that $U(\tilde{y}_j + \delta, \bar{\theta}_j) - U(\tilde{y}_j, \bar{\theta}_j) > U(\tilde{y}_i, \bar{\theta}_i) - U(\tilde{y}_i - \delta, \bar{\theta}_i)$. If $\bar{\theta}_j \leq \bar{\theta}_i$, we note first that by strict concavity of U with respect to income, one has

$$U(\tilde{y}_j + \delta, \bar{\theta}_j) - U(\tilde{y}_j, \bar{\theta}_j) > U(\tilde{y}_i, \bar{\theta}_j) - U(\tilde{y}_i - \delta, \bar{\theta}_j) \tag{A}$$

if $\delta > 0$ and $\tilde{y}_i - \delta \geq \tilde{y}_j + \delta$. Now, since U is differentiable with respect to income at every level of health status, one has, by Cauchy's theorem, that

$$\frac{U(\tilde{y}_i, \bar{\theta}_j) - U(\tilde{y}_i - \delta, \bar{\theta}_j)}{U(\tilde{y}_i, \bar{\theta}_i) - U(\tilde{y}_i - \delta, \bar{\theta}_i)} = \frac{U_y(z, \bar{\theta}_j)}{U_y(z, \bar{\theta}_i)}$$

for some $z \in]\tilde{y}_i - \delta, \tilde{y}_i[$ which, since $U_y(z, \bar{\theta}_j) \geq U_y(z, \bar{\theta}_i)$, implies that

$$U(\tilde{y}_i, \bar{\theta}_j) - U(\tilde{y}_i - \delta, \bar{\theta}_j) \geq U(\tilde{y}_i, \bar{\theta}_i) - U(\tilde{y}_i - \delta, \bar{\theta}_i). \tag{B}$$

Combining inequalities (A) and (B) yields the desired conclusion. Repeating this argument for any finite sequence of distributions of income such that one is obtained from the other by a *RPT* completes the proof of the sufficiency part of the proposition. For necessity, assume that U is either 1) not strictly concave in income for some health status or 2) such that, for some $\theta, \theta' \in \Re_+$ for which $\theta \geq \theta'$, there is some $z \in \Re_+$ satisfying $U_y(z, \theta) > U_y(z, \theta')$. In case 1), we can apply the proof of proposition 1. In case 2) since U is differentiable with respect to income, there exists some $\delta > 0$ such that $\frac{U(z + \delta, \theta) - U(z, \theta)}{\delta} > \frac{U(z, \theta') - U(z - \delta, \theta')}{\delta}$. Consider then a distribution of heath status $\bar{\theta} \in \Re_+^n$ and two distinct income distributions $\hat{y}$ and $\tilde{y} \in \Re_+^n$ such that, for some i, j, $\bar{\theta}_i = \theta' < \bar{\theta}_j = \theta$ and $\tilde{y}_j + \delta = \hat{y}_j = \hat{y}_i = z < \tilde{y}_i = z + \delta$ while, for all other income recipients $h \neq i, j, \tilde{y}_h = \hat{y}_h$. Clearly $(\hat{y}, \bar{\theta})$ is obtained from $(\tilde{y}, \bar{\theta})$ by a *RPT*. Yet $\sum_{h=1}^{n} U(\hat{y}_h, \bar{\theta}_h) - \sum_{h=1}^{n} U(\tilde{y}_h, \bar{\theta}_h) = U(z + \delta, \theta') - U(z, \theta') + U(z, \theta) - U(z - \delta, \theta') < 0$. **QED**

Hence, a utilitarian who believes in the value judgment embodied in the quasi-ordering $\widehat{RPT}$ would not be consistent in claiming that he had a utility function that exhibits a marginal utility of income that is increasing in either income or health status.

3.3 Reconciling utilitarianism with the 'ethics of responsibility'

In the eighties, philosophers such as Dworkin [30] and, after him, Arneson [1] and Cohen [21] have presented informal arguments in favor of a nonwelfarist and egalitarian theory of justice in which the *responsibility* borne by individuals over some of their traits is given an explicit role. The ideas of these philosophers have been recently examined in the context of formal economic models by Bossert [15], Bossert and Fleurbaey [16], Fleurbaey ([33], [34]) and Roemer ([59], [60], ch. 8) among others.

Very roughly, this 'ethics of responsibility' can be summarized as follows. Among the many traits (talents, handicaps, preferences, etc.) that can affect the individual's achievements (whatever those are), one must distinguish traits that are morally relevant from those that are morally irrelevant. Morally relevant traits are those for which the individual can be held responsible. On the other hand the individual is not supposed to be responsible for the value taken by irrelevant traits. In this approach, traits for which the individual is held responsible have reason to affect the individual's achievement. As a result, differences in individuals achievements which can be unambiguously attributed to differences in those traits (and only to difference in those traits) are not compensable at the bar of justice. For if they were, the individual's responsibility would be denied. Using Fleurbaey [35]'s terminology, let us refer to this first principle as to that of *natural reward*. On the other hand, differences of achievements which can be attributed to differences in morally irrelevant traits (and only to those difference) are considered morally irrelevant. For this reason, they should be more or less fully compensable at the bar of justice. This second requirement is called 'principle of *compensation*' by Fleurbaey. The objective of the 'responsibility' approach is to look for rankings of social states that respect simultaneously these two principles. In some economic environment, and for some axiomatic formulation of these principles, such a task has been proved difficult (see e.g. Bossert [15] and Fleurbaey ([33], [34]). On the other hand, there exists social choice correspondences that satisfy other axiomatic expressions of the two principles in some environments. Examples are given in Bossert and Fleurbaey [16] and Roemer ([59], [60]).

Advocates of the 'responsibility' approach insist typically on the fact that it is not welfarist. There are two reasons that justify such a claim. One is the fact the individual's 'achievements' that are considered in the 'responsibility' approach need not be (and typically are not interpreted as) individual utilities. However, this distinction is more semantic than substantial. The standard notion of utility employed in economics is sufficiently flexible (some would say sufficiently ambiguous) to be interpreted as an index of Rawl's 'primary goods' or of Sen's 'functionings' or of any other notion of well-being that one might have in mind. The other factor that distinguishes more substantially the 'responsibility' approach from welfarism is the fact that, in the former, knowledge of the individual's achievement does not suffice for comparing alternative social states from a normative standpoint. Information on the value taken by the relevant and the irrelevant characteristic is also important (not to say relevant) for normative appraisal. Although this distinction is significant, it is nonetheless possible to make welfarism consistent with the 'responsibility' ethics if the individuals 'advantage' functions are made consistent with the principle of compensation and of natural reward. The result of the last subsection is directly relevant for this purpose.

To illustrate, let us consider the following simple model inspired by the 'responsibility' literature. There are two individual traits represented by real variables ρ and θ. ρ is a trait for which individuals are assumed to be responsible. We shall interpret it as indexing the amount of 'effort' undertaken by the individual. In the background, we shall assume that there is a 'natural reward' function that associates to every level of effort the income that is 'naturally' obtained by an individual who responsibly chooses to expand that level of effort. We assume that this 'natural reward' function is increasing with effort. The process by which individuals choose their level of 'effort' (presumably by maximizing some objective function) shall not concern us in this model. As in the last subsection, θ is interpreted as a parameter (say the health) for which the individual is not assumed to be responsible. Individual i's 'achievement' – or utility – function $U_i : \mathfrak{R}^3_+ \to \mathfrak{R}_+$ is assumed to be monotonically increasing in income as well as in health and differentiable (and therefore continuous) with respect to income. Nothing *a priori* is assumed about the relationship between utility and effort. Let $\mathscr{C}^{\mathrm{UEHI}}$ denote the class of all such utility functions for effort, health and income.

In terms of the formal model of Sect. 2, the set Y_n of social states is thus assumed to comprise all income-effort-health allocations (that is $Y_n = \mathfrak{R}^{3n}_+$). As before, rankings of social states are assumed to result from the aggregation of the individual utility functions by the utilitarian functional defined just as in Subsect. 3.1 (using $\mathscr{C}^{\mathrm{UEHI}}$ instead of $\mathscr{C}^{\mathrm{UI}}$ in Definition 3). We now give statements of the principles of natural reward and compensation that an adept of the 'responsibility' approach would like the social ranking to satisfy.

For this task, we introduce the following definitions.

Definition 6. *Given any two income-effort-health allocations* $(\hat{y}, \hat{\rho}, \hat{\theta}), (\tilde{y}, \tilde{\rho}, \tilde{\theta}) \in \mathfrak{R}^{3n}_+$, *we say that* $(\hat{y}, \hat{\rho}, \hat{\theta})$ *is obtained from* $(\tilde{y}, \tilde{\rho}, \tilde{\theta})$ *by a compensating transfer if* $(\hat{\rho}, \hat{\theta}) = (\tilde{\rho}, \tilde{\theta})$ *and there exists two individuals i and j such that* 1) $\hat{\rho}_i = \hat{\rho}_j$ 2) $\hat{\theta}_i \geq \hat{\theta}_j$, 3) $\tilde{y}_j + \delta = \hat{y}_j \leq \hat{y}_i = \tilde{y}_i - \delta$ *for some* $\delta \geq 0$ *and* 4) $\hat{y}_h = \tilde{y}_h$ *for all* $h \neq i, j$.

Definition 7. *Given any two income-effort-health allocations* $(\hat{y}, \hat{\rho}, \hat{\theta}), (\tilde{y}, \tilde{\rho}, \tilde{\theta}) \in \mathfrak{R}^{3n}_+$, *we say that* $(\hat{y}, \hat{\rho}, \hat{\theta})$ *is obtained from* $(\tilde{y}, \tilde{\rho}, \tilde{\theta})$ *by a natural reward transfer if* $(\hat{\rho}, \hat{\theta}) = (\tilde{\rho}, \tilde{\theta})$ *and there exists two individuals i and j such that* 1) $\hat{\rho}_i < \hat{\rho}_j$ 2) $\hat{\theta}_i = \hat{\theta}_j$, 3) $\tilde{y}_j + \delta = \hat{y}_j \leq \hat{y}_i = \tilde{y}_i - \delta$ *for some* $\delta \geq 0$ *and* 4) $\hat{y}_h = \tilde{y}_h$ *for all* $h \neq i, j$.

As in the preceding subsection, we let the notation $\widehat{CT}$ and $\widehat{NRT}$ stand for the transitive closures of the binary relations CT and NRT respectively. In this setting, we define the **principle of compensation** as the requirement for the social ranking be strongly compatible with $\widehat{CT}$. As a matter of fact, this requirement is a very weak version of the principle of compensation discussed above. For this requirement merely says that a finite sequence of income transfers from richer and healthier individuals to poorer and less healthy ones that do not modify the relative ranking of individuals in terms of income is recommendable when the individuals between which the transfers take place exert the same amount of effort. Recall that the compensation principle requires that income should be redistributed in a way which equalizes (or maximin) individual's achievement when the individuals exert the same level of effort. Given that individuals' achievements are assumed to be increasing in both health and income, it is clear that a maximin social aggregation function will be compatible (at least weakly) with $\widehat{CT}$. Compatibility with $\widehat{CT}$ is also weaker than most usual statements of the principle of compensation in the sense that the quasi-ordering $\widehat{CT}$ is restricted to social states with identical distribution of individual traits. This quasi-ordering does not say anything about the relative ranking of social states in which the distribution of individual traits differ. Yet most expressions of the compensation principle found in the literature (for example the axiom of group solidarity considered in Bossert and Fleurbaey [16]) involve comparisons of such social states.

Analogously, we express the **principle of natural reward** as the requirement of strong compatibility of the social ranking with $\widehat{NRT}$. Here again, such a requirement is a weak expression of this principle. If (as it is assumed herein) individual income is increasing with individual effort, and if the individual's exercise of responsibility (in the choice of effort) is to be respected by society, then any redistributive policy should leave an individual which has exerted more effort than another with a higher income when the two individuals are endowed with the same health. $\widehat{NRT}$ is weaker than this requirement since it only claims that a transfer of income from a rich individual who had exerted a low effort to a poorer one who has exerted more effort is a good thing when the two individuals have the same health.

Suppose an individual wants the social ranking to respect the principle of compensation and natural reward (that is, her end-values is the set $\mathscr{E}_n^{\mathrm{CNR}}$ of all binary relations on Y_n that are compatible with both $\widehat{CT}$ and $\widehat{NRT}$) and adopts utilitarianism as her aggregation value. How should she restrict her utility function in this case? The following proposition (whose proof, which extends that of Proposition 2 in a straightforward manner, is omitted) gives the answer to this question.

Proposition 3. *$U \in \mathscr{C}^{UEHI}$ is consistent with the utilitarian aggregation-value $\mathscr{A}_n^U$ and the end values $\mathscr{E}_n^{CNR}$ defined by the compatibility with the principles of compensation and natural reward if and only if it is strictly concave in income and satisfies* 1) $\theta \geq \theta' \Rightarrow U_y(y, \rho, \theta) \leq U_y(y, \rho, \theta')$ *for every* $(y, \rho) \in \Re_+^2$ *and* 2) $\rho \geq \rho' \Rightarrow U_y(y, \rho, \theta) \geq U_y(y, \rho', \theta)$ *for every* $(y, \theta) \in \Re_+^2$.

Hence a utilitarian individual who identifies herself with the two ethical pillars of the 'ethics of responsibility' ('the signal achievement in the field in the last fifteen years' according to Roemer [60]) must, in order to be consistent with her theory, derive extra pleasure from extra money in a way that is decreasing with respect to her health and her income and increasing with the amount of effort.

4 Consistency in an ordinal and interpersonally non-comparable context

4.1 The mismarriage between money-metrics summation and income inequality aversion

This illustration is closely related to the preceding ones (the discussion of the 'ethics of responsibility' notwithstanding) but leads to a restriction of individual *ordinal preferences* (instead of a restriction of the individual utility function). We assume here again that the number of individuals n is fixed and that $X = \Re_+^{l+1}$. We interpret $\Re_+^{l+1}$ (with typical element (p, y)) as the set of all *price-income combinations* that the individual may face when choosing alternative l-dimensional consumption bundles (assuming that individuals treat prices and incomes as parametric to their decision). The class $\mathscr{C}^{\mathrm{IO}}$ of individual criteria relevant for this example is the class of all (indirect) orderings R on $\Re_+^{l+1}$ that have continuous numerical representations (see Debreu ([26]) $V : \Re_+^{l+1} \to \Re_+$ that are non-increasing in prices, non-decreasing in income, quasi-convex in prices and homogenous of degree 0 with respect to all variables. These representations are called indirect utility functions. It is a well known duality result (see Blackorby et al. [12]) that $V : \Re_+^{l+1} \to \Re_+$ is continuous, non-increasing in prices, non-decreasing in income, quasi-convex in prices and homogenous of degree 0 with respect to all variables if and only if there is a function $U : \Re_+^l \to \Re_+$, called the *primal utility function* associated to the indirect ordering R, such that, for all $(p, y) \in \Re_+^{l+1}$, $V(p, y) = \max_{x \in \Re_+^l} U(x)$ subject to $p \cdot x \leq y$. We further assume that the preferences of the individuals are such that, among the primal utility functions, there is at least one that is concave.[11] Given an indirect utility function V representing a preference ordering in $\mathscr{C}^{\mathrm{IO}}$, we can define a so-called *money-metric* utility function $\mu : \Re_+^{2l+1} \to \Re_+$ by:

$$\forall (q, p, y) \in \Re_+^{2l+1}, \quad \mu(q, p, y) = \min_{x \in \Re^l} \; q \cdot x \textit{ subject to } U(x) \geq V(p, y)$$

Intuitively, this function gives the minimal income that an individual needs when facing prices q to be as well-off as in the situation where she was facing price p and had an income of y. The prices vector q is usualy called the *reference-prices* vector. It is well-known (and obvious) that, for a given reference price vector, μ is an increasingly monotonic transform of the image of the function V and, for this reason, represents exactly the same preference ordering as V. For this reason, and because this function can in principle be observed from individual demand behavior (see Deaton [25] Samuelson [62] or Diewert [28]), it has become a standard tool of applied welfare economics analysis.

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[11] Preferences that have a concave utility representation are called concavifiable (see Kannai [50]).

Given this state of affairs, an illustration of the consistency test could be the following. Suppose that, for reason of observability, the only information that the social planner can obtain on the individuals indirect preference orderings on $\Re^{l+1}_{++}$ takes the form of a money metric utility function for some reference prices. Suppose further that it has been agreed in advance that the social rankings of price-incomes distributions (the prices being the same for all individuals) are those that obtains from comparing the values of the *sum* of these individual money-metric functions in alternative price-incomes distributions. That is, we suppose that $Y_n = \Re^{l+n}_{++}$ (with typical element (p, y)) and that the aggregation-values of the individual is the set $\mathscr{A}_n^{\mathrm{SMM}}$ of all rankings of $\Re^{l+n}_{++}$ that result from the summation of the money metric representations of indirect orderings for some reference price. Formally, $\mathscr{A}_n^{\mathrm{SMM}}$ is defined by

Definition 8.

$$\mathscr{A}_n^{SMM} = \Big\{ A_n^q : [\mathscr{C}^{IO}]^n \to \beta(\Re^n_+) : (\exists q \in \Re^l_{++})(\forall (R_1, \ldots, R_n) \in [\mathscr{C}^{IO}]^n),$$

$$(p, y) A_n^q(R_1, \ldots, R_n)(p', y') \Leftrightarrow \sum_{i=1}^{n} \mu_i(q, p, y_i)$$

$$\geq \sum_{i=1}^{n} \mu_i(q, p', y'_i) \forall (p, y), (p', y') \in \Re^{l+n}_{++} \Big\}.$$

As end-values, we assume that the individual dislikes income inequality at identical price configurations. As in Sect. 3.1, we express this inequality aversion by the set $\mathscr{E}_n^{\mathrm{PT}}$ of all rankings of $\Re^{l+n}_{++}$ which are compatible with the Lorenz-domination criterion (or the principle of progressive transfers) as applied to social alternatives that have the same prices configuration. Then we can ask what restrictions of individuals orderings on $\Re^{l+n}_{++}$ are implied by our consistency if we assume that the individual is uncertain about the choice of reference prices that will be made by the social planner. Note carefully the difference with the preceding illustration. In the previous case, we were looking for a restriction of the individual *utility function* which was therefore taken to be interpersonally and interpersonally meaningfully comparable. Here the institution in charge of aggregating individual criteria into collective ranking does not have such a rich information structure. The only meaningful data for aggregation are individual preference orderings and the set of aggregation functions that map these ordinal preferences into social rankings are those consisting in summing the money metric representations of these preference for some unknown level of reference prices. Given the poverty of the informational structure, the restriction of the individual preferences implied by our definition of moral consistency will be more stringent than in the previous section. Indeed the following proposition can be derived as an immediate consequence of Theorem 1 in Blackorby and Donaldson [10].

Proposition 4. *An indirect ordering $R \in \mathscr{C}^{IO}$ is consistent with the summation of money metric aggregation-values $\mathscr{A}_n^{SMM}$ and with the end-value $\mathscr{E}_n^{PT}$ defined by the compatibility with the Pigou-Dalton principle of transfers if and only if the preference represented by primal utility function associated with R is homothetic.*

Proof: $R \in \mathscr{C}^{\mathrm{IO}}$ is consistent with $\mathscr{A}_n^{\mathrm{SMM}}$ and $\mathscr{E}_n^{\mathrm{PT}}$ if and only if, for every reference price vector q, the ordering $A_n^q(R_1, \ldots, R_n)$ of $\mathfrak{R}_{++}^{l+n}$ of Definition 8 is compatible with L. As shown in Proposition 1, for this to be true, it is necessary and sufficient that $\mu(\ )$ be concave in income. But by virtue of Theorem 1 in Blackorby and Donaldson [10] (and of Theorem 3.18 of Diewert [27]), $\mu(\ )$ is concave in income for all reference price vectors if and only if the preference ordering that is represented by the primal utility function associated to R is homothetic. **QED**

That homotheticity of individual ordinal preferences can be interpreted as a normative requirement (rather than a positive assumption) is likely to be the source of some discomfort to many. Yet the reason for this discomfort should be imputed, in our view, more to the implausible assumption that an individual would agree to let collective decisions be taken by summing money metric representations of individual preferences than to our consistency test. To that extent, Proposition 4 reinforces Blackorby and Donaldson's [10] conclusion that income inequality aversion does not go hand in hand with the common practice of evaluating economic policy by summing money metric representation of the individuals ordinal preferences.

4.2 Normative consistency and aggregation of rankings of opportunity sets

The issue of defining and measuring individual freedom of choice has received attention recently (see, i*nter alia*, Arrow [3], Bossert et al. [17], Gravel ([38], [39]) Jones and Sugden [49], Klemisch-Ahlert [52], Pattanaik and Xu [54], Puppe ([55], [56]), Sen ([71], [72], [73]) and Suppes [74]). Typically, this question is handled by studying the consequences of imposing 'plausible' properties, or axioms, on the binary relation induced by the statement 'offers at least as much freedom as' applied to alternative individual *opportunity sets*. In this approach, an opportunity set is interpreted as the set of all *options* to which an individual has access and from which she will, in some latter stage, make a choice.

The consistency test can be applied to this problem by interpreting the framework of Sect. 2 as follows.

Assume that X is the set of all non-empty subsets of some *finite* set Ω of options. A typical element B of X is interpreted as an opportunity set. There is a *variable* number $n \in N_{++}$ of individuals. For any such number n, the set Y_n of social alternatives is assumed to be X^n, the typical element $(B_1, \ldots, B_n)$ of which being interpreted as an *allocation of opportunities*.

The class of admissible individual criteria is the set $\mathscr{C}^{\mathrm{FR}}$ of all freedom-rankings $\succeq$ (with asymmetric and symmetric factors $\succ$ and $\sim$) of individual opportunity sets in X that are **weakly monotonic with respect to set inclusion**. We formally defined this set as follows.

Definition 9. $\mathscr{C}^{FR} = \{\succeq \, \in \beta(X) : \forall B, B' \in X, B' \subseteq B \Rightarrow B \succeq B'$ *and* $\exists \, x \in \Omega$ *such that* $X \succ \{x\}\}$.

This class of rankings of individuals opportunity sets is typically conceived (see for example the discussions in [38], [39], [55] and [56]) as the largest one that is compatible with a minimally acceptable notion of freedom of choice.

We assume that the aggregation-values to which the individual subscribe for this problem is the class $\mathscr{A}^{\mathrm{UN}}$ of social aggregation functions that are **compatible with unanimity**. We define formally this latter notion as follows.

Definition 10. *Given a number n of individuals, a social aggregation function* $A_n : [\mathscr{C}^{FR}]^n \to \beta(X^n)$ *is* **compatible with unanimity** *if for any profile* $(\succeq_1, \ldots, \succeq_n) \in [\mathscr{C}^{FR}]^n$ *of individual rankings and any allocations of opportunities* $(B'_1, \ldots, B'_n)$ *and* $(B_1, \ldots, B_n) \in X^n$ *such that* $B'_i \succeq_i B_i$ *for all individual i and* $B'_j \succ_j B_j$ *for at least one j, we have* $(B'_1, \ldots, B'_n) \; A_n^A(\succeq_1, \ldots, \succeq_n) \; (B_1, \ldots, B_n)\}$ *(where* $A_n^A(\succeq_1, \ldots, \succeq_n)$ *is the asymmetric factor of* $A_n(\succeq_1, \ldots, \succeq_n)$*).*

That is, the only requirement imposed on the social aggregation function is to be strongly consistent with unanimity. If no individual looses freedom and at least someone gains freedom in moving from one allocation of opportunities to another, then the move should be considered as strictly enhancing social freedom.

A key assumption for the result to come is for the domain of social contexts to be larger than that of the preceding examples. Indeed, we assume here that

$$S = \bigcup_{n \in N^+} \{n\} \times \{X\} \times [\mathscr{C}^{\mathrm{FR}}]^n \times \{X^n\}$$

We are therefore interested in aggregating individual ranking of opportunity sets in society of every conceivable finite size.

In order to discuss some plausible end-values that the individual may have on the social ranking of allocations of opportunities, we introduce the following definition.

Definition 11. *Given any number* $n > 1$ *of individuals and any two allocations of opportunities* $(B_1, \ldots, B_n)$ *and* $(B'_1, \ldots, B'_n)$*, we say that* $(B_1, \ldots, B_n)$ *is obtained from* $(B'_1, \ldots, B'_n)$ *by a bilateral transfer of option (denoted* $(B_1, \ldots, B_n)$ *TO* $(B'_1, \ldots, B'_n)$*) if there exists an option* $b \in \Omega$ *and two individuals j and i such that the following holds* (1) $\forall h \neq i, j \; B_h = B'_h$, (2) $b \in B'_j, B_j = B'_j \backslash \{b\}$ *and* (3) $b \notin B'_i, B_i = B'_i \cup \{b\}$.

In words, a bilateral transfer of option a from individual j to individual i is simply a move by which access to option a is transferred from individual j to individual i. In an analogous fashion as in section 3, we can define the transitive closure $\widehat{TO}$ of TO which would then be interpreted as meaning "is obtained from another by a finite sequence of bilateral transfers of option"

Given this, we consider as the individual end-values the set $\mathcal{E}^{\mathrm{NSPR}}$ of all binary relations which exhibits **No Strict Preference for Redistribution** of a given amount of options across a given number of individuals and, this, no matter what this number of individuals is. Formally, we define $\mathcal{E}^{\mathrm{NSPR}}$ as follows.

Definition 12. $\mathcal{E}^{NSPR} = \bigcup_{n \in N_{++}} \{R_n \in \beta(X^n) : \forall (B_1, \ldots, B_n)$ *and* $(B'_1, \ldots, B'_n) \in X^n$, $(B_1, \ldots, B_n)\ \widehat{TO}\ (B'_1, \ldots, B'_n) \Rightarrow \neg((B_1, \ldots, B_n)\ P_n\ (B'_1, \ldots, B'_n)\}$ *(where P_n denotes the asymmetric factor of R_n.*

In words, the normative requirement contained in $\mathcal{E}^{\mathrm{NSPR}}$ *rules out* the possibility for a simple reallocation of the accessible options among a given number of opportunity sets to lead to a strict improvement in social freedom. Although we do not have a wealth of theories telling us how to evaluate social freedom (as opposed to individual one), the principle underlying the definition of $\mathcal{E}^{\mathrm{NSPR}}$ does not seem unreasonable for this purpose. For suppose it was not satisfied. Then, this would imply the possibility of improving social freedom by simply redistributing the access to some (or to all) of the options. But if we simply redistribute the access to every option (by a finite sequence of bilateral transfers of options), it means that, for every individual who *gains* access to a new option, there is another individual who *looses* access to that same option. Provided that freedom of choice is conceived in terms of accessibility to the various options, it seems counterintuitive that society's overall freedom can be enlarged from a mere redistribution in the access to the various options among its members. In order to get a somewhat less abstract picture of the concepts presented so far, it may be helpful to consider the following example (taken from Gravel, Laslier and Trannoy [40]).

Example 1: *Assume a country where the right to vote at national elections is given only to male adults and in which there is as many male and female adults. Without being specific about the way options are defined, one can say that in such a society, every male adult has the option of voting at national election for whoever available candidate that suits his taste best. However, no female adult has this option in her opportunity. Imagine now that, for some reason, it is decided that only the female adult will be given the right to vote. Provided that this decision has no effects other than transferring the option of voting from male to female adults, it is clear that the new allocation of opportunities brought about by the decision has been obtained from the previous one by a series of bilateral transfers. The end-value judgement described above implies that such a change should not be recorded as a strict improvement in society's overall freedom.*

Applying the consistency test to this setting yields the following proposition.

Proposition 5. *An individual ranking of opportunity set* $\succeq \in \mathscr{C}^{FR}$ *based on freedom of choice is consistent with the aggregation value* $\mathscr{A}^{UN}$ *defined by the respect for unanimity and with the end-value* $\mathscr{E}^{NSPR}$ *underlying the refusal of expressing strong preference for redistribution* ***only if*** *there exists a function* $v : \Omega \rightarrow \Re_+$ *such that, for all* $B, B' \in X, B \succeq B' \Leftrightarrow \sum_{x \in B} v(x) \geq \sum_{y \in B'} v(y)$.

Proof: Suppose that an individual criterion $\succeq$ on X is such that there exists no function $v : \Omega \rightarrow \Re_+$ for which $B \succeq B' \Leftrightarrow \sum_{x \in B} v(x) \geq \sum_{y \in B'} v(y)$ for every B and $B' \in X$. Then, as shown in Gravel et al. [40], one can find a number n of individuals and two allocations of opportunities $(B_1, \ldots, B_n)$ and $(B'_1, \ldots, B'_n) \in X^n$ satisfying 1) $(B_1, \ldots, B_n)\ \widehat{TO}\ (B'_1, \ldots, B'_n)$ 2) $B_i \succeq B'_i$ for every individual i and 3) $B_j \succ B'_j$ for at least one individual j. For this reason one has $(B_1, \ldots, B_n)\ A_n^A\ (\succeq_1, \ldots, \succeq_n)\ (B'_1, \ldots, B'_n)$ for any function A_n in $\mathscr{A}^{\mathrm{UN}}$. But then $A_n(\succeq_1, \ldots, \succeq_n) \notin \mathscr{E}^{\mathrm{NSPR}}$. **QED**

In words, the proposition says that the consistency of a freedom-based ranking of individual opportunity sets with an aggregation value respectful for unanimity and with an end-value that refuses to express a strict prefer-ences between allocations of the same number of options among the same number of individuals requires the ranking to have an additive numerical representation. This class of rankings is a generalization of the well-known cardinality rankings of opportunity sets (examined by Pattanaik and Xu [54], Suppes [74], Bossert et al. [17] among many others) by allowing the numerical weights assigned to the different options to differ. We note that the requirement for an individual ranking of opportunity sets to have an additive numerical representation is only necessary for normative consistency. In the absence of further information on the structure of the aggregation functions contained in $\mathscr{A}^{\mathrm{UN}}$, this requirement is not sufficient.

5 Normative consistency and preferences laundering

So far, we have described our method as a test for checking the consistency of a system of normative views held by an individual in a social choice context. In our view, such a test can help in examining further the alleged difficulty of welfarism in producing ‘ethically defensible’ rankings of social states and the related need to launder individuals ‘utilities’ before aggregating them. According to Goodin [37] (p. 76) the problem with welfarism “is that preferences sometimes seem ‘dirty’. Surely it makes sense to see whether they cannot somehow be ‘laundered’ before we discard them altogether”. Yet no convincing method for doing this ‘laundering’ has emerged.

The difficulty of obtaining satisfactory ‘laundering’ methods is, of course, closely related to the difficulty of properly appraising why the examples of expansive tastes, tamed wife, etc. set forth by critics of welfarist are really telling. It is difficult to launder utilities because it is difficult to identifies the dirt that soils them.

Why for instance do we find it ‘ethically unacceptable’ to give more resource to someone with expensive tastes and less resource to someone with ‘humble needs’ as Dworkin [30] has suggested? It is, perhaps because we think that an excessive inequality in *resources* – as opposed to utilities – is bad. Or it is perhaps because we think that individuals should bear responsibility over some aspect of their preferences – or theiry utility function – and should not require resource compensations for differences in terms of the variables for which they are held responsible. Whatever this reason is, we have suggested in this paper that it can plausibly be expressed as a *restriction* of the class of collective rankings of social states that one may wish to consider. For instance, if we think that, other things considered, resources inequality is ethically objectionable, then we require the social ranking to exhibit some sort of resource inequality aversion (as done in Sect. 3.1 and 3.2). If on the other hand we believe that there are variables that individuals should be held responsible for, then we may impose on the social ranking that it somehow ‘respects’ the outcome that results when individuals exercise their responsibility (as done in Sect 3.3). At any rate, it appears clearly important that the alleged difficulty of welfarism to produce social rankings that are ‘ethically defensible’ be expressed in terms of an explicit notion of ‘ethical defensibility’. Our approach offers a framework for doing so. It therefore offers a first step for performing the laundering although many questions remain unsolved.

To see why, consider the first illustration of utilitarian aggregation with income inequality aversion. In this setting, we were led to the conclusion that an individual who is a committed tastes-utilitarian, who dislikes income inequality and who converts income into happiness by a convex function would be inconsistent. Suppose the individual takes our approach seriously and ask us what she can do to develop a more consistent view.

Clearly, our answer will be:

– “You have to give up at least one of the three elements: Utilitarianism, aversion to income inequality or increasing marginal utility for income.”

Suppose then that, after thinking about it for a while, our individual comes back to us and say:

– “Well, I have been convinced by Bentham et al. and by the axiomatization of utilitarianism on economic domains proposed by Donaldson and Roemer [29] or by Bordes et al. [13] that utilitarian aggregation of tastes is the only sensible think to do. Moreover, I am deeply convinced that, everything else being the same, when a rich individual gives a penny to a poor, the world becomes better. Hence the only thing I am willing to change is my utility function. That is, I realize now fully, thanks to your theory and to professor Goodin, that my utility function is dirty and that I should put it in the washing machine. According to your theory it should get out concave. That’s all right but which concave should I take? In so far as I think about it, I derive more extra fun from each extra penny the more money I have. So how should I modify my way of deriving pleasure out of income? Should I take the utility function that is the ‘closest’ to my own? And what ‘closest’ means here?”

– "..."

Our theory does not offer any clue for answering questions such as these. As mentioned, it does not even lead to the conclusion that the individual tastes is the element that should be reformulated. And whenever it does, it provides no inside as to how this reformulation should be done.

6 Conclusion

Various cultures have various ethics and acts regarded as 'moral' by some can be regarded as immoral by others. Yet most ethical systems have in common the property of appraising the morality of individual acts by resorting to an operation of universalization. As recalled above, by such an operation, the *act*or evaluates the consequence of her act in putting herself in the hypothetical situation in which everyone else acts in an equivalent way as she does. In this paper we have suggested with a few illustrations how this idea could lead to non-trivial restrictions of individual criteria in the context of the aggregative method in social choice.

In the present paper, we have interpreted the individual criterion as a 'message' sent by the individual to the social planner for *ethical* reason. The interpretation thus given to the 'message' differs sharply from the one given to that word in the conventional implementation literature. This message should be interpreted as meaning 'I claim that my individual criterion that should be taken into account for defining social good is' rather than 'I claim that my individual criterion is' as in standard implementation theory. As mentioned, our approach contrasts with many methods for restricting the set of admissible individual criteria (or preferences) proposed in the economic and philosophical literature, in that it is not exogenous with respect to the individual. Instead the restriction of individual criteria to which our notion of moral consistency leads results from a 'deliberative process' by which the individual tries to make consistent her (individualistic) criterion with a set of principles for making collective judgement with which she identifies.

If we believe that the endogenization of this process that we have proposed in this paper is a useful preliminary step in the direction of obtaining satisfactory methods for restricting the type of preferences that individuals can have, we are also fully aware that much more need to be done. One direction that remains to be explored is that of coordinating the normative views of distinct individuals. Suppose for instance that we have a group of individuals who are all utilitarian and averse to income inequality and who have adopted a concave utility function. Yet, when these individuals will aggregate their normative views and accept in advance to take collective decision by summing their concave utility functions, they are likely to sum *different* utility functions. Because of this, they may fail to produce a ranking of social states that is compatible with the Lorenz domination criterion. A further line of investigation would be to check how such a group of individuals could solve its coordination problem.

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