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Topological Aggregation of Inequality Preorders*

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Abstract. An inequality preorder is defined as a complete preorder on a simplex which satisfies the properties of continuity and strict Schur-convexity (the mathematical equivalent of Dalton's "principle of transfers"). The paper shows that it is possible to aggregate individual inequality preorders into a collective one if we are interested in continuous anonymous aggregation rules that respect unanimity. The aggregation problem is studied within a topological framework introduced by Chichilnisky.

1. Introduction

Ever since the seminal work of Atkinson [1], the literature on inequality measurement considers that an inequality index should satisfy the property of symmetry and the Dalton's "principle of transfers". Symmetry means that the measure is invariant up to a permutation between the i'th and the j'th components of the vector representing the amount received by each individual in the society. The Dalton's "principle of transfers" means that a finite sequence of transformations transferring income from the rich to the poor, should decrease the value of the inequality measure. A theorem by Hardy, Littlewood and Polya, spelt out in Dasgupta, Sen and Starrett [6], shows that the requirement of Dalton's "principle of transfers" is equivalent to the mathematical property of strict Schur-convexity.

We shall say that a complete preorder defined on a simplex is an inequality preorder if it satisfies the properties of continuity and strict Schur-convexity. Indeed, these two properties imply symmetry (see Sect. 2 for a careful definition of all these terms). It is well known that the set of inequality preorders contains an infinity of elements (see [13]).

This paper deals with the following issue: assuming that the individuals of a given society have preferences toward equality and that these preferences belong to the set

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of inequality preorders, can we aggregate them, in an appropriate and consistent way, to obtain a social preference which is also an element of the set of inequality preorders?

If it is assumed that individual preferences are continuous and strictly Schur-convex, then they can be interpreted as individual opinions on what is socially right. Sen [14] points out that the problem in this approach is in arriving at these distributional judgements rather than in aggregating such judgements. We agree with Sen and do not claim that it can be found a population with such judgements. Instead, our problem is a purely theoretical one: we only investigate the economic meaning of inequality measures.

Let us remark that, since we are dealing with judgement aggregation rather than interest aggregation, the informational basis for collective choice is obtained by ordinally measuring the individual opinions on income distribution and not allowing interpersonal comparisons (on this, see Sen [15]).

In a previous paper [12], we have shown that it is not possible to aggregate individual inequality preorders into a collective one if we are interested in Arrowian aggregation rules. Here we want to tackle the same issue using, instead, the topological approach. This framework has been introduced by Chichilnisky [4] and developped extensively by Chichilnisky and Heal (see, for instance, [5] and [7]). We shall recall briefly the general setting. Let $\mathscr{P}$ be a set of individual preferences over some choice space, i.e. a subset (proper, if there are domain restrictions) of the set of complete preorders over the choice space. A social aggregation rule is a mapping from $\mathscr{P}^m$ (where m is the number of voters) to $\mathscr{P}$. In Chichilnisky [4] it was demonstrated that there is in general no continuous anonymous social aggregation rule that respects unanimity. The anonymity requirement is stronger than Arrow's nondictatorship condition whereas the condition of respect of unanimity is weaker than Arrow's Pareto condition. Given a topology on the space of preferences, it is required that the social aggregation rule be continuous. This assumption takes the role of Arrow's independence of irrelevant alternatives. We do not discuss here the motivations behind the choice of this set of axioms, but we think that this topological framework is suitable to investigate our problem for two reasons: first, the range is restricted in the same way as the domain and second, the continuity condition of the social aggregation rule takes sense here because of the euclidian structure of the choice space, a simplex. Using this approach, the paper investigates the possibility of defining continuous, anonymous and unanimous maps over the space of inequality preorders.

Chichilnisky usually considers preferences which are differentiable, but here, given that the most famous inequality index, the Gini index, is not differentiable, we shall not impose any differentiability condition. This has an important consequence in our work. Under the additional assumption that the space of admissible preferences allows a parafinite parametrization (in short, the space is a parafinite CW complex), Chichilnisky and Heal [5] have proven that a necessary and sufficient condition for the existence of a continuous, anonymous and unanimous social choice rule, for all $m \geqq 3$, is that the space of preferences be contractible (we shall not discuss here the implications of this far-reaching result; for an analysis of the implications of the contractibility property, the reader is referred to the papers previously quoted). The space of inequality preorders does not admit a parafinite parametrization, but the sufficient part of Chichilnisky and Heal's result does not depend heavily on this assumption.

The fact that an inequality preorder may be represented by a retraction map is crucial for our results. This type of representation was introduced first by Chichilnisky [3], viewing a space of smooth preferences as a space of smooth retractions on a neat compact and connected submanifold of the choice space. Following this procedure, Uriarte [17] studied a space of continuous preferences represented by a space of retractions on a compact and connected subset. Here, we follow this procedure to study the aggregation of inequality preorders and a possibility result is obtained. Briefly, the proof of the result goes as follows: we show that the space of inequality preorders is homeomorphic to a convex subset of a normed linear space (and thus contractible) and, from this property, we deduce a positive solution to our aggregation problem.

To close this introduction we summarize the difficulty originated by the ordinal non-comparable informational framework. Let $\mathscr{P}_I$ be the set of complete preorders on the simplex S_h which are continuous and strictly Schur-convex and let $I(S_h, [0, 1])$ be the space of continuous and strictly Schur-convex functions on S_h taking their values in $[0, 1]$. Clearly, it is easy to perform an aggregation operation (continuous, anonymous and unanimous) over the space $I(S_h, [0, 1])$ because of its convexity property. But here, we want to avoid any information of "cardinal type", so that the space to be considered is $\mathscr{P}_I$ rather than $I(S_h, [0, 1])$, i.e. we must consider as equivalent, in $I(S_h, [0, 1])$, any two functions inducing the same preorder on S_h. The difficulty comes from the fact that the algebraic structure of $I(S_h, [0, 1])$ is lost with this identification relation. The main result of this paper is that the convexity property is nevertheless retained (up to a homeomorphism) by the quotient space $\mathscr{P}_I$.

The paper is organized as follows: in Sect. 2, we introduce the notation and definitions needed. Then, in Sect. 3, we define a topology on the space of inequality preorders and prove some useful properties. In Sect. 4, we state and prove the possibility theorem and give an illustration when the distribution process involves three individuals.

2. Notations and Definitions

We are concerned with the distribution of a single, divisible object among h individuals ($h \geqq 3$). The available amount of the object to be distributed will be normalized to one, so that $S_h = \left\{(x_1, \ldots, x_h) \in \mathbb{R}^h_+,\ \sum_{i=1}^{h} x_i = 1\right\}$ denotes the set of feasible distributions. For a given distribution vector x in S_h, x_i denotes the share of the i-th individual. Let $\precsim$ be a complete preorder over S_h. As usual, $\prec$ and $\sim$ are the asymmetric and symmetric parts of $\precsim$, respectively.

Definition 1. *$\precsim$ is said to be continuous if $\forall x \in S_h$, the sets $\{y \in S_h : y \precsim x\}$ and $\{y \in S_h : x \precsim y\}$ are closed.*

Definition 2. *A square matrix of order h, $B = (b_{ij})$, $1 \leqq i, j \leqq h$ is bistochastic if $b_{ij} \geqq 0$, $\forall ij$;*

$$\sum_{i=1}^{h} b_{ij} = 1, \forall j; \qquad \sum_{i=1}^{h} b_{ij} = 1, \forall i.$$

A permutation matrix is a bistochastic matrix which has exactly one positive entry in each row and each column. Let B_h and P_h be the set of bistochastic and permutation matrices of order h, respectively.

Definition 3. $\precsim$ *is said to be symmetric if* $\forall x \in S_h$, $\forall P \in P_h$, *we have*

$$P \cdot x \sim x.$$

Definition 4. $\precsim$ *is said to be strictly Schur-convex if* $\forall x \in S_h$, $\forall B \in B_h$, *we have*

$$B x \prec x$$

when $Bx \neq Px \quad \forall P \in P_h$.

Let $\mathscr{P}_I$ denote the set of complete preorders over S_h which are continuous and strictly Schur-convex. From now on, an element, $\precsim$, of $\mathscr{P}_I$ will be called an inequality preorder and for any $x, y \in S_h$, $x \precsim y$ will mean that "distribution x is at least as equal as distribution y". For a deeper analysis of the motivations behind these formal definitions, see [13] and [16]. Now we show that an inequality preorder is symmetric.

Remark 1. If $\precsim \in \mathscr{P}_I$, then $\precsim$ is symmetric.

Proof. It is well known that any permutation matrix can be seen as a finite product of transposition matrices. Therefore, it is sufficient to prove that if $y = T \cdot x$, where $x \in S_h$ and T is a transposition matrix, then $y \sim x$. Without loss of generality, let us consider

$$T = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & \\ \hline 0 & & I_{h-2} \end{array}\right)$$

where I_{h-2} is the identity matrix of order $h-2$. We have that

$$T = \lim_{\substack{n \to \infty \\ n \geqq 2}} T_n$$

where

$$T_n = \left(\begin{array}{cc|c} \frac{1}{n}, & 1 - \frac{1}{n} & 0 \\ 1 - \frac{1}{n}, & \frac{1}{n} & \\ \hline 0 & & I_{h-2} \end{array}\right).$$

T_n is a bistochastic matrix $\forall n \geqq 2$ and

$$T_n \cdot x = \left(\frac{x_1 + n x_2 - x_2}{n}, \frac{n x_1 - x_1 + x_2}{n}, x_3, \ldots, x_h\right).$$

If $x_1 \neq x_2$ (when $x_1 = x_2$ the result follows trivially), $T_n \cdot x \neq P \cdot x \; \forall P \in P_h$ for n sufficiently large, say $n > N$. Thus, by strict Schur-convexity:

$$T_n \cdot x \prec x \qquad \forall n > N$$

and by continuity, $y \precsim x$. But $T = T^{-1}$ therefore we also have $x \precsim y$ and thus $x \sim y$.

q.e.d.

To any distribution $x \in S_h$, we associate the distribution $x^* = (x_{\sigma(1)}, \ldots, x_{\sigma(h)}) \in S_h$, where σ is a permutation on the set $\{1, 2, \ldots, h\}$ such that

$$x_{\sigma(1)} \leqq x_{\sigma(2)} \leqq, \ldots, \leqq x_{\sigma(n)}.$$

By remark 1, $x \sim x^*$ for any $\precsim \in \mathscr{P}_I$.

For any $x \in S_h$, we defined now the Lorenz curve as the function $\alpha_x(\cdot)$ defined on $[0, 1]$ by

$$\alpha_x(0) = 0$$

$$\alpha_x\left(\frac{k}{n}\right) = \sum_{i=1}^{k} x_i^* \qquad \forall k = 1, \ldots, h$$

and

$$\alpha_x\left(t\frac{(k-1)}{h} + (1-t)\frac{k}{h}\right) = t\,\alpha_x\left(\frac{k-1}{h}\right) + (1-t)\,\alpha_x\left(\frac{k}{h}\right)$$

$\forall t \in]0, 1[$.

We shall need the following useful lemma

Lemma 1. *Let $x, y \in S_h$. The following two conditions are equivalent*

(1) $\alpha_y(t) \leqq \alpha_x(t) \qquad \forall t \in [0, 1]$
and
$\alpha_y(\bar{t}) < \alpha_x(\bar{t})$ *for at least one* $\bar{t} \in]0, 1[$.

(2) $y = B \cdot x$, *where* B *is a bistochastic matrix of order* h, *such that* $B \cdot x \neq P \cdot x \; \forall P \in P_h$.

Proof. See [6], p. 182.

Let $I(S_h, [0, 1])$ be the set of continuous and strictly Schur-convex functions defined on S_h and taking their values in $[0, 1]$. Strict Schur-convexity for functions is defined in a similar way as above (see [2]). An element of $I(S_h, [0, 1])$ will be called an inequality index.

Remark 2. It is easy to show that any element of $\mathscr{P}_I$ admits a numerical representation in $I(S_h, [0, 1])$.

3. Construction of a Topology on the Space of Inequality Preorderings

We want to introduce a "natural" topology on the space of inequality preorders $\mathscr{P}_I$. Following Kannai [9], we can require of a topology on $\mathscr{P}_I$ that, if $x \prec y$ and if $x_n \to x$, $y_n \to y$ and $\precsim_n \to \precsim$ (in the topology of $\mathscr{P}_I$) then, $x_n \prec_n y_n$ for any sufficiently large n.

In terms of open sets, this means that the set $A = \{(x, y, \precsim) \in S_h \times S_h \times \mathscr{P}_I : x \prec y\}$ is open in the product space $S_h \times S_h \times \mathscr{P}_I$.

We shall arrange the rational balls (i.e. the balls with rational center and rational radius) in S_h in a sequence $\{B_j\}_{j \in \mathbb{N}}$. Let $A_{i,j} = \{\precsim \in \mathscr{P}_I : x \prec y$ for all x in $\overline{B_i}$ and y *in* $\overline{B_j}\}$, where a superior bar denotes closure.

Theorem 1. *The minimal topology on $\mathscr{P}_I$ for which the set A is open in $S_h \times S_h \times \mathscr{P}_I$ exists and is equal to the topology whose sub-basis is the class $\{A_{ij}\}_{i,j \in \mathbb{N}}$.*

Proof. This follows from Kannai [9] Theorem 3.1, p. 797, because $\mathscr{P}_I$ is a subset of the set of continuous and complete preorders over S_h (S_h being a locally compact second countable space).

Let us recall that a subset A of a space Y is called a retract of Y if there exists a continuous map $R: Y \to A$ such that R restricted to A is the identity map; the map R is called a retraction of Y onto A.

Now we are going to show that the topology defined in Theorem 1 is metrizable in an easy way. For this, we need the following result.

Lemma 2. *There exists a set C^* included in S_h and a homeomorphism and order-isomorphism β from C^* to* [0, 1], *such that for any $\precsim$ in $\mathscr{P}_I$, there exists one and only one retraction $R_{\precsim}$ from S_h to C^* such that the real-valued function $\beta \circ R_{\precsim}$ is a numerical representation of $\precsim$.*

Proof. Step 1. The set C^* *is defined a follows*:

$$C^* = \left\{ x = (x_1, \ldots, x_h) \in S_h : x_i = \frac{1-y}{h-1} \quad i = 1, \ldots, h-1 \quad \text{and} \quad x_h = y \right.$$

$$\left. \text{with} \quad y \in \left[\frac{1}{h}, 1\right] \right\}.$$

It is easy to verify that this set is homeomorphic to [0,1]. Let β denote this homeomorphism.

Step 2. We denote by $\sim(x)$, the set of elements in S_h indifferent to x i.e.:

$$\sim(x) = \{y \in S_h : y \sim x\}$$

where $\precsim$ belongs to $\mathscr{P}_I$.

We shall show that $|\sim(x) \cap C^*| = 1$ (where $|A|$ denotes the number of elements in the set A).

First: $\sim(x) \cap C^* \neq \emptyset$. For suppose the contrary, i.e. that $\sim(x) \cap C^* = \emptyset$, then (by completeness of $\precsim$) for any $y \in C^*$, we have either $y \prec x$ or $x \prec y$. The sets $\{y \in C^* : y \prec x\}$ and $\{y \in C^* : x \prec y\}$ are open relatively to C^*, and it is easy to show that they are non-empty. As they form a partition of C^*, this contradicts the fact that C^* is connected.

Second: $|\sim(x) \cap C^*| = 1$. Suppose the contrary, i.e., that $|\sim(x) \cap C^*| > 1$. Take y_1 and y_2 in $\sim(x) \cap C^*$, i.e., $y_1 \sim x$ and $y_2 \sim x$. By transitivity we have $y_1 \sim y_2$. Without loss of generality, assume that $y_1 < y_2$. Then, (cf. Fig. 1), $\alpha_{y_2}(t) < \alpha_{y_1}(t)$,

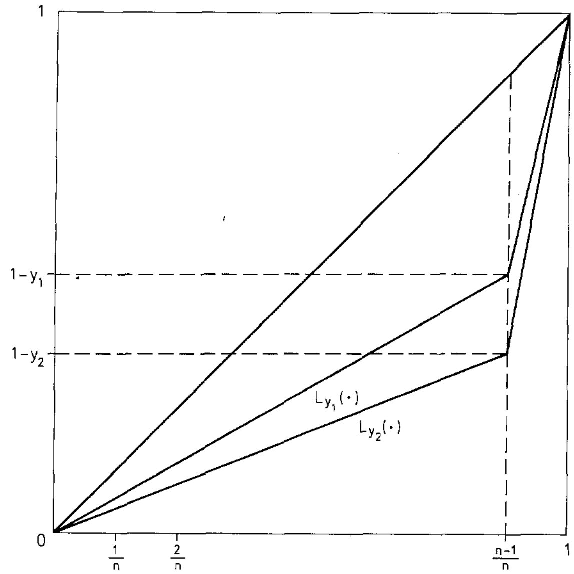

Fig. 1. The Lorenz curves of elements in C^*

$\forall t \in [0,1]$. By Lemma 1 and strict Schur-convexity we deduce that $y_2 \prec y_1$, and we obtain a contradiction. Hence β is also a continuous order-isomorphism (i.e. for any pair $x_1, x_2 \in C^*$, if $x_1 \prec x_2$, $\beta(x_1) < \beta(x_2)$). To complete the proof., it suffices to denote by $R_{\precsim}(x)$ the element $\sim(x) \cap C^*$. To show that $R_{\precsim}$ is continuous, consider $x_n \to x$ in S_h. Consider also a converging subsequence of $\{R_{\precsim}(x_n)\}_{n \in \mathbb{N}}$, – there exists at least one, because C^* is compact-, denoted $\{R_{\precsim}(x_{n_k})\}_{k \in \mathbb{N}}$. Then we have that $R_{\precsim}(x_{n_k}) \sim x_{n_k}$ $\forall k$, and so, by continuity, $\ell \sim x$, where $\ell \underset{\text{def}}{=} \lim_{k \to \infty} R_{\precsim}(x_{n_k})$. Therefore $\ell = R_{\precsim}(x)$. Given that all the converging subsequences have the same limit, $R_{\precsim}(x)$, we deduce that the sequence itself converges to $R_{\precsim}(x)$.

Theorem 2. *The minimal topology on* $\mathscr{P}_I$ *which makes the set* $A = \{(x, y, \precsim) : x \prec y\}$ *open in the product space* $S_h \times S_h \times \mathscr{P}_I$, *is induced by the metric* $d(\precsim_1, \precsim_2) = \sup_{x \in S_h} |\beta \circ R_{\precsim_1}(x) - \beta_0 R_{\precsim_2}(x)|$, ($R_{\precsim_i}$ *is the retraction*: $S_h \to C^*$ *defined in Lemma* 2).

Proof. We only need to adapt, after simplifications (because S_h is compact), the proof of Theorem 3.2 in Kannai [9], p. 798–9. q.e.d.

Remark 3. The topology defined above has been used extensively in mathematical economics (see Hildenbrand [8]). Some of its properties for the space $\mathscr{P}_I$ are described in Le Breton [11].

4. A Possibility Theorem

Assume that there are m voters, whose admissible opinions toward the inequality of the distributions (i.e. elements of S_h) belong to the set $\mathscr{P}_I$. A social aggregation rule is a mapping π from the Cartesian product (m times) of $\mathscr{P}_I$ into $\mathscr{P}_I$.

Definition 5. *Let* $\pi: \underbrace{\mathscr{P}_I \times \mathscr{P}_I \times \ldots \times \mathscr{P}_I}_{m\text{-times}} \to \mathscr{P}_I$ *be a social aggregation rule.*

π *is unanimous if:*

$$\pi(\precsim, \precsim, \ldots, \precsim) = \precsim \qquad \forall \precsim \in \mathscr{P}_I$$

π *is anonymous if:*

$$\pi(\precsim_1, \precsim_2, \ldots, \precsim_m) = \pi(\precsim_{\sigma(1)}, \precsim_{\sigma(2)}, \ldots, \precsim_{\sigma(m)})$$

$\forall (\precsim_1, \ldots, \precsim_m) \in \mathscr{P}_I^m$, $\forall \sigma$ *permutation on the set* $\{1, 2, \ldots, m\}$.

Theorem 3. *There exists a social aggregation rule*

$$\pi: \underbrace{\mathscr{P}_I \times \mathscr{P}_I \times \ldots \times \mathscr{P}_I}_{m\text{-times}} \to \mathscr{P}_I$$

which is continuous, anonymous and unanimous.

To prove Theorem 3, we will use an important property of the "identification map" described in Lemma 2, namely the map defined by

$$\precsim \in \mathscr{P}_I \to \beta \circ R_{\precsim} \in I(S_h, [0, 1]).$$

Let Φ denote this map.

Lemma 3. $\Phi(\mathscr{P}_I)$ *is a convex subset of the space of continuous functions from* S_h to $\mathbb{R}$, $C(S_h, \mathbb{R})$, *and homeomorphic to* $\mathscr{P}_I$ *(when* $C(S_h, \mathbb{R})$ *is endowed with the uniform convergence topology).*

Proof. It is easy to show that the map $\Phi: \mathscr{P}_I \to \Phi(\mathscr{P}_I)$ is a homeomorphism. To see that $\Phi(\mathscr{P}_I)$ is convex, let $\beta \circ R_{\precsim_1}$ and $\beta \circ R_{\precsim_2}$ belong to $\Phi(\mathscr{P}_I)$ and $t \in [0, 1]$. Then $t(\beta \circ R_{\precsim_1}) + (1 - t)(\beta \circ R_{\precsim_2})$ belongs to $I(S_h, [0, 1])$, because $I(S_h, [0, 1])$ is clearly convex. Thus, from Lemma 2, there exists a retraction $R_{\precsim_3}: S_h \to C^*$, such that $\beta \circ R_{\precsim_3}$ represents the element $\precsim_3$ of $\mathscr{P}_I$ induced by $t(\beta \circ R_{\precsim_1}) + (1 - t)(\beta \circ R_{\precsim_2})$. q.e.d.

Proof of Theorem 3. (See also [4], for a similar idea). We exhibit a social aggregation rule π, satisfying the three desired properties. The construction is described by the following diagram.

$$\begin{array}{ccc} \mathscr{P}_I^m & \xrightarrow{\pi} & \mathscr{P}_I \\ \Phi^m \downarrow & & \uparrow \Phi^{-1} \\ (\Phi(\mathscr{P}_I))^m & \xrightarrow[v]{} & \Phi(\mathscr{P}_I) \end{array}$$

where Φ is the "identification map" defined in Sect. 3, $\Phi^m = \underbrace{(\Phi, \Phi, \ldots, \Phi)}_{m\text{-times}}$, and $v(f_1, \ldots, f_m) = \frac{1}{m} \sum_{i=1}^{m} f_i$ with $f_i \in \Phi(\mathscr{P}_I)$ $i = 1, \ldots, m$. The map $\pi = \Phi^{-1} \circ v \circ \Phi^m$ is a continuous, anonymous and unanimous social aggregation rule. q.e.d.

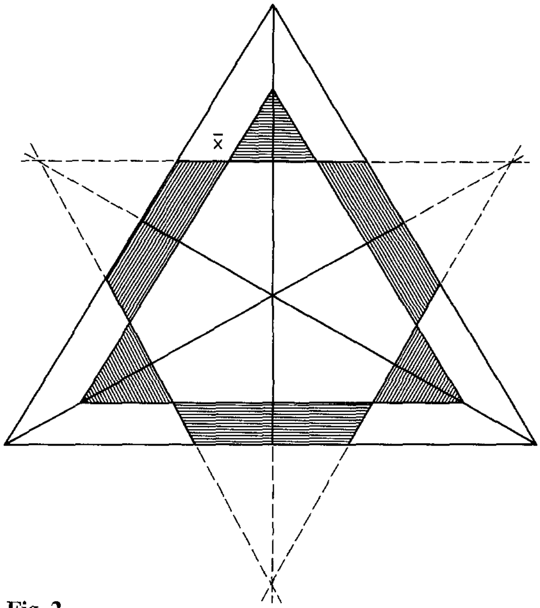

Fig. 2

In order to illustrate the aggregation rule exhibited in the proof of the preceeding theorem (the "convex average"), we consider the case of a distribution process involving three individuals and three voters, thus $h = m = 3$.

We shall consider in S_3 the distribution $(\frac{1}{2}, \frac{35}{100}, \frac{15}{100})$ denoted by $\bar{x}$, and an arbitrary element, $\precsim$, of $\mathscr{P}_I$. By continuity and strict Schur-convexity, it is easy to show that $\sim(\bar{x})$ lies in the hatched part of Fig. 2 and is symmetric.

Now, let us assume (to simplify) that the inequality preorderings of the three voters are generated by the Gini index G (for two of them), and the variance index V (for the other one). We recall that:

$$G(x_1, x_2, x_3) = \frac{1}{6} \sum_{i=1}^{3} \sum_{i=1}^{3} |x_i - x_j|$$

and

$$V(x_1, x_2, x_3) = \frac{1}{3} \sum_{i=1}^{3} |x_i - \tfrac{1}{3}|^2 .$$

We denote by $\precsim_G$ (respectively $\precsim_V$), the inequality preordering generated by G (respectively V). The sets $\sim_G(\bar{x})$ and $\sim_V(\bar{x})$ are depicted in Fig. 3.

Let $\precsim_S$ denote the social inequality preordering $\pi(\precsim_G, \precsim_G, \precsim_V)$. The set $\sim_S(\bar{x})$ is represented in Fig. 4. For instance, in Fig. 4, the set $\sim_S(\bar{x})$ obtained is the intersection of a surface of degree two, whose equation is $\frac{1}{3}(x_1^2 + x_2^2 + x_3^2) + \frac{2}{3}(x_1 + 2x_2 + 3x_3) = a$ (where a is a constant to be determined), with the simplex S_3.

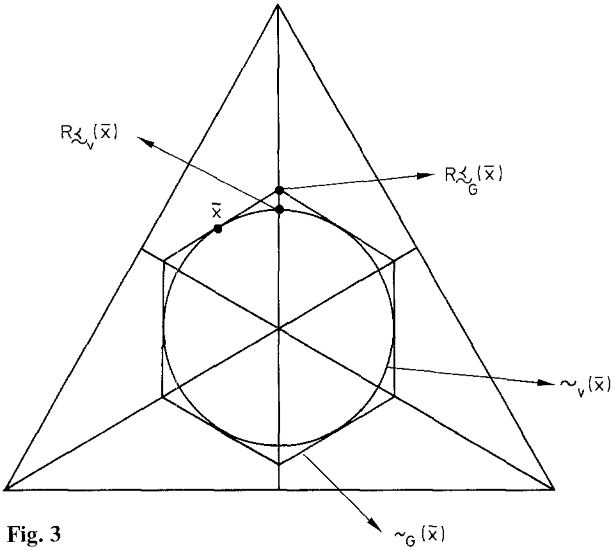

Fig. 3

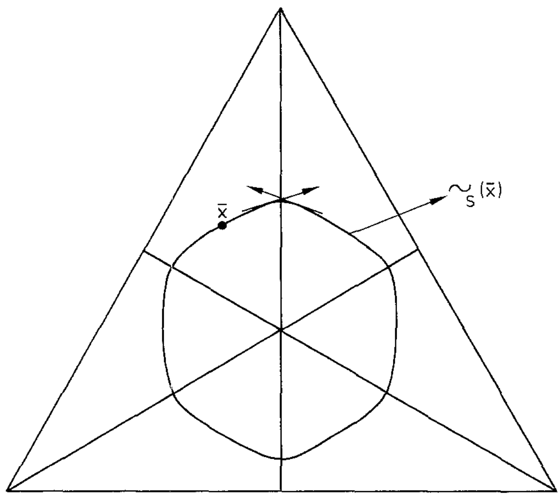

Fig. 4

Concluding Remarks

We have shown that it is possible to aggregate individual inequality preorders into a social inequality preorder if we are interested in continuous anonymous aggregation rules that respect unanimity. Therefore, the restriction on the domain of preferences introduced by strict Schur-convexity is sufficient to avoid Chichilnisky's impossibility theorem. But it must be noticed (as in Chichilnisky and Heal's problems) that there exists an infinity of social choice rules satisfying these conditions. Although they are homotopic (see [7], p. 83), they do not generate the same social ranking of distributions.

The crucial part played by strict Schur-convexity appears in the possibility to represent in a "nice" way the inequality preorderings (Lemma 2). Strict Schur-convexity is sufficient to obtain this result, but it is easy to see that the result does hold for some inequality preorders which are only Schur-convex (for instance, the preorder generated by the relative mean deviation index; for an axiomatic description of this set, see [16]).

In another paper [10], the first author studies the type of restrictions introduced by Schur-convexity in a differential framework.

References

1. Atkinson AB (1970) On the measurement of inequality. J Econ Theory 2: 244–263
2. Berge C (1966) Espaces topologiques. Fonctions multivoques. Dunod, Paris
3. Chichilnisky G (1976) Manifolds of preferences and equilibria. Report No. 27, Project on Efficiency of Decision-Making in Economic Systems, Harvard University
4. Chichilnisky G (1980) Social choice and the topology of spaces of preferences. Adv Math 37: 165–176
5. Chichilnisky G, Heal G (1983) Necessary and sufficient conditions for a resolution of the social choice paradox. J Econ Theory 31: 68–87
6. Dasgupta P, Sen AK, Starrett D (1973) Notes on the measurement of inequality. J Econ Theory 6: 180–187
7. Heal G (1983) Contractibility and public decision-making. In: Pattanaik PK, Salles M (eds) Social choice and welfare. North-Holland, New York
8. Hildenbrand W (1974) Core and equilibria of a large economy. Princeton University Press, Princeton
9. Kannai Y (1970) Continuity properties of the core of a market. Econometrica 38: 791–815
10. Le Breton M (1983) Arrowian and topological approaches to the problem of aggregating differentiable and Schur-convex preorders. D.P. 8304, L.E.M.E. Université de Rennes I
11. Le Breton M (1984) Smooth inequality preorders: an approximation theorem. D.P. 8404, L.E.M.E., Université de Rennes I
12. Le Breton M, Trannoy A (1984) Measures of inequality as an aggregation of individual preferences about income distribution: the arrowian case. S.E.E.D.S., D.P. 24
13. Sen AK (1973) On economic inequality. Oxford University Press, London
14. Sen AK (1974) Informational bases of alternative welfare approaches. J Public Econ 3: 387–403
15. Sen AK (1977) Social choice theory: A re-examination. Econometrica 45: 53–89
16. Trannoy A (1984) Social inequality indices. D.P. 8401 L.E.M.E., Université de Rennes I
17. Uriarte JR (1982) Topological structure of a space of continuous preferences and the aggregation problem. Ph. D. Dissertation, University of Essex