Internet, Literacy and Earnings Inequality

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Abstract

This paper outlines a theoretical framework to think about the role of NIT on earnings inequality at a domestic level. Two main ideas inspired a growth model. First, to be connected is only meaningful for people who are already literate. Second internet, like the invention of printing, permits to increase the part of knowledge that an individual can use. The results are obtained in terms of the Lorenz criterion. The role of some key parameters is emphasized like the elasticity of substitution between talent and knowledge.Two forces are at work. On the one hand, the gap between literate and non literate people will increase. On the other hand, the incentive to become literate increases. Policy implications are derived.

J.E.L : D31, D63, I2, O33.

1 Introduction[1]

The emergence of the knowledge society seems a main feature of developed economies at the start of the 21st century. There is no question that new information technologies (NIT) represent a source of wealth for a society taken as a whole. The question of the impact of these technologies on distribution issues, either at a national level or at an international one remains open. Newspapers, for instance, are full of articles which express the fear of a digital divide between people who are connected and people who are not. The former at the opposite of the latter have access to knowledge which is a source of opportunities and wealth. In the same vein, the idea of an increase of the North-South gap is often mentioned. This paper questions the validity of this fear and investigates the main factors that can in‡uence the evolution of inequality in a given society after the introduction of internet. We organize the discussion around a very simple model which …gures out the adoption of internet in a closed economy. The question is so broad that we have to focus on some issues and to ignore some very important ones intentionaly. Among income sources, capital incomes are omitted. Indeed all inequality decomposition studies agree on the de…nite importance of earnings inequality in industralized countries see for instance Jenkins [8] or Sastre and Trannoy [23]. Hence we restrict our attention to this income component. As a consequence our model is a growth model without capital. Another limitation of the analysis comes from the consideration of a closed economy. The interaction between international trade and IT

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adoption cannot be analysed in such a framework. Therefore the question precisely addressed in this paper is the impact of the IT revolution on earnings inequality at a domestic level.

Two main ideas govern the model. From a qualitative point of view, the digital revolution can be analysed in the same way as two former revolutions in the knowledge technology. The …rst one is the invention of writing, the second one the invention of printing. What are the main characteristics of the di¤usion of knowledge that writing brought to human societies? According to Goody [6], "writing overcame the limitations of memory in oral societies by providing for quasi-permanent storage in material form, which permitted precise communication over time and over space. Writing renders knowledge public in that its publication makes it available to all who can read. Restrictions come on the di¤usion of knowledge before that particular moment. Afterwards it is open to a speed of circulation and to the accumulation and augmentation by others that change the nature of knowledge systems". Clearly, if we analyse the change operated by printing in occidental societies, it enormously extends the bene…ts brought by writing. The digital technology like printing has an impact on the two essential components of the costs borne by information providers, see Shapiro and Varian [25] for developments. It reduces both the reproduction and the distribution costs. This change is captured in the model by a parameter that …gures out the proportion of the knowledge stock of a given society that an individual can mobilize on its own. The value of this parameter increased once with the printing revolution and again with the digital one. A question raised here is whether internet will decrease the cost to be literate as printing surely did. Let us recall the importance of the …rst complete Bible in English published in 1535-1536 for the reading practice in Britain, a fact which is well documented (see Oxford [20]). The evidence that internet will induce such a similar shock on education technology is not obvious for the moment, but it may be still to come (see Gates (1995)[5]). In the reference model, we adopt a pessimistic view, and we assume that it will not produce any productivity gain in the education technology.

The second main idea is that the interest to be connected to internet depends on your literacy. If you are illiterate, the interest of a connection is small if any. Since it is costly …nancially - hardware, software and connection spell- to say nothing of cognitive costs, we can suspect that people with a poor literacy score will not choose to be connected. On the opposite, people with a medium or high literate level will …nd an advantage to be connected to get a better job or a better life. In view of the asymetry between literacy choice and connection choice, it is useful to modelize the decision as a sequential one, …rst to decide to be literate or not, then to be connected or not for those who have chosen to be literate. Then at a personal level, it seems that we can establish a link between literacy and connection decisions. We still have to …nd some empirical evidence of such a link at a more aggregated level. Let us …rst agree on the meaning of literacy.

According to the International Adult Literacy survey (IALS) (see OECD (2000) [17],[2]) literacy is de…ned as "the ability to understand and employ printed information in daily activities, at home, at work and in the community, - to achieve one's goals, and to develop one's knowledge and potential". This broad de…nition encompasses the multiplicity of skills that constitute literacy in advanced countries. This de…nition is make more precise for the sake of measurement and is fragmented into prose literacy, document literacy and quantitative literacy. The …rst one covers "the knowledge and skills needed to understand and use information from texts including editorials, new stories, brochures and instruction manuals". The second one embodies "the knowledge and skills required to locate and use information contained in various formats, including job applications, payroll forms, trans-

[2] See also for previous studies on the same topic OECD 1995 and 1992.

portations schedules, maps, tables and charts, while the third deals with "the knowledge ans skills required to apply arithmetic operations, either alone or sequentially, to numbers embedded in printed materials, such as balancing a chequebook, …guring out a tip, completing an order form or determining the amount of interest on a loan from an advertisement". The IALS stresses that it no longer de…nes literacy in terms of an arbitrary standard of reading performance, distinguishing the few who completely fail the test (the "illiterates") from nearly all the remaining in industrialized countries who reach a minimum threshold "those who are "literate"). Indeed, it de…nes …ve levels of literacy from 1 to 5 according to scores achieved at some tests. Nevertheless it turns out that among the …ve levels of literacy, the …rst two, levels 1 and 2 are considered below a reference line[3]. It is this kind of reference line that we try to take into account here. In our model, we consider that there is a threshold between people who are literate and people who aren't.

In a cross-section analysis made among 20 industrialized countries, it is possible to check roughly the existence of a relation between connection rate and illiteracy rate. The scattered diagram illustrates the relation between the ratio of computers connected to internet (at work and at home) per 1000 inhabitants (source : United Nations [26][4]) and the arithmetic mean of the proportion of people who are below level 3 at prose literacy, document literacy and quantitative literacy tests[5]. Indeed we consider that to be connected mobilize the three types of literacy already mentioned to some degree.(See Data Values for the Table in Appendix).

INSERT FIGURE 1

The …gure captures a potential log-lin relation. The empirical evidence gives some credit to this kind of relation and the results of the regression are displayed below.

$$\begin{aligned} \text{Log(Connection Rate)} &= ¡\,0{:}0233(\text{Iliteracy rate}) + 2.44723 \\ & \quad\ (0{:}0045) \qquad\qquad\qquad\qquad (0.2373) \\ R^2 &= 0{:}595; \qquad F = 26.4465 \qquad DF = 18 \end{aligned}$$

This result[6] does not in…rm the view that there is a signi…cant negative in‡uence of the illiteracy rate on the growth rate of the proportion of people connected to internet. Since this latter variable is linked to an investment in information technology it is a reminiscence of a …nding of Romer [22] which shows that the initial level of literacy does help to predict the subsequent rate of investment.

A more technical remark is in order. Endogeneous growth theory (see for instance Aghion and Howitt [1]) has focused on the crucial role played by the accumulation of technological

[3] See Figure 2.2 p 17 Chapter 2. In describing level 3, it is stated that "it is considering a suitable minimum for coping with the demands of everyday life and work in a complex, advanced society. It denotes roughly teh skill level required for successful secondary school completion and college entry".

[4] Source: Table A1.3 p53.

[5] Source Table 2.2 Annex D OECD (2000).

[6] When one controls for the GDP per capita (PPA), one obtains silly results, the sign of the GDP variable is negative and the sign of illiteracy variable becomes positive. We think that the sample is too small to estimate the role of the two variables correctly. But it is interesting to notice that in a simple regression the …t is better with illiteracy than with GDP. Indeed the results of this second regression are:

$$\begin{aligned} \text{Log(Connection Rate)} &= 5{:}814(\text{GDP/per capita}) + 0.191 \\ & \quad (1{:}24) \qquad\qquad\qquad (0.243) \\ R^2 &= 0{:}552; \qquad F= 22.1949 \qquad DF= 18 \end{aligned}$$

knowledge on the growth process. General interest into questions of how technical change and endogeneous growth affect inequality has been recently revived by new empirical evidence. In particular the possibility of a skill-biased technical progress has been intensively discussed. This bias reveals and enhances new differences in abilities among workers across or within educational cohorts (see Juhn, Murphy and Pierce (1993)). In this burgeoning literature (see for example Aghion, Caroli and Penelosa [2]) one can detect a somewhat irritating feature for the specialist of the measurement of inequality. Very peculiar income distributions have often been considered. For instance the density is assumed to be concentrated on two values: unskilled and skilled wages. Inequality is then easily encapsulated by the ratio of these two numbers. One can kindly remark that the tremendous work of statisticians and economists to establish rigorous measures of inequality measurement comes from the fact that income distributions cannot be summed up in such a simple parabola. Our aim would be to attack the question of the influence of technological shocks on the acquisition of knowledge on economic inequality in incorporating an individual heterogeneity. Results in terms of social dominance tools like Lorenz quasiorderings (see for instance Atkinson[3] and Sen[24]) will be investigated. In view of the generality and robustness of the results it will be worth it to overcome the technical difficulties generated by handling more complex instruments.

I begin in Section 2 with a model of literacy decision in a world where printing already exists. Section 3 describes an extension of the model which analyses literacy and connection choices in a society where internet has been invented. Section 4 contains policy implications and some conclusive comments. Proofs are relegated in Appendix.

2 Literacy and Inequality

At each period $t;\ t = 1;\dots;\bar{T};$ a generation, called the generation t, composed of a continuum of agents, lives one unit of time. Individuals are supposed to be identical with respect to their physical ability, $w_0 > 0$. They are heterogeneous with respect to cognitive ability $\omega:$ This variable is distributed according to a cumulative distribution $F(.)$ which admits a density $f(.)$ over a finite support $[\underline{\omega};\bar{\omega}];\ \underline{\omega} > 0$. The set of such distritbutions is denoted $\mathcal{F}$. Throughout the paper, this distribution is held constant.

The model outlines an artisan economy with no land and capital. The focus is on the role of knowledge in income distribution. Since we want to explain inequality of lifetime incomes, decisions of labour supply are not modelized. Nickell and Layard (1999)[16][7] find that the best predictor at a macroeconomic level of earnings inequality in OECD countries is the inequality of scores obtained at quantitative literacy tests. It is such a finding that the model tries to capture. At each generation, individuals have only one choice to make, to be literate or not. Of course education plays a great part to become literate but remaining literate demands an effort during all your life. In that sense, an individual can confirm or infirm a choice made by his parents while a child.

In case of a negative answer, the lifetime income of an illiterate individual of type ω belonging to generation t denoted $y_t(\omega;0)$; is equal to

$$y_t(\omega;0) = \omega_0. \tag{1}$$

He can only sell his brute physical strength on the labor market as for instance a road worker. In case of a positive answer, earnings are given by a C.E.S return function defined by two inputs, the cognitive ability and the stock of knowledge accumulated at the previous

[7] pp 3077-3078.

period, denoted K_{t-1}. The lifetime income of a literate individual of type ω belonging to generation t, denoted $y_t(\omega;1)$; is equal to

$$\begin{aligned} &(i)\ y_t(\omega;1) = (\omega^{\rho} + (\theta_t K_{t-1})^{\rho})^{1=\rho} && \rho \leq 1 \text{ with } \rho \neq 0 \quad (2) \\ &(ii)\ y_t(\omega;1) = \omega^{\alpha}(\theta_t K_{t-1})^{1-\alpha} && \rho = 0 \text{ with } \alpha \in (0;1) \end{aligned}$$

where θ_t a parameter in (0;1) represents the part of knowledge which an individual can resort to. This formulation tries to capture the key elements which influence the earnings of an intellectual job, let us say for instance, a writer. Her income is generated by the combination of two production factors, a private one, the innate talent, and a public one, the used knowledge of a generation, $\theta_t K_{t-1}$. Among the literates, the natural ranking is preserved, but the public good effect of knowledge mitigates the inborn difference beween individuals. The portion of knowledge that an individual, θ_t, can mobilize for her benefit is dependent on the technology and can change from one generation to another. The elasticity of substitution between talent and knowledge $\sigma = \frac{1}{1-\rho}$ proves to be a crucial parameter in the study of inequality.

Before pursuing, let us establish a link between this expression and the human capital earnings function. It seems to be easier with the Cobb-Douglas formulation. Just for the exercice of comparison, one adopts a double indices notation: y_{it} denotes the earnings of an individual i belonging to generation t. Assuming just for the exercice that θ is specific to an individual i, (ii) is more suitably written

$$\log y_{it} = (1-\alpha)\log\theta_i + (1-\alpha)\log K_{t-1} + \alpha\log\omega_i; \quad (3)$$

Trying to estimate this expression, it seems clear that the last term of RHS is a residual, since ω_i is not observable. The same remark may be made about θ_i, but we can postulate that education and experience increase it. Let us assume that

$$\log\theta_i = a_1 S_i + a_2 E_i + a_3 E_i^2 + \varepsilon_i; \quad (4)$$

where S represents years of completed education, E represents the working experience and ε is a statistical residual.. If we combine the two expressions above, we are back to Mincer's model (1974)[15] for which the log of individual earnings in a given time period can be decomposed into an additive function of a linear education term and a quadratic experience term

$$\log y_{it} = a + b_1 S_i + b_2 E_i + b_3 E_i^2 + e_i; \quad (5)$$

where e is a statistical residual and letting $a = (1-\alpha)\log K_{t-1}$. Knowledge plays the role of a constant within a generation. If we want to explain earnings differentials generations, knowledge by itself must enter as an explanatory variable and the specification to be estimated becomes

$$\log y_{it} = (1-\alpha)\log K_{t-1} + b_1 S_i + b_2 E_i + b_3 E_i^2 + e_i; \quad (6)$$

which allows to infer the value of α: We can conclude that formulation (2) is compatible with standard human capital earnings function and can offer a plausible interpretation of the constant in Mincer's equation. Endogeneous growth theory puts knowledge in the forefront. The expression above suggests that it may be a good idea to do it as well for labor economics.

Going back to the model, utility is assumed to be quasi-linear in income and in case of a illiterate person, his lifetime utility is given by his income. A parameter c_t enters in the lifetime utility of a literate individual and it figures out the financial cost to be literate as well as a monetary appraisal of the cognitive effort implied by such a learning. Since we do not want to cope with two parameters of individual heterogeneity, we assume that this learning cost does not vary across individuals. With obvious notations we define

$$U_t(\omega,0) = y_t(\omega,0), \tag{7}$$
$$U_t(\omega,1) = y_t(\omega,1) - c_t. \tag{8}$$

Innovations in information technologies, educative training or government intervention through for instance free compulsory public education or vouchers, can reduce the learning cost c. An individual decides to become literate iff

$$U_t(\omega,1) \geq U_t(\omega,0). \tag{9}$$

Hence, at each generation, a threshold in terms of cognitive talent, ω_t^*, is implicitly defined between those with a cognitive ability larger or equal who will choose to become literate and those with a strictly smaller cognitive value who find this effort unvaluable. This threshold is defined by

$$\omega_t^* = \max(\underline{\omega}, ((\omega_0 + c_t)^\rho - (\theta_t K_{t-1})^\rho)^{1/\rho} \qquad \rho \neq 0, \tag{10}$$
$$\omega_t^* = \max(\underline{\omega}, \frac{(\omega_0 + c_t)^{1/\alpha}}{(\theta_t K_{t-1})^{(1-\alpha)/\alpha}}) \qquad \rho = 0. \tag{11}$$

Quite naturally, this threshold increases in learning cost and decreases in knowledge as well as in the proportion of knowledge absorbed by an individual. The income cumulative distribution of a generation which presents a point mass in ω_0, can be easily deduced

$$G_F(y_t,\rho) = 0, \text{ for } y_t < \omega_0, \tag{12}$$
$$G_F(y_t,\rho) = F(\omega_t^*), \text{ for } \omega_0 \leq y_t < \omega_0 + c_t, \tag{13}$$
$$G_F(y_t,\rho) = F((y_t^\rho - (\theta_t K_{t-1})^\rho)^{1/\rho}), \text{ for } y_t \geq \omega_0 + c_t, \qquad \rho \neq 0 \tag{14}$$
$$G_F(y_t,\rho) = F(\frac{(y_t)^{1/\alpha}}{(\theta_t K_{t-1})^{(1-\alpha)/\alpha}}), \text{for } y_t \geq \omega_0 + c_t, \qquad \rho = 0. \tag{15}$$

When a generation is fully literate, the income distribution is described by one of the last two equations.

To end the description of the model, we have to specify the law of accumulation of knowledge. We assume that only literate people can extend the knowledge of a society. Moreover we assume that knowlege cannot become obsolete. Knowledge grows at a constant rate $\beta \in (0,1)$ in a fully literate society. Since the literacy rate of generation t is equal to $1 - F(\omega_t^*)$, we write

$$K_t = K_{t-1}(1 + \beta[1 - F(\omega_t^*)]). \tag{16}$$

The dynamics across generations of such an economy can be easily expressed if we assume that the initial stock of knowledge, K_0, accumulated by the oral tradition is strictly positive. Moreover we suppose that

$$\bar{\omega} > \omega_1^*. \tag{17}$$

The invention of writing by itself proves that in the history of mankind there was an individual who satisfied this inequality[8].

Proposition 2.1 Let c_t and μ_t be constant over time. Under the assumptions, there is a period t^* from which the society is fully literate, i-e, $\forall t < t^*;\ \omega^*_t > \underline{\omega};\ \forall t \geq t^*;\ \omega^*_t = \underline{\omega}$.

Proof. See Appendix A ■

Hence in case of a stability of the parameters of the economy, each generation becomes more literate than its precursor until a generation becomes fully literate. From this generation, knowledge grows at a constant rate. Then two periods can be distinguished, a period of fully literate generations,a mature period, and an initial period where illiterate and literate people coexist, a transition period.

First, we begin with the analysis of the evolution of income inequality for the mature period. More specifically, it is instructive to learn the consequences of choosing a particular value for the elasticity of substitution between talent and knowledge on the shape of the evolution of income inequality.

Inequality is measured by an index of inequality consistent with the Lorenz criterion. The Lorenz ordering of distributions involves the comparison of the income shares accruing to different fractions of the population. Given a cumulative distribution function G defined on a support $X = [\underline{\omega}; \overline{\omega}]$; its mean is defined by its $\mu_G = \int_{\underline{\omega}}^{\overline{\omega}} x dG(x)$ and its left inverse distribution function is defined by

$$G^{-1}(p) = \operatorname{Inf}\{x \in X \mid G(x) \geq p\} \qquad \forall p \in [0; 1]: \qquad (18)$$

The Lorenz curve of a distribution G is given by

$$L_G(p) = \frac{\int_0^p G^{-1}(s)ds}{\mu_G} \qquad \forall p \in [0; 1]: \qquad (19)$$

Actually, $L_G(p)$ represents the proportion of total income possessed by the px100% poorest income units in configuration G.

Definition 2.1 Given F and G two distribution functions, we say that F weakly dominates G in the (relative) Lorenz sense, which we write $F \geq_L G$ if $L_F(p) \geq L_G(p)$ for all $p \in [0; 1]$:

We denote as $>_L$ the asymetric component of $\geq_L$: Sometimes, it is more suitable to present the results in terms of inequality indices.

Definition 2.2 Let $\mathcal{F}$ be the set of distribution function on X. A relative inequality index is a real valued function I defined on $\mathcal{F}$ which is Lorenz consistent, i-e, $I(F) \geq I(G) \iff F \geq_L G$ and which is equal to zero in case of a point mass.

The set of relative inequality indices will be denoted $\mathcal{I}$.

Our second proposition gathers some results about the earnings inequality in fully literate societies. In this particular case, the income distribution G_F is derived from the distribution of talents F through the following relation

[8] The assumption that the distribution of talent is unbounded above is identical at this stage but it unecessary complicates the study of the inequality dynamics.

$$
\begin{aligned}
G_F(y_t;\rho) &= F[(y_t^{\rho} - (\theta_t K_{t-1})^{\rho}]^{1/\rho} & \rho \neq 0; \qquad (20)\\
G_F(y_t;0) &= F\left[\frac{y_t^{1/\alpha}}{(\theta_t K_{t-1})^{(1-\alpha)/\alpha}}\right] & \rho = 0.
\end{aligned}
$$

When we make comparisons of inequality, we would like their domain of validity to be as extensive as possible, namely, that they do not depend on the talent distribution. Here we stick to this requirement, which is justified by our ignorance of the true distribution of skills. But we have to recognize that this care about robustness has a cost. In some circumstances, it can be impossible to conclude to an increase (or a decrease) in inequality whatever the distribution of talents. From a formal point of view, this investigation relies on results obtained about the progressivity of taxation schemes, see Jakobsson [7], Eichhorn Funke and Richter [4], Le Breton Moyes and Trannoy [10].

Proposition 2.2 *Let $t \geq t^*$. It is composed of five statements valid for all $I \in \mathcal{I}$ and for all $F \in \mathcal{F}$.*

(i) If $K_{t-1} = 0$; the income inequality is null for all $\rho \leq 0$ and the income inequality is equal to the natural inequality, namely, $I(G_F) = I(F)$ for the case $0 < \rho \leq 1$.

(ii) Whatever the values of K_{t-1} and ρ,

$$I(G_F(y_t;\rho) \leq I(F). \tag{21}$$

(iii) In the Cobb-Douglas case, inequality is invariant to the stock of knowledge, provided it is positive.

(iv) Let $\theta_t = \theta$. Then,

$$K_t > K_{t-1} \Rightarrow I(G_F(y_{t+1};\rho)) > I(G_F(y_t;\rho)) \text{ for all } \rho < 0; \tag{22}$$

$$K_t > K_{t-1} \Rightarrow I(G_F(y_{t+1};\rho)) < I(G_F(y_t;\rho)) \text{ for all } 0 < \rho \leq 1. \tag{23}$$

(v) Let $\theta_t = \theta$. Then,

$$\lim_{K_{t-1} \to +\infty} I(G_F(y_{t+1};\rho)) = I(F) \text{ for all } \rho < 0; \tag{24}$$

$$\lim_{K_{t-1} \to +\infty} I(G_F(y_{t+1};\rho)) = 0 \text{ for all } 0 < \rho \leq 1. \tag{25}$$

Proof. See Appendix B ■

Figure (2) illustrates the evolution of inequality according to the value of the elasticity of substitution which is the key parameter. If talent and knowledge are rather substitute, a fully literate society will converge toward a fully equal society. If talent and knowledge are rather complementary, the inequality of talents will become the dominant factor in the long run for a fully literate society. The Cobb-Douglas case provides a unique evolution, the steady state is reached immediately. A gain in knowledge increases the income of each literate person in the same proportion. In the following, we will refer to the inequality in a Cobb-Douglas economy as the Cobb-Douglas inequality.

Insert Figure 2

In view of these results, the plausibility of all scenarii does not appear to be the same. It seems clear that the case for the substitution is rather weak. Let us now examine the Cobb-Douglas and complementary cases. For obvious reasons the data available on earnings inequality on the long run, since for example the invention of printing, are scarce. A noticeable exception is Britain for which we have access to statistical elements from the late eighteenth century (Williamson [27]). Lindert [12][9] estimates on the basis of the more recent articles that "It is hard to say there was any rise-fall pattern in pay gaps within the non-farm sector across the nineteenth century". The beginning of the twentieth century corresponds surely to a configuration where almost all Britons received a compulsory education. Piketty [21] finds a similar empirical evidence of a more or less constant earnings inequality over the twentieth century for France. Hence there is no strong empirical evidence against the Cobb-Douglas case and for this reason it will occupy a proeminent place in the following. To simplify the notations, from now on $G_F(y_t;\rho) \equiv G_F(y_t)$:

Now we study the evolution of income inequality in the transition period. On the one hand, we would like to compare inequality of income distribution within the generation t^* and within a generation $t < t^*$ and on the other hand we would like to compare the inequality between two transition generations. The first question raised is about the comparison of a fully literate society and a partially literate society, while the second question addressed is : Does the extension of literacy bring inequality in uncomplete literate societies? As stated by the next proposition a conclusion independent of the distribution of talents is impossible to achieve.

Proposition 2.3 Let c_t and μ_t be constant over time.(i) It is impossible to obtain a ranking of the Lorenz curves associated to the income distribution of generation t^* and to the income distribution of a generation t with $t < t^*$ valid for all $F \in \mathcal{F}$.

(ii)It is impossible to obtain a ranking of the Lorenz curves associated with the income distribution of generation t and to the income distribution of a generation t' with $t < t' < t^*$valid for all $F \in \mathcal{F}$.

Proof. See Appendix C ■

The proof of the above proposition teaches us that the trouble comes from the discontinuity of the income function at ω_t^* which jumps from ω_0 to $\omega_0 + c$. Hence the discontinuity introduced by the literacy cost produces such an impossibility to rank income distributions from an inequality point of view[10]. Unfortunately the obtention of positive ones implies the restriction of the domain of talent distributions. The next proposition follows this route. Hence we can expect that a condition requiring the discontinuity to be not too large will help to obtain explicit comparisons. Indeed one of the conditions which emerges bounds the ratio $\frac{c}{\omega_0}$: Here our aim is not to find necessary and sufficient conditions to be able to rank earnings distributions. We will be pleased to find sufficient conditions which allow to perform a comparison between the income distribution in a partially literate generation and in a fully literate generation.

Let us denote

$$\mu_F(\alpha) = \int_{\underline{\omega}}^{\overline{\omega}} \omega^{\alpha} dF(\omega): \qquad (26)$$

[9] p 182.

[10] Income distributions are obviously ranked accordingly a welfare criterion like the Generalized Lorenz one. Welfare is improving along time.

Since α is the elasticity of the return function to the talent, we term ω^α the "dollar-talent" and $\mu_F(\alpha)$ the "dollar-talent" average. The dollar-talent average up to ω is equal to

$$\mu_F(\alpha;\omega) = \frac{\int_{\underline{\omega}}^{\omega} z^\alpha dF(z)}{F(\omega)}. \tag{27}$$

The dollar-talent ratio up to ω is defined as

$$T(\omega) = \frac{\int_{\underline{\omega}}^{\omega} z^\alpha dF(z)}{\omega^\alpha F(\omega)}; \qquad \omega \neq \underline{\omega}. \tag{28}$$

This ratio is bounded by 1 and $\lim_{\omega \to \underline{\omega}} T(\omega) = 1$. Then if $T(\omega)$ is monotone, it can only be monotone decreasing. Indeed, it is at least the case with a uniform continuous and a Pareto probability distribution.

Proposition 2.4 Let c_t and θ_t be constant over time. Let $F \in \mathcal{F}$ be such that $T(\omega)$ is decreasing. Then for any such $F \in \mathcal{F}$, there exists a period $T(\omega)$ with $1 \leq t_F < t^*$ such that for any t with $t < t_F$,

$$G_F(y_t) >_L G_F(y_{t^*}). \tag{29}$$

Moreover t_F is decreasing with $\frac{c}{\omega_0}$.

Proof. See Appendix D ■

Hence the lower the literacy cost is, the more complete the ranking of income distributions is. The most plausible dynamics is that starting from a complete equal income distribution, the invention of writing or printing introduces inequality, albeit many partially literate generations experiment a level of inequality strictly smaller than the level characterizing a fully literate society. It may be the case that inequality is higher in final transition periods than in the steady state. It is still possible that beyond t_F not definite conclusion is obtained. Let us recall that these findings concern the Cobb-Douglas case.

3 The Connection Decision and Inequality

We provide an extension of the model[11] which captures the invention of internet. Individuals have the possibility to be connected to internet at a cost c_t'. It figures out the financial cost to be connected (personal computer, connection costs) augmented by cognitive costs associated to the learning period. Albeit individuals can choose to be connected whatever their literacy mastery is, we assume that the benefits to do so are substantial only if they are fully literate. In this version of the model we capture these benefits through a parameter θ_t' with $1 > \theta_t' > \theta_t$ which represents the part of the knowledge that individuals can mobilize with internet. Therefore $\theta_t' - \theta_t$ represents the informational gain associated to internet.

Hence the lifetime income of a connected literate individual of type ω belonging to generation t, denoted $y_t(\omega;1;1)$, is equal to

$$y_t(\omega;1;1) = \omega^\alpha(\theta_t' K_{t-1})^{1-\alpha}. \tag{30}$$

[11] The Cobb-Douglas case is only treated.

Since a rational illiterate person has clearly no interest to connect, the choice of an individual is between three options; to be illiterate and unconnected, to be literate and unconnected and to be literate and connected. The utility associated to the first option is defined by

$$U_t(\omega;0;0) = \omega_0; \tag{31}$$

the utility of the second by

$$U_t(\omega;1;0) = \omega^{\alpha}(\theta_t K_{t-1})^{1-\alpha} - c_t; \tag{32}$$

and the utility of the third by

$$U_t(\omega;1;1) = \omega^{\alpha}(\theta'_t K_{t-1})^{1-\alpha} - (c_t + c'_t). \tag{33}$$

An individual decides to become literate and connected iff

$$U_t(\omega;1;1) \geq U_t(\omega;1;0) \text{ and } U_t(\omega;1;1) \geq U_t(\omega;0;0). \tag{34}$$

The first inequality defines a threshold

$$\omega_t^{**} = \max\left(\underline{\omega}; \left[\frac{(c'_t)}{((\theta'_t)^{1-\alpha} - \theta_t^{1-\alpha})K_{t-1}^{(1-\alpha)}}\right]^{1/\alpha}\right); \tag{35}$$

as well as the second inequality

$$\omega_t^{***} = \max\left(\underline{\omega}; \left(\frac{\omega_0 + c_t + c'_t}{(\theta'_t K_{t-1})^{(1-\alpha)}}\right)^{1/\alpha}\right). \tag{36}$$

An individual becomes literate and connected iff

$$\omega \geq \max(\omega_t^{**}; \omega_t^{***}); \tag{37}$$

while an individual chooses to become literate and unconnected iff

$$\omega \geq \omega_t^{*} \text{ and } \omega < \omega_t^{**}. \tag{38}$$

Finally an individual remains illiterate iff

$$\omega < \min(\omega_t^{*}; \omega_t^{***}). \tag{39}$$

Two regimes can be distinguished according to the respective values of this three thresholds.

Proposition 3.1 (i) First Regime. If the following condition holds,

$$\left[\frac{\theta'_t}{\theta_t}\right]^{1-\alpha} - 1 \geq \frac{c'_t}{\omega_0 + c_t}; \tag{40}$$

then, in any transition period, there only exists two kinds of individuals, the literate and connected ones for which $\omega \geq \omega_t^{***}$ and the illiterate ones for which $\omega < \omega_t^{***}$.

(ii) Second Regime. Otherwise, in any transition period, there exists three groups of individuals, the literate and connected for which $\omega \geq \omega_t^{**}$; the literate and unconnected ones for which $\omega \geq \min[\omega_t^{*}; \omega_t^{**})$; and the illiterate ones for which $\omega < \omega_t^{*}$.

Proof. See Appendix E. ■

The condition stated in this proposition means that the connection benefit is larger than the connection cost relatively to their respective values associated to literacy. If this condition holds, we are going back to the configuration studied in the second section, except that the threshold value is different. If this condition does not stand, there are three groups, a regime reflecting the present configuration in many countries. We start by studying inequality evolution in the simplest case of a fully literate society.

3.1 The Advanced Country Case

The period at which internet appears is assumed to be posterior to t^*. W.l.o.g, we will suppose that internet is discovered in t^*. Therefore an individual is connected if

$$\omega \geq \omega_t^{**}; \tag{41}$$

and unconnected otherwise. Even if no society can be considered as fully literate in the sense given in the introduction, this case proves to be instructive as a benchmark. We assume that all parameters are constant through time and that internet does not speed up the growth rate of knowledge. Admitting that it represents a pessimistic view, the law of accumulation of knowledge is still given by

$$K_t = K_{t-1}(1+\beta) \quad \forall t \geq t^*. \tag{42}$$

Proposition 3.2 Let c_t, c_t' and θ_t, θ_t' be constant over time. Generations become more and more connected and there is a period t^{**} from which society is fully connected, i-e, $\forall t < t^{**}$; $\omega_t^{**} < \underline{\omega}$; $\forall t \geq t^{**}$; $\omega_t^{**} = \underline{\omega}$.

Proof. See Appendix F. ■

The evolution of inequality in this case is described in the next proposition. The first statement compares the dynamics of inequality with internet and without internet. A superscript equal to 1 refers to the situation "without", a superscript 2 to the situation "with". The evolution of the knowledge stock is the same in the two configurations. In the second one, we already know that inequality will remain constant beyond t^{**}. The second statement compares the inequality for two generations living in the period of transition between a fully literate society and a fully literate connected society.

Proposition 3.3 (i) For all $t^* < t < t^{**}$ and for all $F \in \mathcal{F}$

$$G_F(y_t^1) >_L G_F(y_t^2). \tag{43}$$

(ii) The Lorenz curves associated to $G_F(y_t)$ and to $G_F(y_{t'})$ with $t^* < t < t^{**}$ and $t^* < t' < t^{**}$ intersect.

Proof. See Appendix G. ■

In a fully literate society, the introduction of internet generates inequality for the transition period but it is impossible to rank income distributions of the period of transition. Indeed both the poorest and the richest individuals experiment a decrease of their income shares with the diffusion of internet.

3.2 Developing Country Case

We assume that the internet invention is anterior to t^* and occurs in period t_I. We start by the analysis of the first regime.

Proposition 3.4 Let c_t, c_t' and θ_t, θ_t' be constant over time. Under assumption prevailing in first regime, there is a period t^{***} from which the society is fully literate and connected, i-e, $\forall t < t^{***}$; $\omega_t^{***} < \underline{\omega}$; $\forall t \geq t^{***}$; $\omega_t^{***} = \underline{\omega}$. Moreover $t^{***} \leq t^*$.

Proof. The proof of the first statement is similar to that of proposition 2.1. The second statement derives from the fact that $\omega_t^{***} \le \omega_t^{*}\ \forall t$. ■

The transition period is shorter with internet. It speeds up the convergence process to a fully literate society. Since with a Cobb-Douglas return function a fully literate is more unequal than any partial literate society, we can expect a greater inequality for the transition period. Indeed the next proposition shows that this intuition proves to be true provided the connecting cost is sufficiently large. With the same notations than with the advanced country case we state the following result.

Proposition 3.5 Let $c_t; c_t^0$ and $\mu_t; \mu_t^0$ be constant over time and assume that the first regime holds. Let $F \in \mathcal{F}$ satisfying the following condition

$$\frac{\frac{c+c^0}{\omega_0+c+c^0}}{\frac{c}{\omega_0+c}} > \frac{\omega_t^{*}F(\omega_t^{*})}{\omega_t^{***}F(\omega_t^{***})} \text{ for any } t \text{ such } t_l \le t < t^{***}. \quad (44)$$

Then for any t with $t_l \le t < t^{***}$;

$$G_F(y_t^1) >_L G_F(y_t^2). \quad (45)$$

Proof. See Appendix H. ■

In this first scenario (see Figure 4), the two costs boil down to a generalized literacy cost. If the ratio of the relative literacy cost - the literacy cost relative to the minimum wage - is larger than the ratio of illiteracy rates weighted by their respective thresholds, then the comparison is unambiguous. This condition means that the configurations have to be sufficiently distinct in order to be able to rank the respective income distributions.

We now turn to the second regime.

Proposition 3.6 Let $c_t; c_t^0$ and $\mu_t; \mu_t^0$ be constant over time. Under assumption prevailing in the second regime, there is a period t^{*} from which the society is fully literate and a period t^{**} from which the society is fully connected. Moreover $t^{**} \ge t^{*}$.

Proof. The first statement is a consequence of propositions 2.1 and 3.2. The second statement derives from the fact that $\omega_t^{**} > \omega_t^{*}\ \forall t$. ■

The inequality evolution in this second regime is more in tune with the common wisdom. Internet will generate more inequality at each transition period up to the first fully literate and connected generation.

Proposition 3.7 Let $c_t; c_t^0$ and $\mu_t; \mu_t^0$ be constant over time and assume that the second regime holds. Then

$$G_F(y_t^1) >_L G_F(y_t^2) \qquad \forall t_l \le t < t^{**}. \quad (46)$$

Proof. See Appendix I. ■

4 Policy implications

The teachings of the model are the following. They concern the Cobb-Douglas case, a case where the elasticity of substitution between talent and knowledge is equal to one. This case is attractive since in a fully literate society inequality remains constant over time as knowledge increases. We show that starting from a totally illiterate society, earnings inequality will increase gradually as the illiterate rate diminishes and at some point can itself exceed its stationary value.

In a fully literate society the internet revolution produces a temporary upsurge of the earnings inequality like any innovation technology. Inequality will follow an inverted-U curve, a Kuznets curve, as per capita income rises. But in the long run, inequality will return to its stationary path.

When we move to the case where internet is introduced in an incomplete literate society, a case which can surely describe the situation of developing countries, two configurations must be distinguished. In the first one the relative benefit of internet, in this model a larger access to knowledge, is so high to its relative cost that every literate individual connects. In this case internet rises the interest in being literate and the illiteracy rate decreases at a faster speed than the one which will be observed without internet. Thanks to internet such a society will converge to the inequality stationary state, experimenting a shorter transition period. The most impressive rise of inequality during this period will largely be a by product of this reduction of the transition period. In this case the impact of internet is ambiguous. On the one hand in the short run inequality increases. On the other hand, internet speeds up the convergence process of developing countries toward a fully literate society.

A more pessimistic case has also been investigated where internet has only bad effects on the inequality dynamics. This time the relative cost of internet is higher than its relative benefit in comparison with literacy; by way of consequence only a fraction of the literate population decides to connect. For a long time - the transition period which lasts until everyone is literate and connected- inequality will rise comparatively to a reference situation without internet.

As we move into the information age, policy-makers are increasingly concerned about the role played by knowledge in enhancing productivity growth and innovation. In view of the results they should also be concerned by its role in shaping inequalities. A public policy can prevent the occurence of the worrying scenario. On the one hand, providing free training public programs to internet and organizing the competition on the market of providers to internet can decrease the generalized connection costs. On the other hand, supplying the ADSL network on the whole territory like in Sweden can improve the benefits brought by internet. Such a policy acknowledges the public good effect played by knowledge which mitigates the effect of talent if both factors are not too complementary in the return function. The less literate a society is, the less favorable the impact of internet will be on the inequality dynamics The efficiency of the education system to innoculate the basic knowledge and know-how proves to be more crucial than it has been at every prior period. In this respect the scores obtained for instance in France at the entrance of junior high school are rather worrying. Only 68% (respectively 64%)of pupils in average pass a prose literacy (resp. quantitative) literacy test (Le Monde [11]). It is difficult to accept a vision in which 35% of the population will be left over the cognitive progress. Obviously, internet can provide an improvement of the educational methods, an aspect which is not modelized here but as Bill Gates admits (Gates (1995)[5]), we are still on the sides of the road ahead to this respect. For sure educational sofware will increase earnings inequality through a rise of the gains associated with intellectual property before they maybe contribute to a reduction of the illiteracy rate.

The model built is certainly a prototype and can be supplemented in several directions. Apart from considering the potential impact of internet on educative technology, the direct impact of internet on the speed of accumulation of the knowledge stock can also be incorporated in the model. An increase of this speed can be viewed as plausible. For instance, Lyman and Varian [14] estimate that the growth rate of the worldwide production of books of original content is about 2 percent[12]. It will be interesting to see whether this rate grows in the near future. More immediate extensions would be to investigate other cases than the Cobb-Douglas one and to try to make a calibration of the model. We have modelized the literacy and the connection decision as a deterministic discrete choice. Introducing uncertainty will smooth the earnings distributions and make them closer to those observed. All these directions are matters for further research but we think that the main message is already provided by the model. Two forces drive the earnings inequality with internet. On the one hand, the gap between literate and non literate people will increase. On the other hand, the incentive to become literate increases. The …rst one will surely be dominant for a preliminary period. It is a matter of hope that the second one will prevail in the future.

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APPENDIX

A Proof of proposition 2.1

Proof. Since $K_0 > 0$; $y_1(!;1) > 0$; for all ½: Thanks to the above assumption, there always exists an individual with a talent larger than $!^{¤}_1$ in generation 1. Therefore $1 ¡ F(!^{¤}_1) > 0$; which implies $K_1 > K_0$: Hence the sequence $hK_{t¡1}i$ is strictly increasing.

The sequence $\langle \omega_t^* \rangle$ is bounded below by $\underline{\omega}$. Furthermore, since $\langle K_{t-1} \rangle$ is strictly increasing and $\frac{\partial \omega_t^*}{\partial K_{t-1}} < 0$, $\langle \omega_t^* \rangle$ is strictly decreasing. Therefore it converges.

It remains to prove that this limit is $\underline{\omega}$ which is reached within a finite number of periods. Let us build the sequence $\langle \tilde{\omega}_t \rangle$ whose general term is defined by

$$\tilde{\omega}_t = ((\omega_0 + c)^\rho - (\theta K_0)^\rho (1 + \beta[1 - F(\omega_1^*)])^{\rho(t-1)})^{1/\rho} \qquad \rho \neq 0, \qquad (47)$$

$$\tilde{\omega}_t = \left(\frac{(\omega_0 + c)}{(\theta_t K_0)^{(1-\alpha)} (1 + \beta[1 - F(\omega_1^*)])^{(1-\alpha)(t-1)}} \right)^{1/\alpha} \qquad \rho = 0. \qquad (48)$$

$\tilde{\omega}_1 = \omega_1^*$ and $\tilde{\omega}_t > \omega_t^*$ $\forall t > t_1$ since $\frac{\partial \omega_t^*}{\partial K_{t-1}} < 0$. Moreover $\langle \tilde{\omega}_t \rangle$ reaches $\underline{\omega}$ in a finite number of periods. Indeed let us define $\tilde{t} = \min \{ t \in N^+ \mid \underline{\omega} \geq \tilde{\omega}_t \}$. Denoting $[x]$ the greatest integer in x, we find

$$\tilde{t} = \left[1 + \frac{\log\left(\frac{(\omega_0 + c)^\rho - \underline{\omega}^\rho}{(\theta K_0)^\rho}\right)}{\rho \log(1 + \beta(1 - F(\omega_1^*)))} \right] + 1 \qquad \rho \neq 0; \qquad (49)$$

$$\tilde{t} = \left[1 + \frac{\log\left(\frac{(\omega_0 + c)}{\underline{\omega}^\alpha (\theta K_0)^{1-\alpha}}\right)}{(1 - \alpha) \log(1 + \beta(1 - F(\omega_1^*)))} \right] + 1 \qquad \rho = 0 \qquad (50)$$

except in the case where the expression inside [ ] is already the integer investigated. Therefore $t^* < \tilde{t}$. ■

B Proof of Proposition 2.2

Proof. (i) For all $\rho \leq 0$, $K_{t-1} = 0 \Rightarrow y_t(\omega; 1) = 0$. For all $0 < \rho \leq 1$, $K_{t-1} = 0 \Rightarrow y_t(\omega; 1) = \omega$.

(ii) $y_t(\omega; 1)$ is increasing with ω. Furthermore

$$\left(\frac{y_t(\omega; 1)}{\omega}\right)' = -(\theta_t K_{t-1})^\rho ((\omega^\rho + (\theta_t K_{t-1})^\rho)^{\frac{1-\rho}{\rho}} < 0. \qquad (51)$$

Hence $\frac{y_t(\omega;1)}{\omega}$ is decreasing with ω over the support $[\underline{\omega}; \overline{\omega}]$. By applying Proposition 3.1 in Le Breton, Moyes and Trannoy [10] which is a generalization of the Jakobsson's theorem on progressivity [7], the result follows.

(iii) In the Cobb-Douglas case, inspection of formula 2 reveals that knowlege intervenes in a multiplicative way on the individual income. Therefore the relative inequality holds constant.

(iv) The elasticity of income relatively to knowledge $\varepsilon_{y/K}$ is increasing in talent if $\rho < 0$ and decreasing if $0 < \rho \leq 1$ since

$$(\varepsilon_{y/K})'_\omega = -\rho(\theta K_{t-1})^\rho (\omega^{\rho-1})(\omega^\rho + (\theta_t K_{t-1})^\rho)^{-2}. \qquad (52)$$

Therefore the ratio

$$\frac{y_t(\omega'; 1)}{y_t(\omega; 1)} \quad \text{with } \omega' > \omega \qquad (53)$$

increases with K_{t-1} if $\rho < 0$, which leads to an increase of inequality according to the relative Lorenz criterion. The opposite holds for $0 < \rho \leq 1$.

(v) It follows from the fact that

$$\lim_{K_{t-1} \to +\infty} y_t(\omega;1) = \omega \text{ for } \rho < 0; \tag{54}$$

and that

$$\lim_{K_{t-1} \to +\infty} y_t(\omega;1) = \theta K_{t-1} \text{ for } 0 < \rho \leq 1. \tag{55}$$

■

C Proof of Proposition 2.3

Proof. (i)The earnings in generation t are given by:

$$\begin{aligned} y_t(\omega) &= \omega_0 && \text{for } \omega < \omega_t^*; \\ y_t(\omega) &= \omega^{\alpha}(\theta K_{t-1})^{1-\alpha} && \text{for } \omega \geq \omega_t^*. \end{aligned} \tag{56}$$

The earnings in generation t^* are given by

$$y_{t^*}(\omega) = \omega^{\alpha}(\theta K_{t^*-1})^{1-\alpha} \quad \text{for all } \omega \in [\underline{\omega}; \overline{\omega}]. \tag{57}$$

Let us define the function $H(y_{t^*})$ which transforms income of generation t^* into income of generation t

$$H(y_{t^*}(\omega)) = y_t(\omega) \quad \text{for all } \omega \in [\underline{\omega}; \overline{\omega}]. \tag{58}$$

Precisely

$$y_{t^*}(\underline{\omega}) \leq y_{t^*} < y_{t^*}(\omega_t^*) \Rightarrow H(y_{t^*}) = \omega_0; \tag{59}$$

$$y_{t^*} \geq y_{t^*}(\omega_t^*) \Rightarrow H(y_{t^*}) = y_{t^*}\left(\frac{(K_{t-1})}{(K_{t^*-1})}\right)^{1-\alpha}. \tag{60}$$

$H(y_{t^*})$ is discontinuous at $y_{t^*}(\omega_t^*)$; since $H(y_{t^*}(\omega_t^*)) = \omega_0 + c$. According to Propositions 3.2 and 3.3 in Le Breton, Moyes and Trannoy [10] the continuity of H over $[y_{t^*}(\underline{\omega}); y_{t^*}(\overline{\omega})]$ is a necessary condition for obtaining a ranking of the Lorenz curves of $G_F(y_t)$ $G_F(y_{t^*})$ valid for all $G_F(y_{t^*}); F \in \mathcal{F}$.

(ii)The earnings in generation t' are given by

$$y_{t'}(\omega) = \omega_0 \quad \text{for } \omega < \omega_{t'}^*; \tag{61}$$

$$y_{t'}(\omega) = \omega^{\alpha}(\theta K_{t'-1})^{1-\alpha} \quad \text{for } \omega \geq \omega_{t'}^*. \tag{62}$$

Let us define the function $H'(y_{t'})$ which transforms income of generation t' into income of generation t.

$$H'(y_{t'}(\omega)) = y_t(\omega) \text{ for all } \omega \in [\underline{\omega}; \overline{\omega}] \tag{63}$$

Precisely

$$y_{t'} = \omega_0 \Rightarrow H'(y_{t'}) = y_{t'}; \tag{64}$$

$$\omega_0 < y_{t'} < y_{t'}(\omega_t^*) \Rightarrow H'(y_{t'}) = \omega_0; \tag{65}$$

$$y_{t'} \geq y_{t'}(\omega_t^*) \Rightarrow H'(y_{t'}) = y_{t'}\left(\frac{(K_{t-1})}{(K_{t'-1})}\right)^{1-\alpha}. \tag{66}$$

$H'(y_{t'})$ is discontinuous at $y_{t'}(\omega_t^*)$ since $H'(y_{t'}(\omega_t^*)) = \omega_0 + c$. According to Propositions 3.2 and 3.3 in Le Breton, Moyes and Trannoy [10], the continuity of H' over $[y_{t'}(\underline{\omega}); y_{t'}(\bar{\omega})]$ is a necessary condition for obtaining a ranking of the Lorenz curves of $G_F(y_t)$ $G_F(y_{t'})$ valid for all $G_F(y_{t'}); F \in \mathcal{F}$. ■

D Proof of Proposition 2.4

Proof. To save notations $G_F(y_t(\omega)) \equiv G_t$. We recall that the slope of a Lorenz curve of a distribution G which admits a density over the support $[\underline{\omega}; \bar{\omega}]$ at $p \in [0; 1]$ is given by

$$\frac{x}{\mu_G}; \tag{67}$$

with

$$x = G^{-1}(p); \tag{68}$$

where at end points the slope must be interpreted as the left or right slope, see Lambert [13]. The Lorenz curves of $G_F(y_t(\omega))$ for $t < t^*$ are not differentiable at $\omega = \omega_t^*$. The left slope corresponding at $p = G_F(y_t(\omega_t^*))$ is equal to

$$\frac{\omega_0}{\mu_{G_t}}; \tag{69}$$

while the right slope at that point is equal to

$$\frac{\omega_0 + c}{\mu_{G_t}}. \tag{70}$$

Fact. For any t, the income functions defined by expressions (56) and (57) are rank preserving, namely, they are weakly increasing in ω. Then the proportion of the population which is poorer than or equal to an individual of type ω is always equal to $F(\omega)$ for any t. Therefore the slope of the Lorenz curve of $G_F(y_t(\omega))$ evaluated at $p = F(\omega)$ is equal to :

$$\frac{y_t(\omega)}{\mu_{G_t}} \quad \forall t. \tag{71}$$

Step1. We prove that if

$$\frac{\omega_0}{\omega_0 + c} > T(\omega_t^*); \tag{72}$$

then[13]

$$\forall \omega \in [\omega_t^*; \bar{\omega}]; \quad \frac{y_{t^*}(\omega)}{\mu_{G_{t^*}}} > \frac{y_t(\omega)}{\mu_{G_t}}; \tag{73}$$

[13] The slope at ω_t^* is the right hand slope.

and

$$L_{G_t}(p) > L_{G_{t^*}}(p) \qquad \text{for } p = F(\omega_t^*). \tag{74}$$

Indeed, using the fact

$$(\theta K_{t-1})^{1-\alpha} = \frac{\omega_0 + c}{(\omega_t^*)^{\alpha}}, \tag{75}$$

we obtain

$$\frac{y_t(\omega)}{\pi_{G_t}} = \frac{\omega^{\alpha}(\theta K_{t-1})^{1-\alpha}}{\omega_0 F(\omega_t^*) + (\theta K_{t-1})^{1-\alpha}\int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)} = \frac{\omega^{\alpha}}{\frac{\omega_0}{\omega_0+c}(\omega_t^*)^{\alpha}F(\omega_t^*) + \int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}; \tag{76}$$

while

$$\frac{y_{t^*}(\omega)}{\pi_{G_{t^*}}} = \frac{\omega^{\alpha}}{\int_{\underline{\omega}}^{\omega_t^*} \omega^{\alpha} dF(\omega) + \int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}. \tag{77}$$

Then

$$\frac{y_{t^*}(\omega)}{\pi_{G_{t^*}}} > \frac{y_t(\omega)}{\pi_{G_t}} \;\Leftrightarrow\; \int_{\underline{\omega}}^{\omega_t^*} \omega^{\alpha} dF(\omega) < \frac{\omega_0}{\omega_0 + c}(\omega_t^*)^{\alpha}F(\omega_t^*); \tag{78}$$

which gives the condition expressed in (72). Moreover

$$1 - L_{G_t}(p) = \frac{(\theta K_{t-1})^{1-\alpha}\int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}{\pi_{G_t}} = \frac{\int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}{\frac{\omega_0}{\omega_0+c}(\omega_t^*)^{\alpha}F(\omega_t^*) + \int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}; \tag{79}$$

and

$$1 - L_{G_{t^*}}(p) = \frac{\int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}{\int_{\underline{\omega}}^{\omega_t^*} \omega^{\alpha} dF(\omega) + \int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}. \tag{80}$$

Then

$$L_{G_t}(p) > L_{G_{t^*}}(p) \;\Leftrightarrow\; \frac{\omega_0}{\omega_0 + c}(\omega_t^*)^{\alpha}F(\omega_t^*) > \int_{\underline{\omega}}^{\omega_t^*} \omega^{\alpha} dF(\omega); \tag{81}$$

again the condition expressed in (72).

Step 2. We now prove that

$$L_{G_t}(p) > L_{G_{t^*}}(p). \qquad \forall p \in [F(\omega_t^*), 1). \tag{82}$$

Suppose for a contradiction that

$$\exists\, p' \in [F(\omega_t^*), 1) \mid L_{G_{t^*}}(p) \geq L_{G_t}(p). \tag{83}$$

Combined with (74) we obtain

$$L_{G_{t^*}}(p) > L_{G_t}(p) \quad \forall\, p > p'; \tag{84}$$

which contradicts

$$L_{G_{t^{¤}}}(1) = L_{G_t}(1): \tag{85}$$

Step 3. We now prove that

$$L_{G_t}(p) > L_{G_{t^{¤}}}(p) \qquad 8p \; 2 \; (0; F(!_t^{¤})): \tag{86}$$

Suppose for a contradiction that

$$9 \; p^0 \; 2 \; (0; F(!_t^{¤})) \; j \; L_{G_{t^{¤}}}(p) \; ¸ \; L_{G_t}(p): \tag{87}$$

We already know that the slope of L_{G_t} is constant and equal to $\frac{!_0}{^1G_t}$ on $(0; F(!_t^{¤}))$: Moreover $L_{G_{t^{¤}}}$ is strictly convex. Therefore

$$L_{G_{t^{¤}}}(p) > L_{G_t}(p) \quad 8 \; p > p^0; \tag{88}$$

which contradicts

$$L_{G_t}(p) > L_{G_{t^{¤}}}(p) \qquad \text{for } p = F(!_t^{¤}): \tag{89}$$

Step 4. Let t_F be the …rst period such $T(!_t^{¤}) \; ¸ \; \frac{!_0}{!_0+c}$: Thanks to decreasingness of $T(!_t)$, it must be the case that $T(!_t^{¤}) > \frac{!_0}{!_0+c}$ for any t beyond t_F, since $!_t^{¤}$ is strictly decreasing in t. Step 1 proves that if $T(!_t^{¤}) \; ¸ \; \frac{!_0}{!_0+c}$; then $L_{G_{t^{¤}}}(p) \; ¸ \; L_{G_t}(p)$ for $p = F(!_t^{¤})$. Thanks to the same assumption, it must be case that $\frac{!_0}{!_0+c} > T(!_t^{¤})$ for any t before t_F: Steps 1, 2 and 3 prove that if this condition holds, then $L_{G_t}(p) > L_{G_{t^{¤}}}(p)$ for all $p \; 2 \; (0; 1)$: ■

E Proof of Proposition 3.1

Proof. (i) The category of literate and unconnected ones vanishes when $!_t^{¤¤} \; · \; !_t^{¤}$. In order to do so, the parameters of the model must satisfy the following condition

$$[\frac{µ_t^0}{µ_t}]^{1 \, ¡ \, ®} \; ¡ \; 1 \; ¸ \; \frac{c_t^0}{!_0 + c_t}: \tag{90}$$

Therefore

$$[\frac{µ_t^0}{µ_t}]^{1 \, ¡ \, ®} \; ¸ \; \frac{!_0 + c_t + c_t^0}{!_0 + c_t}; \tag{91}$$

which implies that

$$!_t^{¤¤¤} \; · \; !_t^{¤}: \tag{92}$$

Now we establish that in this case

$$!_t^{¤¤} \; · \; !_t^{¤¤¤}: \tag{93}$$

Indeed $!_t^{¤¤¤}$ is de…ned by

$$\frac{U_t(!_t^{¤¤¤}; 1; 1)}{U_t(!_t^{¤¤¤}; 1; 0)} \frac{U_t(!_t^{¤¤¤}; 1; 0)}{U_t(!_t^{¤¤¤}; 0; 0)} = 1: \tag{94}$$

By definition

$$\frac{U_t(\omega_t^{**};1;1)}{U_t(\omega_t^{**};1;0)} = 1; \tag{95}$$

while $\omega_t^{**} \leq \omega_t^{*}$ associated to the increasingness of the below ratio in ω implies

$$\frac{U_t(\omega_t^{**};1;0)}{U_t(\omega_t^{**};0;0)} \leq 1; \tag{96}$$

which proves

$$\frac{U_t(\omega_t^{**};1;1)}{U_t(\omega_t^{**};1;0)}\frac{U_t(\omega_t^{**};1;0)}{U_t(\omega_t^{**};0;0)} \leq 1: \tag{97}$$

Since this ratio is increasing in ω, it proves that $\omega_t^{**} \leq \omega_t^{***}$. Hence illiterate people are characterized by

$$\omega < \omega_t^{***}; \tag{98}$$

while literate connecting individuals are characterized by

$$\omega \geq \omega_t^{***}: \tag{99}$$

(ii) The literate connected category does not vanish if $\omega_t^{**} > \omega_t^{*}$ requiring

$$\frac{c_t'}{\omega_0 + c_t} + 1 > [\frac{\theta_t'}{\theta_t}]^{1-\alpha}: \tag{100}$$

The increasingness of the below ratio in ω combined with $\omega_t^{**} > \omega_t^{*}$ implies

$$\frac{U_t(\omega_t^{**};1;0)}{U_t(\omega_t^{**};0;0)} > 1; \tag{101}$$

which induces

$$\frac{U_t(\omega_t^{**};1;1)}{U_t(\omega_t^{**};1;0)}\frac{U_t(\omega_t^{**};1;0)}{U_t(\omega_t^{**};0;0)} > 1: \tag{102}$$

Since this ratio is increasing in ω, it proves that $\omega_t^{**} > \omega_t^{***}$. Moreover by definition of ω_t^{*}

$$\frac{U_t(\omega_t^{*};1;0)}{U_t(\omega_t^{*};0;0)} = 1: \tag{103}$$

Since

$$1 + \frac{c_t'}{c_t} > 1 + \frac{c_t'}{\omega_0 + c_t} \tag{104}$$

combined with (100) implies

$$1 + \frac{c_t'}{c_t} > [\frac{\theta_t'}{\theta_t}]^{1-\alpha}; \tag{105}$$

we deduce that $\varphi'(\omega) > 0$ with $\varphi(\omega) = \frac{U_t(\omega;1;1)}{U_t(\omega;1;0)}$. Therefore $\omega_t^{**} > \omega_t^{*}$ implies

$$\frac{U_t(\omega_t^*;1;1)}{U_t(\omega_t^*;1;0)} < 1; \tag{106}$$

which induces

$$\frac{U_t(\omega_t^*;1;1)}{U_t(\omega_t^*;1;0)}\frac{U_t(\omega_t^*;1;0)}{U_t(\omega_t^*;0;0)} < 1. \tag{107}$$

Therefore the increasingness of the above ratio in ω implies that $\omega_t^{***} > \omega_t^{*}$. Hence we deduce that an individual becomes literate and connected iff

$$\omega \geq \omega_t^{**}; \tag{108}$$

while an individual chooses to become literate and unconnected iff

$$\omega \geq \min[\omega_t^{*}; \omega_t^{**}). \tag{109}$$

Finally an individual remains illiterate iff

$$\omega < \omega_t^{*}. \tag{110}$$

∎

F Proof of Proposition 3.2

Proof. The sequence $\langle \omega_t^{**} \rangle$ is bounded below by $\underline{\omega}$. Furthermore, since $\langle K_{t-1} \rangle$ is strictly increasing and since ω_t^{**} is decreasing in K_{t-1}, $\langle \omega_t^{**} \rangle$ is strictly decreasing. Therefore it converges. It remains to prove that this limit is $\underline{\omega}$ which is reached within a finite number of periods. We can write

$$\omega_t^{**} = \left(\frac{(c^0)}{((\theta^0)^{1-\alpha} - \theta^{1-\alpha})(1+\beta)^{(t-t^*)(1-\alpha)} K_{t^*}^{(1-\alpha)}} \right)^{1/\alpha}. \tag{111}$$

Define $t^{**} = \min\{t \in N^+ \mid \underline{\omega} \geq \omega_t^{**}\}$. We find

$$t^{**} = \left[t^* + \frac{\log\left(\frac{c^0}{((\theta^0)^{1-\alpha} - \theta^{1-\alpha})\underline{\omega}^{\alpha}(\theta K_0)^{1-\alpha}}\right)}{(1-\alpha)\log(1+\beta)} \right] + 1; \tag{112}$$

except in the case where the expression inside [ ] is already the integer investigated. ∎

G Proof of Proposition 3.3

Proof. (i) First it is obvious that

$$L_{G_t^1}(p) > L_{G_t^2}(p) \qquad \forall p \in (0; F(\omega_t^{**})); \tag{113}$$

since the incomes of the p poorest individuals who are literate and unconnected do not change by assumption, while the average income is larger in situation 2 than in situation 1. Now we prove that

$$L_{G_t^1}(p) > L_{G_t^2}(p) \qquad \forall p \in [F(\omega_t^{**}), 1]. \tag{114}$$

Let $p = F(\omega')$. It comes

$$1 - L_{G_t^1}(p) = \frac{(\theta K_{t-1})^{1-\alpha}\int_{\omega_t'}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}{\mu_{G_t^1}}, \tag{115}$$

$$1 - L_{G_t^2}(p) = \frac{(\theta' K_{t-1})^{1-\alpha}\int_{\omega'}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}{\mu_{G_t^2}}. \tag{116}$$

Hence

$$\frac{1 - L_{G_t^1}(p)}{1 - L_{G_t^2}(p)} = \left(\frac{\theta}{\theta'}\right)^{1-\alpha} \frac{\mu_{G_t^2}}{\mu_{G_t^1}} \tag{117}$$

and

$$\frac{\mu_{G_t^2}}{\mu_{G_t^1}} = M(\omega_t^{**}) + \left(\frac{\theta'}{\theta}\right)^{1-\alpha}(1 - M(\omega_t^{**})) \tag{118}$$

with

$$M(\omega) = \frac{\int_{\underline{\omega}}^{\omega} x^{\alpha} dF(x)}{\int_{\underline{\omega}}^{\bar{\omega}} \omega^{\alpha} dF(\omega)} < 1. \tag{119}$$

Finally we obtain

$$\frac{1 - L_{G_t^1}(p)}{1 - L_{G_t^2}(p)} = 1 + M(\omega_t^{**})\left(\left(\frac{\theta}{\theta'}\right)^{1-\alpha} - 1\right) < 1. \tag{120}$$

(ii) The initial slope of the Lorenz curve decreases with t. Indeed

$$\frac{y_t(\underline{\omega})}{\mu_{G_t}} = \frac{\underline{\omega}^{\alpha}}{\int_{\underline{\omega}}^{\omega_t^{**}} \omega^{\alpha} dF(\omega) + \left(\frac{\theta'}{\theta}\right)^{1-\alpha}\int_{\omega_t^{**}}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}. \tag{121}$$

Since $\frac{\theta'}{\theta} > 1$, this ratio is increasing in ω_t^{**}, which is itself decreasing in t.

The final slope of the Lorenz curve decreases with t. Indeed

$$\frac{y_t(\bar{\omega})}{\mu_{G_t}} = \frac{\bar{\omega}^{\alpha}}{\left(\frac{\theta}{\theta'}\right)^{1-\alpha}\int_{\underline{\omega}}^{\omega_t^{**}} \omega^{\alpha} dF(\omega) + \int_{\omega_t^{**}}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}. \tag{122}$$

Since $\frac{\theta}{\theta'} < 1$, this ratio is increasing in ω_t^{**}, which is itself decreasing in t. ∎

H Proof of Proposition 3.5

Proof. Its structure is similar to the proof of proposition 2.4.

Step 1. We prove that if for some t

$$\frac{\frac{c+c'}{\omega_0+c+c'}}{\frac{c}{\omega_0+c}} > \frac{\omega_t^{*}F(\omega_t^{*})}{\omega_t^{***}F(\omega_t^{***})}, \tag{123}$$

then

$$\forall \omega \in [\omega_t^{***}; +\infty]; \quad \frac{y_t(\omega)}{\mu_{G_t^2}} > \frac{y_t(\omega)}{\mu_{G_t^1}} \tag{124}$$

and

$$L_{G_t^1}(p) > L_{G_t^2}(p) \qquad \text{for } p = F(\omega_t^{***}). \tag{125}$$

We already know that

$$\frac{y_t(\omega)}{\mu_{G_t^1}} = \frac{\omega^{\alpha}}{\frac{\omega_0}{\omega_0+c}(\omega_t^{*})^{\alpha}F(\omega_t^{*}) + \int_{\omega_t^{*}}^{+\infty}\omega^{\alpha}dF(\omega)}. \tag{126}$$

Equivalently

$$\frac{y_t(\omega)}{\mu_{G_t^2}} = \frac{\omega^{\alpha}}{\frac{\omega_0}{\omega_0+c+c'}(\omega_t^{***})^{\alpha}F(\omega_t^{***}) + \int_{\omega_t^{***}}^{+\infty}\omega^{\alpha}dF(\omega)}. \tag{127}$$

Then

$$\frac{y_{t^*}(\omega)}{\mu_{G_{t^*}}} > \frac{y_t(\omega)}{\mu_{G_t}} \Leftrightarrow \frac{\omega_0}{\omega_0+c}(\omega_t^{*})^{\alpha}F(\omega_t^{*}) < \frac{\omega_0}{\omega_0+c+c'}(\omega_t^{***})^{\alpha}F(\omega_t^{***}) + \int_{\omega_t^{***}}^{\omega_t^{*}}\omega^{\alpha}dF(\omega). \tag{128}$$

Integrating by parts the last term of the RHS and rearranging, we obtain

$$\frac{y_{t^*}(\omega)}{\mu_{G_{t^*}}} > \frac{y_t(\omega)}{\mu_{G_t}} \Leftrightarrow (\omega_t^{*})^{\alpha}F(\omega_t^{*})[\frac{c}{\omega_0+c}] - (\omega_t^{***})^{\alpha}F(\omega_t^{***})[\frac{c+c'}{\omega_0+c+c'}] - \alpha\int_{\omega_t^{***}}^{\omega_t^{*}}\omega^{\alpha-1}dH(\omega) < 0; \tag{129}$$

with

$$H(\omega) = \int_{\underline{\omega}}^{\omega} F(\zeta)d\zeta. \tag{130}$$

The conclusion regarding (124) follows. The same token is used to prove (125).

Step 2. Using the same argument as in step 2 in Proposition 2.4, we deduce that

$$L_{G_t^1}(p) > L_{G_t^2}(p) \qquad \forall p \in [F(\omega_t^{*}); 1). \tag{131}$$

We now prove that

$$L_{G_t^1}(p) > L_{G_t^2}(p) \qquad \forall p \in [F(\omega_t^{***}); F(\omega_t^{*})). \tag{132}$$

Suppose for a contradiction that

$$\exists\, p' \in [F(\omega_t^{***}); F(\omega_t^{*})) \mid L_{G_t^2}(p) \geq L_{G_t^1}(p). \tag{133}$$

Combined with (124) we obtain

$$L_{G_t^2}(p) > L_{G_t^1}(p) \quad \forall\, p > p'; \tag{134}$$

which contradicts (131).

Step 3. We now prove that

$$L_{G_1}(p) > L_{G_2}(p) \qquad\qquad \forall p \in (0; F(\omega_t^{***})). \tag{135}$$

It can be deduced from

$$\frac{y_t(\omega)}{\mu_{G_t^1}} > \frac{y_t(\omega)}{\mu_{G_t^2}} \qquad \forall \omega \in [\underline{\omega}; \omega_t^{***}); \tag{136}$$

which prevails since

$$y_t(\omega) = \omega_0 \text{ and } \mu_{G_t^2} > \mu_{G_t^1}. \tag{137}$$

∎

I Proof of Proposition 3.6

Proof. For t such that $t^* \leq t < t^{**}$; it is a consequence of Proposition 3.3. For $t < t^*$ the structure is similar to the proof of proposition 3.5.

Step 1. We prove

$$\forall \omega \in [\omega_t^*; \bar{\omega}]; \ \frac{y_t(\omega)}{\mu_{G_t^2}} > \frac{y_t(\omega)}{\mu_{G_t^1}}; \tag{138}$$

and

$$L_{G_t^1}(p) > L_{G_t^2}(p) \qquad \text{for } p = F(\omega_t^*). \tag{139}$$

Since for $\omega \in [\omega_t^*; \bar{\omega})$

$$\frac{y_t(\omega)}{\mu_{G_t^2}} = \frac{\omega^{\alpha}}{\frac{\theta}{\theta'}\frac{\omega_0}{\omega_0+c}(\omega_t^*)^{\alpha}F(\omega_t^*) + \frac{\theta}{\theta'}\int_{\omega_t^*}^{\omega_t^{**}} \omega^{\alpha} dF(\omega) + \int_{\omega_t^{**}}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}; \tag{140}$$

and

$$\frac{y_t(\omega)}{\mu_{G_t^1}} = \frac{\omega^{\alpha}}{\frac{\omega_0}{\omega_0+c}(\omega_t^*)^{\alpha}F(\omega_t^*) + \int_{\omega_t^*}^{\bar{\omega}} \omega^{\alpha} dF(\omega)}; \tag{141}$$

we deduce (138).(139) is obtained using the same argument.

Step 2. See proof of step 2 in proposition 3.5.

Step 3. For $\omega \in [\underline{\omega}; \omega_t^*)$; adapting the proof of proposition 3.5 allows to deduce that

$$L_{G_1}(p) > L_{G_2}(p) \qquad\qquad \forall p \in (0; F(\omega_t^*)). \tag{142}$$

For $\omega \in [\omega_t^*, \omega_t^{**})$ the same relation holds. Indeed

$$\frac{\omega^\alpha}{\frac{\omega_0}{\omega_0+c}(\omega_t^*)^\alpha F(\omega_t^*) + \int_{\omega_t^*}^{\bar{\omega}} \omega^\alpha dF(\omega)} > \frac{\omega^\alpha}{\frac{\omega_0}{\omega_0+c}(\omega_t^*)^\alpha F(\omega_t^*) + \int_{\omega_t^*}^{\omega_t^{**}} \omega^\alpha dF(\omega) + \frac{\theta'}{\theta}\int_{\omega_t^{**}}^{\bar{\omega}} \omega^\alpha dF(\omega)} \tag{143}$$

implies

$$\frac{y_t(\omega)}{\psi_{G_t^1}} > \frac{y_t(\omega)}{\psi_{G_t^2}} \quad \text{for} \quad \omega \in [\omega_t^*, \omega_t^{**}). \tag{144}$$

Combined with (142), it proves

$$L_{G_1}(p) > L_{G_2}(p) \qquad \forall p \in [F(\omega_t^*), F(\omega_t^{**})). \tag{145}$$

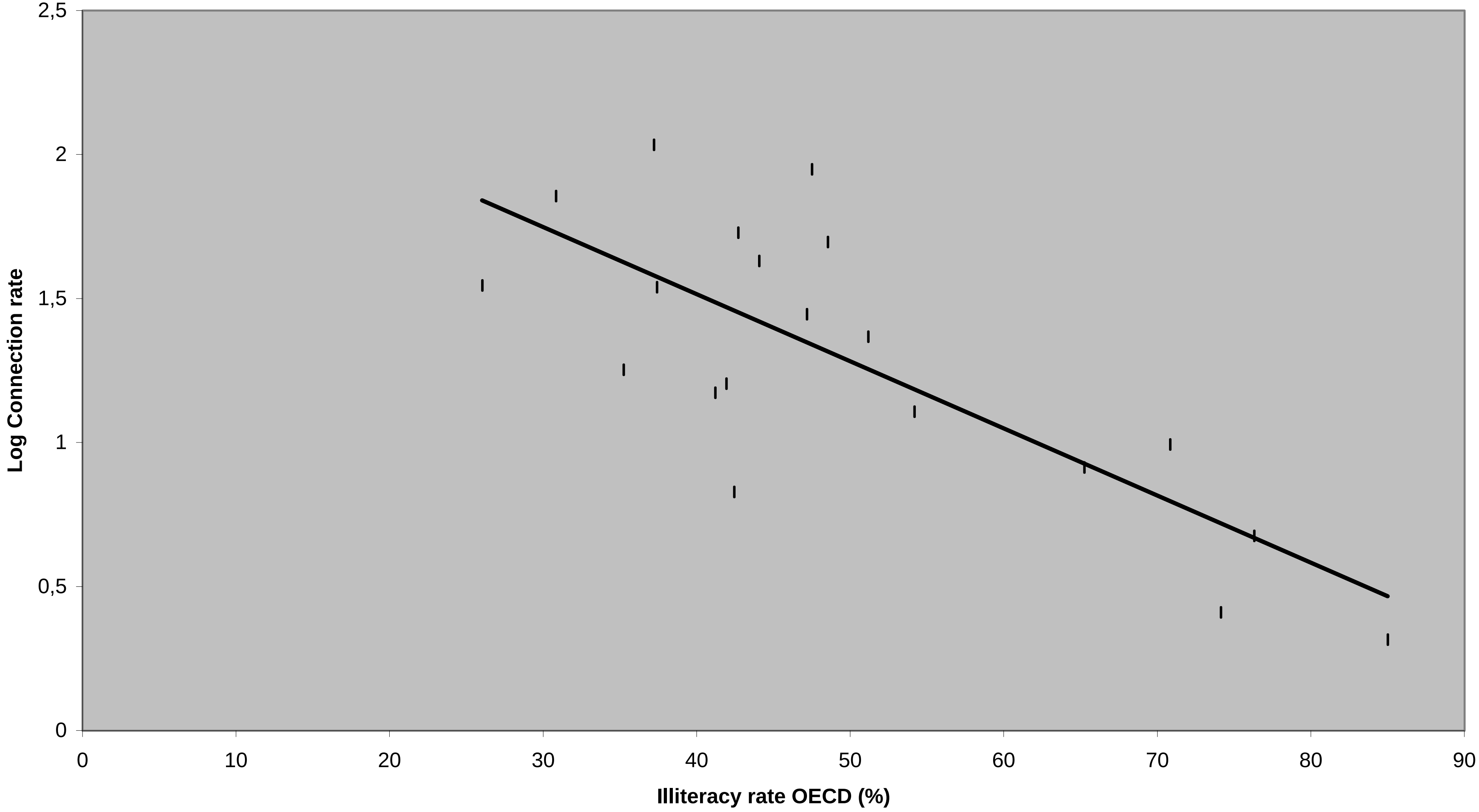
■

DATA APPENDIX

TABLE 1: CONNECTION AND ILLITERACY RATE IN INDUSTRALIZED COUNTRIES

	A	B	C	D	E	F
1	**COUNTRIES**	**% ILL Prose**	**% ILL Document**	**% ILL Quantitative**	**Average**	**Connection Ratio**
2	Canada	42,2	42,9	43	42,7	53,5
3	Germany	48,6	41,7	33,3	41,2	14,9
4	Ireland	52,4	57	53,1	54,1666667	12,8
5	Netherlands	40,6	35,8	35,8	37,4	34,6
6	Poland	77,1	76,1	69,2	74,1333333	2,57
7	Sweden	27,8	25,1	25,2	26,0333333	35,1
8	Switzerland	54,2	47	40,3	47,1666667	27,9
9	US	46,6	49,6	46,3	47,5	88,9
10	Australia	44,1	44,8	43,3	44,0666667	42,7
11	Belgium (Flanders)	46,6	39,5	39,7	41,9333333	16
12	New Zealand	45,7	50,6	49,3	48,5333333	49,7
13	United Kingdom	52,1	50,4	51	51,1666667	23,3
14	Chile	85,1	86,9	83	85	2,07
15	Czech	53,8	42,3	31,2	42,4333333	6,73
16	Denmark	46	32	27,7	35,2333333	17,9
17	Finland	36,7	36,7	38,2	37,2	108
18	Hungary	76,5	67,1	52,1	65,2333333	8,2
19	Norway	33,2	29,6	29,7	30,8333333	71,8
20	Portugal	77	80,1	71,8	76,3	4,74
21	Slovenia	76,7	72,7	63,1	70,8333333	9,85

Figure 1: Connection to Internet and Illiteracy among Industralized Countries



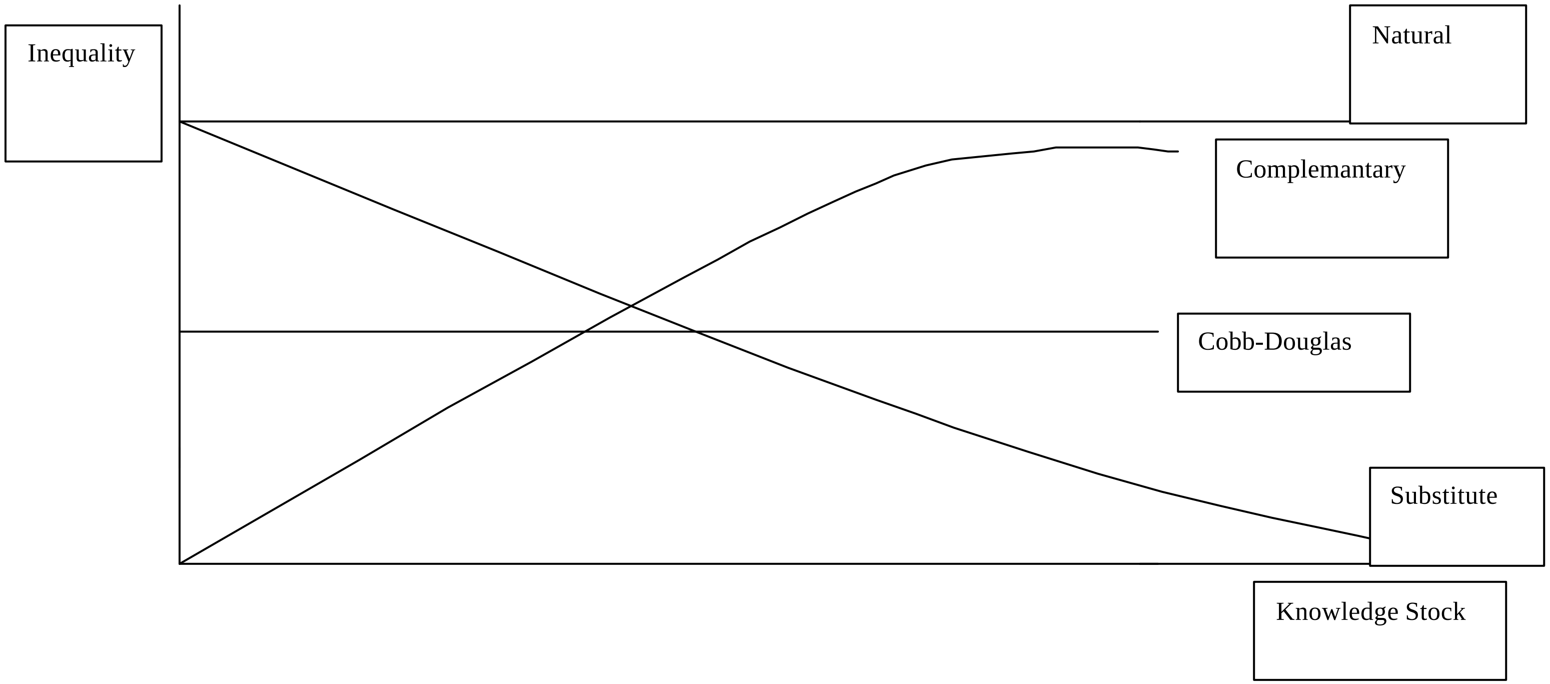


Figure 2 : The Evolution of the earnings inequality in a fully literate society .