Welfare Comparisons With Bounded Equivalence Scales [1]

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Abstract

The paper considers the problem of comparing income distributions for heterogeneous populations. The first contribution of this paper is a precise dominance criterion combined with a simple algorithm. This criterion is shown to be equivalent to unanimity among utilitarian social planners whose objectives are compatible with given intervals of equivalence scales. The second contribution of the paper is to show that this criterion is equivalent to dominance for two different families of social welfare functions, one inspired by Atkinson and Bourguignon [3], in which household utility is a general function of income and needs, and a second family inspired by Ebert [16], in which household utility is a function of equivalent incomes. Finally, we extend our results to the case where the distributions of needs differ between the two compared populations.

JEL classification: D31, D63

Key Words: heterogeneous population, dominance, equivalence scale, welfare comparison.

Résumé

Cet article examine le problème de la comparaison des distributions de revenu pour des populations hétérogènes. La première contribution de cet article repose sur la définition d'un critère de dominance combiné avec un algorithme d'implémentation. Il est montré que ce critère est équivalent à l'unanimité des planificateurs sociaux dont l'objectif est compatible avec des intervalles d'échelle d'équivalence. La seconde contribution est la démonstration que ce critère est équivalent à la dominance pour deux familles différentes de fonctions de bien-être social, l'une inspirée d'Atkinson-Bourguignon [3], pour lesquels l'utilité d'un ménage est une fonction générale du revenu et des besoins, l'autre inspirée d'Ebert [16], pour lequel l'utilité d'un ménage est une fonction du revenu équivalent. Enfin, nous étendons nos résultats au cas où la distribution des besoins n'est pas identique dans les deux populations comparées.

JEL: D31, D63

Mots clé: population hétérogène, dominance, échelle d'équivalence, comparison de bien-être.

1. INTRODUCTION

In the field of inequality and welfare comparisons, the focus of researchers is shifting away from the study of income distribution among identical agents to the study of populations of agents differing in non-income attributes like family size, age, sex, health, or more generally, needs. A growing number of papers deal with this issue since the landmark article by Atkinson and Bourguignon [3]. A non exhaustive list would include Bourguignon [6], Jenkins and Lambert [22], Shorrocks [29], Ebert [13, 14, 15, 16], Moyes [24], Chambaz and Maurin [9], Ok and Lambert [25], Cowell and Mercader-Prats [11]. As the last authors wrote: 'At the heart of any distributional analysis, there is the problem of allowing for differences in people's non income characteristics' (abstract).

When needs differ across agents, there are basically two opposite ways to perform inequality or welfare comparisons. The first one makes use of equivalence scales and has been axiomatized by Ebert [14, 16]. The second one investigates dominance criteria by considering a wide class of household utility functions and the pioneers are without doubt Atkinson and Bourguignon [3], who refused to make welfare judgments depend on equivalence scales, which they described as a floppy notion. The aim of this paper is to conciliate these two views by performing a dominance analysis over a range of equivalent scales. Let us present the advantages and drawbacks of these two solutions before going into the details of our proposal.

Equivalence scales are a means of converting ordinary incomes for households with different needs into comparable quantities called equivalent incomes. Once an agreement about the choice of a specific equivalence scale has been reached one can perform a usual dominance analysis like Lorenz dominance (Atkinson [2]) for an inequality comparison, or Generalized Lorenz dominance (Shorrocks [28]) for a welfare comparison, based on the vectors of equivalent incomes. The Lorenz curve is a helpful graphical device and retains its normative meaning, in terms of inequality and welfare, in this context : if the Lorenz curve (resp. Generalized Lorenz curve) corresponding to an initial equivalent income distribution is above to the Lorenz curve (resp. Generalized Lorenz curve) corresponding to a final one, then the inequality (resp. welfare) has increased (resp. decreased), and reciprocally. This approach, analysed by Ebert [16], is straightforward but the choice of a particular equivalence scale is always controversial. Many equivalence scales have been proposed (Buhmann et al. [8] list thirty- four) and it has been admitted that this multiplicity does not come from statistical problems but stems from a basic difficulty lying at the heart of the concept of equivalence scales (see the discussion of Pollak and Wales [26], and Blundell and Lewbel [5]).

Since there is not one "correct" equivalence scale, Atkinson and Bourguignon [3] defend the idea that the use of such information has to be avoided in making welfare comparisons. They explore less informationally demanding methods, which rest upon general assumptions on derivatives of utility functions, and exhibit a dominance condition, the Sequential Generalized Lorenz (SGL) criterion. The main assumptions about utilities are that marginal utility increases with need, and that the smaller the need is, the slower the marginal utility decreases. These assumptions imply that social welfare is increased when a household makes a transfer of income in favor of another household with less incomes and more needs, and that social welfare increases all the more as, when a household makes a transfer to a poorer household with identical needs, these households have greater needs (see Ebert [15]). Bourguignon [6] has proposed a criterion that does not rely on the latter assumption and therefore covers a broader class of utility functions.

There is a cost to generality and the SGL criterion will fail to compare many income distributions. The ranking generated is much more partial than the order generated by the Lorenz curve for equivalent incomes. This is even more the case for Bourguignon's criterion, which is strictly more partial than the SGL criterion. This drawback mainly comes from the fact that these criteria pay attention to utility functions which may give any order of magnitude to the priority of households with greater needs, even though many of these utility functions would be considered as unreasonable by all practitioners. Consider for instance utility functions such that a single has the same marginal utility, or equivalently, the same social priority, as a couple with ten times the same income. These utility functions are unreasonable because it would be easy to argue that the couple should not have a greater priority when it has more than twice the single's income. But they do belong to the class of utility functions on which the SGL and the Bourguignon criteria rely.

To admit that equivalence scales cannot be precisely measured is one thing, another is to deny any empirical value to the equivalence scales widely used in applied studies. For instance all the equivalence scales exhibit returns to scale in household size so that, for example, a family of two adults does not require twice the income of a single person. The value 2 can be thus considered as a fairly large upper bound for computing the equivalent income of a couple (in terms of a single's income) while $2^{1/4}$ can be seen as the lowest bound possible according to subjective equivalence scales (see Buhmann et al. [8] for details). As shown below, this kind of information can be used in order to sharpen the class of admissible utility functions.

Therefore, between the Generalized Lorenz criterion applied to equivalent incomes proposed by Ebert [16], and the Sequential Generalized Lorenz criterion of Atkinson and Bourguignon [3], there is room for a middleway criterion whose properties are explored in this paper. The need for an analysis of the sensitivity of the Generalized Lorenz criterion, when applied to equivalent incomes, to the choice of the equivalence scale has already been emphasized by Kakwani [23] and Jenkins [20], who compare the results obtained with different equivalence scales. The more specific idea of considering intervals of acceptable equivalence scales has been promoted by Cowell and Mercader-Prats [11] and Bradbury [7]. The first contribution of this paper is a precise dominance criterion combined with a simple algorithm for implementing the criterion. This criterion is shown to be equivalent to unanimity among utilitarian social planners whose objectives are compatible with given intervals of equivalence scales. The second contribution of the paper is to show that this criterion is equivalent to dominance for two different families of social welfare functions, one inspired by Atkinson and Bourguignon, in which household utility is a general function of income and needs, and a second family inspired by Ebert, in which household utility is a function of equivalent incomes. In this way, this paper builds a bridge between these two approaches, which have so far been studied separately.

It should be stressed that the criterion uncovered in this paper does not imply any choice among the various possible ways of constructing intervals of equivalence scales. Whether they are based on more or less arbitrary value judgments or on empirical data about subjective welfare does not affect the validity of the criterion, which only requires the intervals to be precisely defined. Similarly, the utilitarian shape of the social objective considered in this paper does not imply any commitment to the utilitarian philosophy. The only restriction is additive separability of the social objective, and the utility functions referred to in the sequel can be viewed as representing either the households' actual utility functions, or the planner's valuation functions embodying ethical principles such as a degree of inequality aversion.

The paper begins with the presentation of the framework and the introduction and discussion of the families of utility functions appearing in the social welfare function, as well as of our central concept of social dominance. This is followed in Section 3 by the main equivalence theorem featuring the dominance criterion, and the presentation of the related algorithm. Section 4 shows that the dominance criterion is also equivalent to Generalized Lorenz dominance applied to equivalent incomes, with bounded equivalent scales. An extension to the case where the distributions of needs can differ between the distributions being compared is provided in Section 5 with a reexamination of assumptions and tools developed up to now. In particular, in line with the spirit of the paper, we make use of a condition which bounds the difference of utility levels across groups.

2. FRAMEWORK AND ETHICAL ASSUMPTIONS

2.1. Basic definitions

In this section and the next one, we will deal with a framework introduced by Bourguignon [6] and we retain most of the notations. The population is divided into K types, or groups of needs, where needs are ranked on a scale such that $k = 1$ corresponds to the least needy group and $k = K$ denotes the most needy group (the reader may think of k as an index of needs, such as family size). The living standard of a household which belongs to group k and has income y is evaluated by the utility function $V(y, k)$, which is supposed to be finite at 0 and twice continuously differentiable in y for all k, while y is assumed to belong to $\mathbb{R}_+$. In the sequel, $V_y(y, k)$ and $V_{yy}(y, k)$ will respectively denote the first and second order partial derivatives of V with respect to y. Social welfare is evaluated by a utilitarian function such that the social welfare associated with an income distribution f and a utility function V is:

$$W_V = \sum_{k=1}^{K} p_k \int_0^{s_k} f_k(y)V(y, k)dy, \tag{1}$$

where p_k is the subgroup k's population share, and $f_k(y)$ the income density function of group k, with a finite support $[0, s_k]$. Let $f = (f_1, ..., f_K)$ denote the income distribution.

Consider two income distributions f and f^*. If we suppose that the distribution of needs is the same in the two populations, the difference in social welfare between f and f^* is given and denoted by:

$$\triangle W_V = \sum_{k=1}^{K} p_k \int_0^{\overline{s}_k} \triangle f_k(y)V(y, k)dy, \tag{2}$$

with $\triangle f_k(y) = f_k(y) - f_k^*(y)$ and $\overline{s}_k = \max(s_k, s_k^*)$, where s_k^* is the upper bound of the support of f_k^*.

Dominance is defined as unanimity for a family of social welfare functions based on different utility functions.

Definition 1. f dominates f^* for a family $\mathcal{V}$ of utility functions if and only if $\triangle W_V \geq 0$ for all utility functions V in $\mathcal{V}$. This is denoted $f \ D_{\mathcal{V}} \ f^*$.

The family of utility functions on which we focus in this paper is introduced in the next subsection.

2.2. Family of utility functions

In this subsection we deal with the general family of functions à la Atkinson-Bourguignon, and introduce assumptions on marginal utilities based on various ethical principles. These assumptions determine the subfamily of utility functions with respect to which dominance will be studied in the next section.

The first assumption simply means that more income is better, and can be viewed as reflecting a kind of Pareto principle with respect to incomes, implying that giving more income to any part of the population is socially better.

U1: $V_y(y,k) \geq 0, \;\; \forall y \in \mathbb{R}_+, \forall k \in \{1,...,K\}$.

The second assumption states that marginal utility is decreasing, and can be related to the Pigou-Dalton transfer principle, according to which it is a good thing to make transfers from rich to poor households of the same type.

U2: $V_{yy}(y,k) \leq 0\,, \;\; \forall y \in \mathbb{R}_+, \forall k \in \{1,...,K\}$.

Our next assumptions have to do with comparisons of marginal utilities for households of different types. It may be easier to start with an example. Suppose that the experts' opinions about the relative needs for couples, taking singles as the reference type of household, fall down into the interval $[1.3, 1.7]$. This means that, with respect to a single with an income of \$10,000, all experts agree that a couple with income less than \$13,000 is poorer, or should be given a higher priority (social marginal value), and they all agree that a couple with income above \$17,000 is richer, and should have a lower priority. On the other hand, all incomes between these two bounds, for the couple, raise disagreement between experts about which household should be considered as poorer.

Generalizing from this example, we will assume that, in the comparison between households from groups k and $k-1$, there is agreement about two bounds $1 \leq \alpha_k \leq \beta_k$, such that households from group k and income less than α_k times greater (resp. more than β_k times greater) are considered unambiguously poorer (resp. richer) than households from group $k-1$. Figure 1 illustrates this configuration with a given household H in group $k-1$.

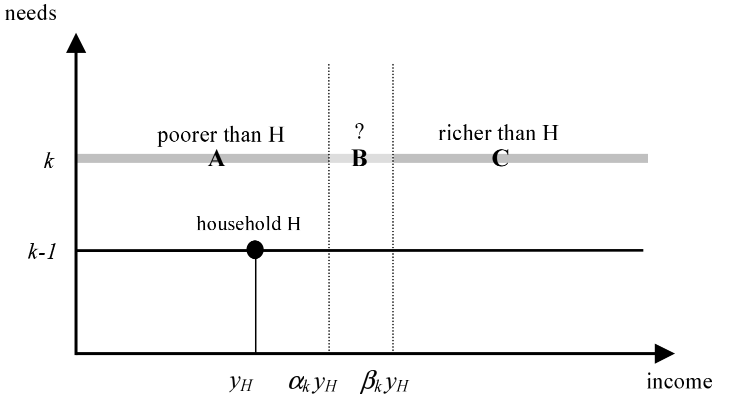

FIG. 1 Comparison between households from groups k and $k-1$

The implications of these considerations in terms of social marginal value are presented in Figure 2. The grey areas indicate the zone in which the social marginal value of households from group k have to be when one compares them to the household H from group $k-1$ having an income y_H.

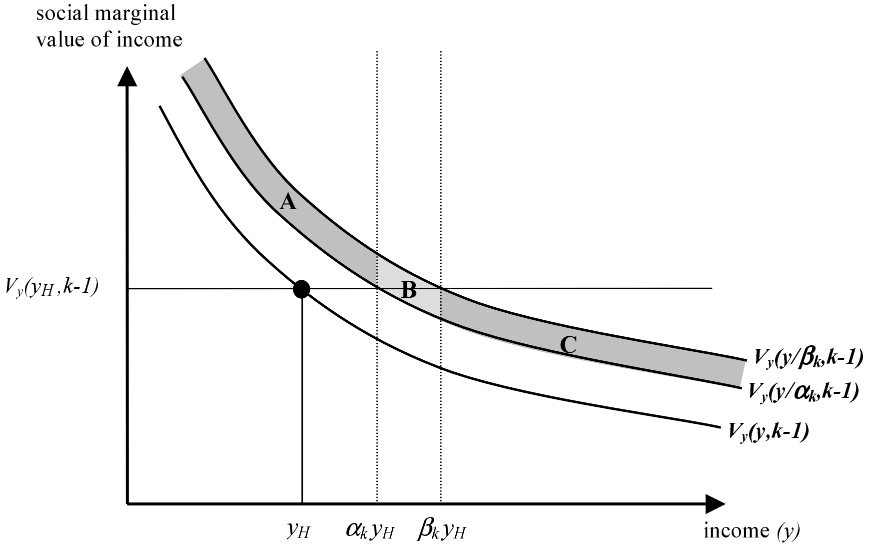

FIG. 2 Social marginal value of income in group k relatively to social marginal value of income in group $k-1$

In view of U2, this discussion leads to the following pair of assumptions. When the situation is examined for an income level y_H, the first is related to the limits of area A, while the second is associated with region C. The disagreement between experts is represented by area B.

U3: $V_y(\alpha_k y, k) \geq V_y(y, k-1),\ \forall\, y \in \mathbb{R}_+,\ \forall k \in \{2, ..., K\}.$

U4: $V_y(\beta_k y, k) \leq V_y(y, k-1),\ \forall\, y \in \mathbb{R}_+,\ \forall k \in \{2, ..., K\}.$

U3 and U4 imply that $V_y(\frac{y}{\alpha_k}, k-1) \leq V_y(y, k) \leq V_y(\frac{y}{\beta_k}, k-1)$, which is illustrated in Figure 2.

We will focus on the family of utility functions $V(y, k)$ satisfying assumptions U1 through U4, letting $\mathcal{V}(\alpha, \beta)$ denote this family, with $\alpha = (\alpha_2, ..., \alpha_K)$ and $\beta = (\beta_2, ..., \beta_K)$.

Our assumptions generalize those proposed by Bourguignon [6], in a context where k represents household size. He considered two families of utility functions. The first one is the family of functions satisfying U1, U2 and

U3$_\mathbf{B}$: $V_y(y, k) \geq V_y(y, k-1),\ \forall\, y \in \mathbb{R}_+,\ \forall k \in \{2, ..., K\}.$

The second one is the family of functions satisfying U1, U2, U3$_\mathrm{B}$ and

U4$_\mathbf{B}$: $V_y(y,k) \leq V_y(\frac{k-1}{k}y, k-1),\ \forall\, y \in \mathbb{R}_+,\ \forall k \in \{2, ..., K\}$.

One can see that U3$_\mathrm{B}$ is a particular case of U3, taking $\alpha_k = 1$, while U4$_\mathrm{B}$ is a particular case of U4, taking $\beta_k = k/(k-1)$. Because, for any $\alpha_k \geq 1$, U2 and U3 imply U3$_\mathrm{B}$, the first family considered by Bourguignon is a superset of $\mathcal{V}(\alpha, \beta)$. Similarly, if $1 \leq \alpha_k \leq \beta_k \leq k/(k-1)$, the same holds for the second family[2]. Therefore, our approach allows us to restrict considerably the relevant family of utility functions, and this makes it possible to obtain a much less partial ranking of distributions on the basis of the dominance criterion.

3. THE DOMINANCE CRITERION

Our main result provides a necessary and sufficient condition for ranking two distributions of income f and f^* when dominance $D_{\mathcal{V}(\alpha,\beta)}$ is required.

THEOREM 1.

$$f\ D_{\mathcal{V}(\alpha,\beta)} f^* \tag{A}$$

$$\Updownarrow$$

$$\sum_{k=1}^{K} p_k \triangle H_k(x_k) \leq 0 \quad \forall (x_k)_{k=1,\dots,K} \ \textit{such that} \tag{B}$$

$$\alpha_k x_{k-1} \leq x_k \leq \beta_k x_{k-1} \ \ \forall k = 2, ..., K,$$

$$\textit{and } 0 \leq x_1 \leq \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2}, \frac{\overline{s}_3}{\beta_2\beta_3}, ..., \frac{\overline{s}_K}{\beta_2\beta_3...\beta_K}),$$

where $\triangle H_k(x) = \int_0^x \int_0^y \Delta f_k(z) dz dy$.

Proof. Sufficiency: (B) implies (A). Let us introduce for all k the following functions, defined on $\mathbb{R}_+$:

$$V^n(y,k) = V(y,k) + \frac{\lambda_k}{n} \log(\frac{y}{\lambda_k} + 1), \tag{3}$$

with $\lambda_1 \geq 1$, $\alpha_k \lambda_{k-1} \leq \lambda_k \leq \beta_k \lambda_{k-1}$ for all $k = 2, ..., K$, and $n > 0$.

The first and second derivatives of $V^n(y,k)$ are respectively:

$$V_y^n(y,k) = V_y(y,k) + \frac{1}{n(\frac{y}{\lambda_k} + 1)}$$

and

$$V_{yy}^n(y,k) = V_{yy}(y,k) - \frac{1}{n\lambda_k(\frac{y}{\lambda_k} + 1)^2}.$$

[2] In other words, U3$_\mathrm{B}$ and UB4$_\mathrm{B}$ imply a larger disagreement between experts, which would be illustrated in Figures 1 and 2 by a larger area B.

One checks that, for all functions $V(y,k)$ belonging to the family $\mathcal{V}(\alpha,\beta)$, $V^n(y,k)$ satisfy assumptions U3, U4, and the strict versions of U1 and U2, respectively denoted U1* and U2*, for all $n > 0$.

We will first prove that the condition (B) is sufficient for dominance for any utility functions $V^n(y,k)$ satisfying U1*, U2*, U3 and U4. Let us fix n.

Rewriting expression (2) for functions $V^n(y,k)$, we have:

$$\triangle W_{V^n} = \sum_{k=1}^{K} p_k \int_0^{\overline{s}_k} \triangle f_k(y) V^n(y,k)dy. \tag{4}$$

Integrating by parts expression (4) and using the finiteness of $V_y^n(y,k)$ at 0 and the fact that $\triangle F_k(0) = \triangle F_k(\overline{s}_k) = 0\ \forall k$, we obtain:

$$\triangle W_{V^n} = -\sum_{k=1}^{K} \int_0^{\overline{s}_k} V_y^n(y,k) p_k \triangle F_k(y)dy. \tag{5}$$

Since $V_y^n(y,k)$ is continuous and strictly monotonous for all k, assumptions U3 and U4 are satisfied if and only if there exists for all $k \in \{2,...,K\}$, a continuous function $\varphi_k(y)$ such that:

$$\alpha_k y \leq \varphi_k(y) \leq \beta_k y\ \forall y \in \mathbb{R}_+, \tag{6a}$$

$$\text{and } V_y^n(y,k-1) = V_y^n(\varphi_k(y),k)\ \forall y \in \mathbb{R}_+. \tag{6b}$$

Notice that functions $V_y^n(y,k)$ can then be written, for all $k \in \{1,...,K-1\}$,

$$\begin{aligned} V_y^n(y,k) = V_y^n(\varphi_{k+1}(y),k+1) &= V_y^n(\varphi_{k+2}\circ\varphi_{k+1}(y),k+2) = ... \\ &= V_y^n(\varphi_K\circ\varphi_{K-1}\circ ...\circ\varphi_{k+1}(y),K) \end{aligned} \tag{7}$$

Moreover, $\varphi_k(y)$ is differentiable because $V_y^n(y,k)$ is. So that expression (6b) implies $V_{yy}^n(y,k-1) = \varphi_k'(y)V_{yy}^n(\varphi_k(y),k)$. Thus, U2* requires $\varphi_k'(y) > 0$, $\forall y \in \mathbb{R}_+$, $\forall k \in \{2,...,K\}$.

Since $\triangle F_k(y) = 0\ \forall y \geq \overline{s}_k$, $\forall k$, expression (5) can be written:

$$\triangle W_{V^n} = -\sum_{k=1}^{K} \int_0^{b_k} V_y^n(y,k) p_k \triangle F_k(y)dy, \tag{8}$$

with $b_k \geq \overline{s}_k$ for all k.

Let us take $b_1 = \max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K})$ and $b_k = \varphi_k(b_{k-1})$, $\forall k \in \{2,...,K\}$. These expressions combined with the condition (6a) guaranty that $b_k \geq \overline{s}_k$ for all k.

Let $\varphi_1(y) = y$ and define $\psi_k(y) = \varphi_k\circ\varphi_{k-1}\circ ...\circ\varphi_1(y)$ for all $k > 1$ and $\psi_1(y) = \varphi_1(y)$. Thus, we can write $b_k = \psi_k(b_1)$ for all k. Moreover, one can remark that $\psi_k(0) = 0$ for all k. Thus, introducing formula (7) in expression (8) leads to:

$$\begin{aligned}
\triangle W_{V^n} &= -\sum_{k=1}^{K-1} \int_{\psi_k(0)}^{\psi_k(b_1)} p_k \triangle F_k(y).V_y^n(\varphi_K \circ \varphi_{K-1} \circ ... \circ \varphi_{k+1}(y), K)dy \\
&\quad - \int_{\psi_k(0)}^{\psi_K(b_1)} p_K \triangle F_K(y).V_y^n(y, K)dy.
\end{aligned}$$

By a change of variable, and according to the fact that $\varphi_K \circ \varphi_{K-1} \circ ... \circ \varphi_{k+1}(\psi_k(y)) = \psi_K(y)$, it follows:

$$\triangle W_{V^n} = -\sum_{k=1}^{K} \int_0^{b_1} p_k \triangle F_k(\psi_k(y)).V_y^n(\psi_K(y), K).\psi_k^{'}(y)dy.$$

Integrating by parts leads to:

$$\begin{aligned}
\triangle W_{V^n} &= -\sum_{k=1}^{K} p_k \triangle H_k(\psi_k(b_1)).V_y^n(b_1, K) \\
&\quad + \sum_{k=1}^{K} \int_0^{b_1} p_k \triangle H_k(\psi_k(y)).\psi_K^{'}(y).V_{yy}^n(\psi_K(y), K)dy \\
&= -V_y^n(b_1, K) \sum_{k=1}^{K} p_k \triangle H_k(\psi_k(b_1)) \\
&\quad + \int_0^{b_1} \psi_K^{'}(y).V_{yy}^n(\psi_K(y), K) \sum_{k=1}^{K} p_k \triangle H_k(\psi_k(y))dy.
\end{aligned}$$

Since $\varphi_k(y)$ is strictly increasing for all k, $\psi'_K(y) > 0$. Consequently, a sufficient condition for f to dominate $f^*$for all utility functions V^n satisfying U1*, U2*, U3 and U4 is:

$$\sum_{k=1}^{K} p_k \triangle H_k(\varphi_k \circ \varphi_{k-1} \circ ... \circ \varphi_2(y)) \leq 0, \tag{9}$$

$$\text{for all } y \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K})],$$

and all functions φ_k such that $\alpha_k y \leq \varphi_k(y) \leq \beta_k y$, $\forall k \in \{2, ..., K\}$.

By setting $x_1 = y$ and $x_k = \varphi_k(x_{k-1})$ for all $k \in \{2, ..., K\}$, it follows that this expression is equivalent to:

$$\begin{aligned}
&\sum_{k=1}^{K} p_k \triangle H_k(x_k) \leq 0 \quad \forall (x_k)_{k=1,\ldots,K} \text{ such that} \\
&0 \leq x_1 \leq \max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, \ldots, \frac{\overline{s}_K}{\alpha_2\alpha_3\ldots\alpha_K}), \\
&\text{and } \alpha_k x_{k-1} \leq x_k \leq \beta_k x_{k-1} \;\; \forall k = 2, \ldots, K.
\end{aligned}$$

Moreover, since functions $\triangle H_k$ are constant above $\overline{s}_k$, it is not necessary to check the conditions for some x_1 larger than $\max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2}, \frac{\overline{s}_3}{\beta_2\beta_3}, \ldots, \frac{\overline{s}_K}{\beta_2\beta_3\ldots\beta_K})$. Therefore, condition (B) is sufficient.

It remains to prove that (B) is also sufficient when we consider functions $V(y,k)$ in $\mathcal{V}(\alpha,\beta)$. For this, we apply a corollary of the dominated convergence theorem (see Apostol [1, Theorem 10.29, p.273]).

According to expression (3), it is clear that $\lim_{n\to\infty} V^n(y,k) = V(y,k)$ $\forall y$, $\forall k$. Since by assumption, $V(y,k)$ and functions $\frac{\lambda_k}{n}\log(\frac{y}{\lambda_k}+1)$ are bounded for y belonging to $[0,\overline{s}_k]$, so are $V^n(y,k)$. It is also the case for functions $\triangle f_k(y)$. Then, the dominated convergence theorem implies that

$$\lim_{n\to\infty} \int_0^{\overline{s}_k} V^n(y,k) \triangle f_k(y) dy = \int_0^{\overline{s}_k} V(y,k) \triangle f_k(y) dy, \text{ for all } k.$$

Consequently, $\lim\limits_{n\to\infty} \triangle W_{V^n} = \triangle W_V$. Moreover, if condition (B) holds then $\triangle W_{V^n} \geq 0$ for all $n > 0$ and therefore (B) is a sufficient condition for f to dominate f^* for all utility functions V satisfying U1, U2, U3 and U4.

Necessity: (A) implies (B). Suppose $f \; D_{\mathcal{V}(\alpha,\beta)} f^*$ and condition (B) is not verified. Thus, there exists a K-vector $(e_1, e_2, \ldots, e_K)$ such that:

$$e_1 \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2}, \frac{\overline{s}_3}{\beta_2\beta_3}, \ldots, \frac{\overline{s}_K}{\beta_2\beta_3\ldots\beta_K})], \tag{10a}$$

$$\alpha_k e_{k-1} \leq e_k \leq \beta_k e_{k-1} \text{ for all } k = 2, \ldots, K, \tag{10b}$$

$$\text{and } \sum_{k=1}^{K} p_k \triangle H_k(e_k) > 0. \tag{10c}$$

Consider the following function:

$$V(y,k) = e_k U(\frac{y}{e_k}), \tag{11}$$

with $\alpha_k e_{k-1} \leq e_k \leq \beta_k e_{k-1}$ for all $k = 2, \ldots, K$, $e_1 > 0$, and $U(x)$ a twice differentiable function such that $U'(x) \geq 0$ and $U''(x) \leq 0$ for all $x \geq 0$. One checks that $V(y,k)$ satisfies assumptions U1 to U4.

Now, consider a function $U_0(x)$ such that:

$$U_0'(x) = \begin{cases} 1 \text{ if } x \leq 1-\varepsilon \\ \frac{1}{\varepsilon}(1-x) \text{ if } x \in (1-\varepsilon, 1] \\ 0 \text{ if } x > 1 \end{cases} \tag{12}$$

where $\varepsilon \in (0, 1]$.

Recall that, since $\triangle f_k(y) = 0 \ \forall \ y > \overline{s}_k$, $\forall k$, one can write

$$\triangle W_{V_0} = \sum_{k=1}^{K} p_k \int_0^{\overline{s}_k} \triangle f_k(y) e_k U_0(\frac{y}{e_k}) dy = \sum_{k=1}^{K} p_k \int_0^{b_k} \triangle f_k(y) e_k U_0(\frac{y}{e_k}) dy,$$

with $b_k \geq \overline{s}_k$. In particular, we can choose b_k such that $e_k \leq b_k$ for all k. Thus, integrating twice by parts gives

$$\triangle W_{V_0} = -\sum_{k=1}^{K} \frac{1}{\varepsilon e_k} \int_{e_k(1-\varepsilon)}^{e_k} p_k \triangle H_k(y) dy,$$

which tends to $-\sum_{k=1}^{K} p_k \triangle H_k(e_k)$ when ε tends to 0. Therefore there exists ε such that

$$\triangle W_{V_0} < -\lambda \sum_{k=1}^{K} p_k \triangle H_k(e_k), \tag{13}$$

where $0 < \lambda < 1$.

U_0 is not twice differentiable, but for any ε and any λ, one can find a twice differentiable function U arbitrarily close to U_0, so that:

$$|\triangle W_V - \triangle W_{V_0}| < \lambda \sum_{k=1}^{K} p_k \triangle H_k(e_k). \tag{14}$$

Since $\triangle W_V - \triangle W_{V_0} \leq |\triangle W_V - \triangle W_{V_0}|$, we deduce from (13) and (14) that one can find functions $V(y,k)$ satisfying assumptions U1-U4 such that $\triangle W_V < \triangle W_{V_0} + \lambda \sum_{k=1}^{K} p_k \triangle H_k(e_k) < 0$, in contradiction with $f \ D_{\mathcal{V}(\alpha,\beta)} f^*$. ∎

Our condition generalizes the two conditions obtained by Bourguignon [6]:

$$\sum_{k=1}^{K} p_k \triangle H_k(x_k) \leq 0 \quad \forall (x_k)_{k=1,...,K} \text{ such that } x_k \geq x_{k-1} \ (k = 2, ..., K)$$
$$\text{and } x_1 \in [0, \overline{s}_1],$$

$$\sum_{k=1}^{K} p_k \triangle H_k(x_k) \leq 0 \quad \forall (x_k)_{k=1,...,K} \text{ such that } x_{k-1} \leq x_k \leq \frac{k}{k-1} x_{k-1}$$

$$(k = 2, ..., K) \text{ and } x_1 \in [0, \max(\bar{s}_1, \frac{1}{2}\bar{s}_2, ..., \frac{K-1}{K}\bar{s}_K)].$$

The first criterion corresponds, in our framework, to the case when $\alpha_k = 1$ and $\beta_k \to +\infty$ for all $k = 2, ..., K$. Regarding the second condition, it concerns the case $\alpha_k = 1$ and $\beta_k = \frac{k}{k-1}$. A byproduct of the proof of Theorem 1 is to provide a more direct proof of Theorem p. 73 in Bourguignon [6].

The Atkinson-Bourguignon [3] criterion deals with the family of utility functions satisfying assumptions U1, U2, $\mathrm{U3_B}$ and the following additional condition:

$\mathbf{U_{AB}}$**:** $V_{yy}(y, k-1) \geq V_{yy}(y, k), \ \forall y \in \mathbb{R}_+, \forall k \in \{2, ..., K\}$

The criterion itself is that:

$$\sum_{k=1}^{K} p_k \triangle H_k(x_k) \leq 0 \quad \forall (x_k)_{k=1,...,K} \text{ such that, for some } l,$$

$$0 = x_1 = ... = x_{l-1} \leq x_l = ... = x_K. \quad (15)$$

This criterion is not a particular case of ours, although it covers a very large family of utility functions.

Condition (B) is nothing else than the second degree stochastic dominance condition restricted to income intervals. It can be given an intuitive interpretation by recalling that (after integrating by part)

$$\sum_{k=1}^{K} p_k \triangle H_k(x_k) = \sum_{k=1}^{K} p_k \int_0^{x_k} (x_k - y) \triangle f_k(y) dy$$

and that

$$\int_0^{x_k} (x_k - y) f_k(y) dy$$

is the absolute poverty gap for households of group k, taking x_k as the poverty line. This link between second degree stochastic dominance and the absolute poverty gap has been emphasized by Foster and Shorrocks [18]. Our condition thus states that the absolute poverty gap for the whole population must never be higher for f than for f^*, for all poverty lines $(x_1, ..., x_K)$ satisfying $\alpha_k x_{k-1} \leq x_k \leq \beta_k x_{k-1}$ for all $k = 2, ..., K$. In contrast, Bourguignon's first criterion refers to the poverty lines satisfying $x_{k-1} \leq x_k$ for all $k = 2, ..., K$. And the Atkinson-Bourguignon approach deals with poverty lines such that $0 = x_1 = ... = x_{k-1} \leq x_k = ... = x_K$ for some k.

Unfortunately, condition (B) is not implementable since it leads to checking an infinity of conditions. One more step allows us to propose a more tractable condition.

For $K = 2$, condition (B) is written:

$$p_1 \triangle H_1(x_1) + p_2 \triangle H_2(x_2) < 0 \quad \forall x_1, x_2 \text{ such that}$$
$$\alpha_2 x_1 \le x_2 \le \beta_2 x_1 \text{ and } x_1 \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2})]$$

It is straightforward to show that this condition is equivalent to the following:

$$p_1 \triangle H_1(x_1) + \max_{x_2 \in [\alpha_2 x_1, \beta_2 x_1]} \{p_2 \triangle H_2(x_2)\} \le 0 \;\; \forall x_1 \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2})].$$

This implementable condition can be generalized in the following way.

THEOREM 2. *Define the following functions:*

$$Z_K(x) = \max_{z \in [\alpha_K x, \beta_K x]} \{p_K \triangle H_K(z)\},$$
$$Z_k(x) = \max_{z \in [\alpha_k x, \beta_k x]} \{p_k \triangle H_k(z) + Z_{k+1}(z)\} \quad \forall k = 2, ..., K-1.$$

Then a necessary and sufficient condition for $f \ D_{\mathcal{V}(\alpha,\beta)} f^*$ is:

$$p_1 \triangle H_1(x) + Z_2(x) \le 0 \;\; \forall x \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2}, \frac{\overline{s}_3}{\beta_2\beta_3}, ..., \frac{\overline{s}_K}{\beta_2\beta_3...\beta_K})]. \quad \text{(C)}$$

Proof. Condition (C) can be written:

$$p_1 \triangle H_1(x) + \max_{x_2 \in [\alpha_2 x, \beta_2 x]} \left\{ p_2 \triangle H_2(x_2) + \max_{x_3 \in [\alpha_3 x_2, \beta_3 x_2]} \{p_3 \triangle H_3(x_3) + ...\} \right\}$$
$$\le 0 \;\; \forall x \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2}, \frac{\overline{s}_3}{\beta_2\beta_3}, ..., \frac{\overline{s}_K}{\beta_2\beta_3...\beta_K})]. \quad (16)$$

This condition is clearly a particular case of condition (B), then by Theorem 1, the necessity part is proved.

For the converse, suppose that condition (B) is not verified, thus there exist a K-vector $(\overline{x}_1, \overline{x}_2, ..., \overline{x}_K)$ such that $\sum_{k=1}^{K} p_k \triangle H_k(\overline{x}_k) > 0$ with $\alpha_k \overline{x}_{k-1} \le \overline{x}_k \le \beta_k \overline{x}_{k-1} \quad \forall k = 2, ..., K$ and $\overline{x}_1 \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2}, \frac{\overline{s}_3}{\beta_2\beta_3}, ..., \frac{\overline{s}_K}{\beta_2\beta_3...\beta_K})]$.

Now, write the condition (16) for $x = \overline{x}_1$:

$$p_1 \triangle H_1(\overline{x}_1) + \max_{x_2 \in [\alpha_2 \overline{x}_1, \beta_2 \overline{x}_1]} \left\{ p_2 \triangle H_2(x_2) \right.$$
$$\left. + \max_{x_3 \in [\alpha_3 x_2, \beta_3 x_2]} \{p_3 \triangle H_3(x_3) + ...\} \right\} \le 0 \qquad (17)$$

Since $\overline{x}_k \in [\alpha_k \overline{x}_{k-1}, \beta_k \overline{x}_{k-1}]$ $\forall k = 2, ..., K$, and because of the max conditions, we have:

$$
\begin{aligned}
& p_1 \triangle H_1(\overline{x}_1) + \max_{x_2 \in [\alpha_2 \overline{x}_1, \beta_2 \overline{x}_1]} \left\{ p_2 \triangle H_2(x_2) + \max_{x_3 \in [\alpha_3 x_2, \beta_3 x_2]} \{ p_3 \triangle H_3(x_3) + ... \} \right\} \\
& \geq p_1 \triangle H_1(\overline{x}_1) + p_2 \triangle H_2(\overline{x}_2) + \max_{x_3 \in [\alpha_3 \overline{x}_2, \beta_3 \overline{x}_2]} \{ p_3 \triangle H_3(x_3) + ... \} \\
& \geq p_1 \triangle H_1(\overline{x}_1) + p_2 \triangle H_2(\overline{x}_2) + p_3 \triangle H_3(\overline{x}_3) + \max_{x_4 \in [\alpha_4 \overline{x}_3, \beta_4 \overline{x}_3]} \{ p_4 \triangle H_4(x_4) + ... \} \\
& \geq ... \geq \sum_{k=1}^{K} p_k \triangle H_k(\overline{x}_k)
\end{aligned}
$$

Thus, $p_1 \triangle H_1(\overline{x}_1) + Z_2(\overline{x}_1) > 0$. Consequently (C) implies (B) and by Theorem 1, (C) implies $f \ D_{\mathcal{V}(\alpha,\beta)} f^*$. ∎

Theorem 2 thus yields a simple algorithm for the implementation of social dominance $D_{\mathcal{V}(\alpha,\beta)}$.

4. DOMINANCE WITH BOUNDED EQUIVALENCE SCALES

We now turn our attention to a second framework, proposed by Ebert [16]. It is based on equivalence scales and is a particular case of the first framework.

An equivalence scale is a list of numbers e_k for $k = 1, ..., K$, such that $e_1 \leq ... \leq e_K$, and these numbers are interpreted in the following way. A household from group k and with income y will be said to have an equivalent income equal to y/e_k, and equivalent incomes are assumed to be directly comparable across types of households. It is usual, in applied studies, to choose a reference group k_0, which amounts to letting $e_{k_0} = 1$. For instance, taking the group of singles as the reference, one can then view e_k as the number of "equivalent adults" in households of group k, and y/e_k is then the equivalent adult's average income in the household.

When a particular equivalence scale $e = (e_1, ..., e_K)$ is chosen, social welfare can be computed by aggregating the utility levels of equivalent incomes over the population, and Ebert [16] proposed to adopt the following household utility function:

$$V(y,k) = e_k U\left(\frac{y}{e_k}\right),$$

where U is a twice differentiable real-valued function, which leads to the following formula for the social welfare:[3]

$$W_{U,e} = \sum_{k=1}^{K} p_k \int_0^{\overline{s}_k} f_k(y) e_k U\left(\frac{y}{e_k}\right) dy.$$

Now, a particular case of our approach in the previous sections is when $\alpha_k = \beta_k$ for all $k = 2, ..., K$. In this case, choose e_1 arbitrarily, and for $k = 2, ..., K$, compute $e_k = \alpha_k e_{k-1}$. One then obtains an equivalence scale $e = (e_1, ..., e_K)$, and assumptions U3 and U4 imply that for all $y \geq 0$,

$$V_y(y, k) = V_y(\frac{e_{k-1}}{e_k} y, k-1) = ... = V_y(\frac{e_1}{e_k} y, 1). \tag{18}$$

Define

$$U(y) = \frac{1}{e_1} V(e_1 y, 1).$$

By integrating (18), up to a constant, one gets Ebert's formula:

$$V(y, k) = e_k U(\frac{y}{e_k}).$$

In other words, this second approach based on a precise equivalence scale is just a particular case of our approach in the first framework, and is obtained when the intervals $[\alpha_k, \beta_k]$ boil down to points. In this section we study the relationship between this approach in terms of equivalence scales, and the approach studied in the first two sections, with non degenerate intervals.

The comparison of two distributions f and f^*, when a particular equivalence scale is chosen, amounts to calculating the following difference:

$$\triangle W_{U,e} = \sum_{k=1}^{K} p_k \int_0^{\overline{s}_k} \triangle f_k(y) e_k U\left(\frac{y}{e_k}\right) dy, \tag{19}$$

where $\triangle f_k(y)$ is defined as previously. This expression has been written supposing that the equivalence scale is known, but we can generalize it to the case when there is some uncertainty on the values of equivalence scales.[4] Let Θ be a set of equivalence scales $e = (e_1, ..., e_K)$, that is, a subset of vectors e from $\mathbb{R}^K_{++}$, such that $e_1 \leq ... \leq e_K$.

Now, consider the following new definition of social welfare dominance. Let $\mathcal{U}$ denote a family of real-valued functions defined on $\mathbb{R}_+$.

[3]Several forms of social welfare function have been discussed in the literature. Here, each household is weighted by its equivalence scale. Other authors, like Glewwe [19], make use of a social welfare function in which the weight is the number of persons in the household. But, as shown in Ebert [14], weighting by equivalence scales is necessary and sufficient for a social welfare function to satisfy the condition that a household with greater equivalent-income has a lower social priority (social marginal value of income).

[4]Recall that we suppose that there may exist an uncertainty on the values of equivalence scales, but not on the determination of groups.

DEFINITION 2. f dominates f^* for a family $\mathcal{U}$ of utility functions and a set Θ of equivalence scales if and only if $\triangle W_{U,e} \geq 0$ for all utility functions U in $\mathcal{U}$ and all K-vectors e in Θ. This is denoted $f\ D_{\mathcal{U},\Theta}\ f^*$.

As noticed above, one has $\triangle W_{U,e} = \triangle W_V$ when $V(y,k) = e_k U\left(\frac{y}{e_k}\right)$. Consequently, the dominance $D_{\mathcal{U},\Theta}$ is related to dominance $D_{\mathcal{V}}$. It turns out that, for some appropriate families of functions, these two dominance conditions are equivalent. In order to obtain a precise statement of this fact, consider the class $\mathcal{U}2$ of increasing and concave utility functions, namely the family of functions satisfying the following assumptions:

$\widetilde{\mathbf{U1}}$: $U'(y) \geq 0\ \ \forall y \geq 0$.

$\widetilde{\mathbf{U2}}$: $U''(y) \leq 0\ \ \forall y \geq 0$.

We propose to consider a particular set Θ defined in the following parametric way:

$$\Theta(\alpha,\beta) = \left\{(e_1,...,e_K)\,|\,\alpha_k e_{k-1} \leq e_k \leq \beta_k e_{k-1}\ \ \forall k = 2,...,K\right\}, \tag{20}$$

where $1 \leq \alpha_k \leq \beta_k$ are given.

As we have remarked in the necessity part of the proof of Theorem 1, the functions $e_k U\left(\frac{y}{e_k}\right)$ are under these conditions a subclass of $\mathcal{V}(\alpha,\beta)$. Hence $f\ D_{\mathcal{V}(\alpha,\beta)} f^*$ implies $f\ D_{\mathcal{U}2,\Theta(\alpha,\beta)} f^*$.[5] Conversely, because the necessity part of the proof of Theorem 1 is built on particular functions of $\mathcal{U}2$, it appears that $f\ D_{\mathcal{U}2,\Theta(\alpha,\beta)}\ f^*$ implies (B), and by Theorem 1, implies $f\ D_{\mathcal{V}(\alpha,\beta)} f^*$. This discussion can be summarized in the following proposition:

THEOREM 3. $f\ D_{\mathcal{U}2,\Theta(\alpha,\beta)}\ f^* \Longleftrightarrow f\ D_{\mathcal{V}(\alpha,\beta)} f^*$.

This theorem is interesting in three different ways. First, it bridges the gap between two different approaches which have so far remained separated

[5]Therefore, by Theorem 1, (B) implies $f\ D_{\mathcal{U}2,\Theta(\alpha,\beta)} f^*$. A direct proof of this fact can be given. By a change of variable, expression (19) can be written:

$$\triangle W_{U,e} = \sum_{k=1}^{K} p_k \int_0^{\bar{s}_k/e_k} \triangle f_k(e_k y) e_k^2 U\,(y)\,dy.$$

Integrating twice by part this expression, we obtain:

$$\triangle W_{U,e} = -\sum_{k=1}^{K} p_k \triangle H_k(\bar{s}_k) U'\,(\bar{s}_k) + \sum_{k=1}^{K} p_k \int_0^{\bar{s}_k/e_k} \triangle H_k(e_k y) U''\,(y)\,dy.$$

By posing $x_k = e_k y$, (B) is a sufficient condition for $\triangle W_{U,e} \geq 0$ under the assumptions that U is increasing and concave.

in the literature, and shows that the general approach in terms of functions $V(y,k)$ essentially amounts to considering wide sets of equivalence scales, rather than merely abandoning the concept of equivalence scales. Second, in view of Theorem 1, it provides a dominance criterion for the equivalence scale approach when some uncertainty prevails about the equivalence scale. And third, it provides a more intuitive reading of the dominance concept $D_{\mathcal{V}(\alpha,\beta)}$, if one thinks that it is easier to understand the conditions defining $\Theta(\alpha,\beta)$ than the assumptions U3 and U4.

Ebert [16] has proved that a necessary and sufficient condition for the dominance with given equivalence scales is the Generalized Lorenz dominance on equivalent incomes. Thus, according to Theorem 3, an alternative implementation of our criterion could be the comparison of Lorenz curves for all equivalence scales satisfying the chain conditions defined by (20). But this procedure, which in principle relies on an infinity of comparisons, would be very cumbersome. Indeed, even by taking only n values in each interval, it would amount to performing n^{K-1} comparisons of Lorenz curves. Consequently, the graphical interest of Lorenz curves would be lost. Furthermore, assuming that a comparison of two Lorenz curves spends one second on a powerful computer, an empirical application with 10 groups of needs and $n = 10$ would take more than 30 years!

One can, however, wonder whether there might be a kind of monotonicity of the dominance in equivalence scales, in the sense that it would be sufficient to consider only the bounds of the intervals of equivalence scales. The following example proves that this is not the case.[6]

Example 1. Consider a population composed by two groups of needs with 10 households in each group. Table 1 gives two distributions of income having the same mean.

TABLE 1
A counter-example

distribution f		distribution f^*	
income	number of households	income	number of households
group 1			
1	1	1	3
1.5	5	1.75	6
2	4	3	1
group 2			
2	9	2	8
3	1	2.5	2

[6] Using the HBAI income data and a parametric function of equivalence scales, Jenkins and Cowell [21] obtain the same kind of result on inequality and poverty indices.

When the equivalence scale e_2 is given, a necessary and sufficient condition for f to dominate f^* is[7] $p_1 \triangle H_1(x) + p_2 \triangle H_2(e_2 x) \leq 0 \; \forall x$.

The Figure 3 provides the curve representing the function $p_1 \triangle H_1(x) + p_2 \triangle H_2(e_2 x)$ for $e_2 = 1.2$, $e_2 = 1.4$ and $e_2 = 1.7$. f dominates f^* if and only if the curve is always below the horizontal axis. We remark that the dominance occurs when the equivalence scale is equal to 1.2 or 1.7, but not for 1.4.

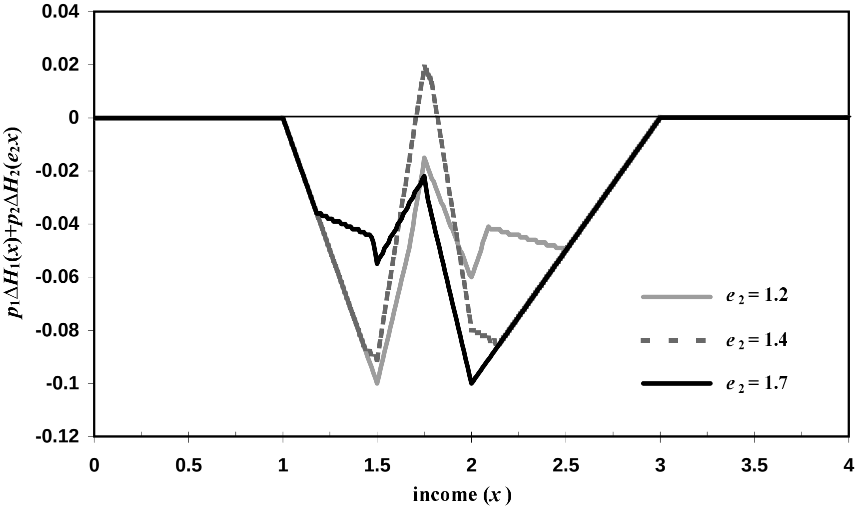

FIG. 3 No monotonicity in equivalence scales

The condition on equivalence scales that is expressed in the definition of $\Theta(\alpha, \beta)$ is a chain condition which may seem less convenient for empirical applications than the following one, in which bounds are defined with respect to the reference type of household (assumed to be the first type here, in order to fix ideas):

$$\bar{\Theta}(\underline{e}, \bar{e}) = \{(e_1, ..., e_K) \,|\, e_1 = 1, \; e_{k-1} \leq e_k \text{ and } \underline{e_k} \leq e_k \leq \bar{e}_k \; \forall k = 2, ..., K\}.$$

Fortunately, by a simple adaptation of our previous results one can derive a dominance criterion, for this new set of equivalence scales.

[7] According to Ebert [16], checking this condition is equivalent to checking the Lorenz dominance on equivalent incomes. Here, because the Lorenz curves are very close to each other, the stochastic dominance condition gives a better visual illustration.

THEOREM 4.

$$f\ D_{\mathcal{U}2,\bar{\Theta}(\underline{e},\overline{e})}f^* \qquad (\widetilde{\mathrm{A}})$$

$$\Updownarrow$$

$$\sum_{k=1}^{K} p_k \triangle H_k(x_k) \leq 0 \quad \forall (x_k)_{k=1,...,K} \textit{ such that} \qquad (\widetilde{\mathrm{B}})$$

$$x_1 \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\overline{e}_2}, \frac{\overline{s}_3}{\overline{e}_3}, ..., \frac{\overline{s}_K}{\overline{e}_K})],$$

$$0 \leq x_{k-1} \leq x_k \leq \overline{s}_k \textit{ and } \underline{e}_k x_1 \leq x_k \leq \overline{e}_k x_1\, \forall k = 2, ..., K.$$

Proof. Sufficiency: $(\widetilde{\mathrm{B}})$ implies $(\widetilde{\mathrm{A}})$. The proof is exactly the same as the proof of sufficiency part of Theorem 3 given in footnote 5.

Necessity: $(\widetilde{\mathrm{A}})$ implies $(\widetilde{\mathrm{B}})$. Suppose condition $(\widetilde{\mathrm{B}})$ is not satisfied. There exists a real number x, and a vector $(e_1, e_2, ..., e_K)$ such that $e_1 = 1$, and for all $k = 2, ..., K$, $e_{k-1} \leq e_k$ and $\underline{e}_k \leq e_k \leq \overline{e}_k$, such that $\sum_{k=1}^{K} p_k \triangle H_k(e_k x) > 0$. By a similar method as in the necessity part of the proof of theorem 1, one can find a function U in $\mathcal{U}2$, such that $\triangle W_{U,e} < 0$, in contradiction with $f\ D_{\mathcal{U}2,\bar{\Theta}(\underline{e},\overline{e})}f^*$. ∎

An algorithm for implementing condition $(\widetilde{\mathrm{B}})$ is the following. Define

$$Q_K(x,y) = \max_{z \in [\underline{e}_K x, \overline{e}_K x] \cap [y, +\infty[} \{p_K \triangle H_K(z)\}$$

$$Q_k(x,y) = \max_{z \in [\underline{e}_k x, \overline{e}_k x] \cap [y, +\infty[} \{p_k \triangle H_k(z) + Q_{k+1}(x,z)\} \text{ for } k = 2, ..., K-1.$$

Then a necessary and sufficient condition for $f\ D_{\mathcal{U}2,\bar{\Theta}(\alpha,\beta)}f^*$ is:

$$p_1 \triangle H_1(x) + Q_2(x,x) \leq 0 \quad \forall x \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\overline{e}_2}, \frac{\overline{s}_3}{\overline{e}_3}, ..., \frac{\overline{s}_K}{\overline{e}_K})].$$

The proof of this fact is similar to that of Theorem 2, and relies on the fact that $(\widetilde{\mathrm{C}})$ is equivalent to:

$$p_1 \triangle H_1(x) + \max_{x_2 \in [\underline{e}_2 x, \overline{e}_2 x]} \Big\{ p_2 \triangle H_2(x_2) + \max_{x_3 \in [\underline{e}_3 x, \overline{e}_3 x] \cap [x_2, +\infty[} \{p_3 \triangle H_3(x_3) + ...\} \Big\} \leq 0,$$

$$\forall x \in [0, \max(\overline{s}_1, \frac{\overline{s}_2}{\overline{e}_2}, \frac{\overline{s}_3}{\overline{e}_3}, ..., \frac{\overline{s}_K}{\overline{e}_K})] \text{ and } x_k \in [0, \overline{s}_k] \text{ for } k = 2, ..., K-1.$$

5. EXTENSION TO THE CASE WHERE DISTRIBUTION OF NEEDS DIFFER

When one wishes to make some intertemporal or inter-country comparisons of welfare, it is necessary to have a dominance criterion which allows to compare distributions with different needs. Consider two income distributions f and f^* respectively associated to the population shares vectors $(p_k)_{k=1,...,K}$ and $(p_k^*)_{k=1,...,K}$.

First, rewrite expression (2) in this context:

$$\triangle W_V = \sum_{k=1}^{K} \int_0^{\overline{s}_k} \triangle[p_k f_k(y)] V(y,k) dy, \tag{21}$$

with $\triangle[p_k f_k(y)] = p_k f_k(y) - p_k^* f_k^*(y)$.

Assumptions U1 to U4 introduced in Section 2 are not sufficient to obtain a similar characterization to Theorem 1. Until now, the informational basis required by the aggregation process are captured by the *cardinal unit*-comparability invariance axiom defined by d'Aspremont and Gevers [12]. As noted by Atkinson and Bourguignon [4], extending dominance results when the marginal distributions of needs differ requires a stronger invariance axiom known in the social choice literature as the *full*-comparability one, in which, additionally to the fact that comparing utility differences is meaningful, the utility *levels* can be compared. In the literature, two kinds of assumptions have been proposed in this vein to extend the Atkinson-Bourguignon's criterion. Before presenting our assumption, we discuss the merit of the proposals made by Jenkins and Lambert [22] and Moyes [24]. The former authors introduce a number $\overline{a} \geq \overline{s}_k \forall k$, which is interpreted as the maximum conceivable income (or income limit), and state that all households face the same utility level for an income just equal to $\overline{a}$, i.e.

$\mathbf{U_{JL}}$**:** $\exists \overline{V},\ V(\overline{a},k) = \overline{V}\ \ \forall k.$

To make this assumption meaningful, one has to consider it jointly with the other assumptions on utility functions and in particular $\mathrm{U3_B}$, stating that the larger the need, the larger the marginal utility. Posed together, $\mathrm{U3_B}$ and $\mathrm{U_{JL}}$ capture two ideas. First, for a given income belonging to $[0,\overline{a})$, the smaller the need, the larger the utility level. Second, when the household income is very large the importance of the difference in needs is negligeable for a welfare analysis. Under $\mathrm{U_{JL}}$, social welfare is invariant to transfers of population across groups of needs, at income level $\overline{a}$.

Assumption $\mathrm{U_{JL}}$ has been criticized by Moyes [24] on the ground it is too strong. He proposes to consider only the first of the two previous ideas. Then, instead of $\mathrm{U_{JL}}$, he makes the following assumption:

$\mathbf{U_M}$**:** $V(y,k-1) \geq V(y,k)\ \ \forall y \in [0,\overline{a}], \forall k \in \{2,...,K\}.$

Considering the family of utility functions satisfying U1, U2, $\mathrm{U3_B}$, $\mathrm{U_{AB}}$ and $\mathrm{U_{JL}}$, Jenkins and Lambert show[8] that a simple generalization of the

[8]Notice that Jenkins and Lambert [22] only prove the sufficiency part of the result. The necessity part is given by Chambaz and Maurin [9].

dominance condition of Atkinson and Bourguignon [3], recalled in equation (15), is valid in the case where the distribution of needs differ. This condition is written:

$$\sum_{k=j}^{K} \triangle[p_k H_k(x)] \leq 0 \;\; \forall x \in [0, \overline{a}], \forall j = 1, ..., K, \tag{22}$$

where $\triangle[p_k H_k(x)] = \int_0^x \int_0^y [p_k f_k(z) - p_k^* f_k^*(z)]\, dzdy$.

Considering a larger family of utility functions leads to a more partial criterion of dominance. Indeed, Moyes [24] proves that f dominates f^* for the family of utility functions satisfying assumptions U1, U2, $\mathrm{U3_B}$, $\mathrm{U_{AB}}$ and $\mathrm{U_M}$ if and only if

$$\sum_{k=j}^{K} \triangle[p_k H_k(x)] \leq 0 \;\; \forall x \in [0, \max(\overline{s}_1, ..., \overline{s}_K)], \forall j = 1, ..., K, \tag{23a}$$

$$\text{and} \; \sum_{k=j}^{K} [p_k - p_k^*] \leq 0 \; \forall j = 2, ..., K-1. \tag{23b}$$

This last condition means that the proportion of needy people, evaluated in a sequential way, is at least as great in the dominated configuration than in the dominating one. This condition restricts the set of income distributions to which the comparative test can be performed, and therefore, passing from $\mathrm{U_{JL}}$ to $\mathrm{U_M}$ implies a loss of the discriminating power of the dominance criterion.

Assumption $\mathrm{U_M}$ implies a demographic condition like (23b) because the difference between the utility levels for two different groups of needs can be arbitrarily large, so that the proportion of more needy groups in the population becomes the only relevant information in the comparison of income distributions. Symmetrically, assumption $\mathrm{U_{JL}}$ means that the difference in utility vanishes totally for large incomes. But it is worth noticing that Jenkins and Lambert's criterion contains a demographic condition as well. This condition becomes more and more pregnant as $\overline{a}$ is large. At the limit, the demographic condition is Moyes' one. More precisely, we can state:

Remark 1. When $\overline{a}$ goes to infinity, Jenkins' and Lambert's criterion boils down to Moyes' one. Indeed, for $x \geq \overline{s}_k$ one can write:

$$\begin{aligned}
\triangle[p_k H_k(x)] &= \triangle[p_k H_k(\overline{s}_k)] + p_k \int_{\overline{s}_k}^{x} F_k(y)dy - p_k^* \int_{\overline{s}_k}^{x} F_k^*(y)dy \\
&= p_k \int_0^{\overline{s}_k} (\overline{s}_k - y) f_k(y)dy - p_k^* \int_0^{\overline{s}_k} (\overline{s}_k - y) f_k^*(y)dy \\
&\quad + (p_k - p_k^*)(x - \overline{s}_k) \\
&= p_k(\overline{s}_k - \mu_{f_k}) - p_k^*(\overline{s}_k - \mu_{f_k^*}) + (p_k - p_k^*)(x - \overline{s}_k) \\
&= p_k^* \mu_{f_k^*} - p_k \mu_{f_k} + (p_k - p_k^*)x,
\end{aligned}$$

where μ_{f_k} and $\mu_{f_k^*}$ represent the average incomes relative to f_k and f_k^*. Therefore, for $x \geq \max(\overline{s}_1, ..., \overline{s}_K)$, condition (22) can be written

$$\sum_{k=j}^{K} \triangle[p_k H_k(x)] = \sum_{k=j}^{K} [p_k^* \mu_{f_k^*} - p_k \mu_{f_k}] + x \sum_{k=j}^{K} (p_k - p_k^*) \ \ \forall j = 1, ..., K.$$

The function $\sum_{k=j}^{K} \triangle[p_k H_k(x)]$ is monotone on the interval $[\max(\overline{s}_1, ..., \overline{s}_K), \infty)$ for all j. Then, checking the two following conditions is necessary and sufficient to verify Jenkins' and Lambert's criterion.

$$\sum_{k=j}^{K} \triangle[p_k H_k(x)] \leq 0 \ \ \forall x \in [0, \max(\overline{s}_1, ..., \overline{s}_K)], \forall j = 1, ..., K, \tag{24a}$$

$$\sum_{k=j}^{K} [p_k^* \mu_{f_k^*} - p_k \mu_{f_k}] + \overline{a} \sum_{k=j}^{K} (p_k - p_k^*) \leq 0 \ \forall j = 2, ..., K. \tag{24b}$$

When $\overline{a} \to \infty$, a necessary and sufficient condition to verify (24b) is the condition (23b).

Then, Moyes' criterion corresponds to Jenkins' and Lambert's one when differences of utility between groups only vanish at the limit.

Assuming $\mathrm{U_M}$ or $\mathrm{U_{JL}}$ in addition to the previous assumptions U1 to U4 does not raise a contradiction, but we prefer to consider an assumption which is more in tune with the previous ones. Indeed, in view of our approach, one can interpret $\mathrm{U_{JL}}$ as meaning that differences of utility between groups of unequal needs may vanish, and $\mathrm{U_M}$ as meaning that they are not bounded above. Considering that the differences of utility due to differential needs are bounded[9], we propose a condition that is somehow intermediate between the two previous ones. The utility functions must be such that there exist income levels for which differences of utility and differences of marginal utility disappear across groups. Like in Jenkins and Lambert, our condition is parameterized by an income limit which, here, is the income limit for a reference group, w.l.o.g. group 1, a_1. Let a_1 be given. We introduce the following assumption:

U5: There exist $a_2, ..., a_K, \overline{V}$ and $\overline{V}^{'}$ such that:
(i) $V(a_k, k) = \overline{V} \ \forall k$,
(ii) $V_y(a_k, k) = \overline{V}^{'} \ \forall k$.

In contrast with $\mathrm{U_{JL}}$ and $\mathrm{U_M}$ the second part of U5 introduces a restriction on marginal utility for income levels equal to the income limits a_k.

[9]Let us recall that the utility functions are defined on $\mathbb{R}_+$. Assumptions on utility functions beyond the support have an impact about welfare comparisons performed over the support.

Composed with assumptions U2 to U4, assumption U5(ii) allows to deduce that $(a_k)_{k=1,...,K}$ satisfy $\alpha_k \leq \frac{a_k}{a_{k-1}} \leq \beta_k$, $k = 2, ..., K$. In view of that restriction, assumption U5(i) states that the differences in the utility level disappear across groups when the income level in each group is just equal to their own income limit provided that they satisfy the chain condition stating that the ratio between two adjacent income limits is included in the interval of admissible equivalence scales. In other words, U5(i) implies that social welfare is invariant to transfers of population across groups of needs at the $(a_k)_{k=1,...,K}$ income levels, while U5(ii) implies that social welfare is invariant to transfers of income at the same income levels. For an illustration, see Figure 4.

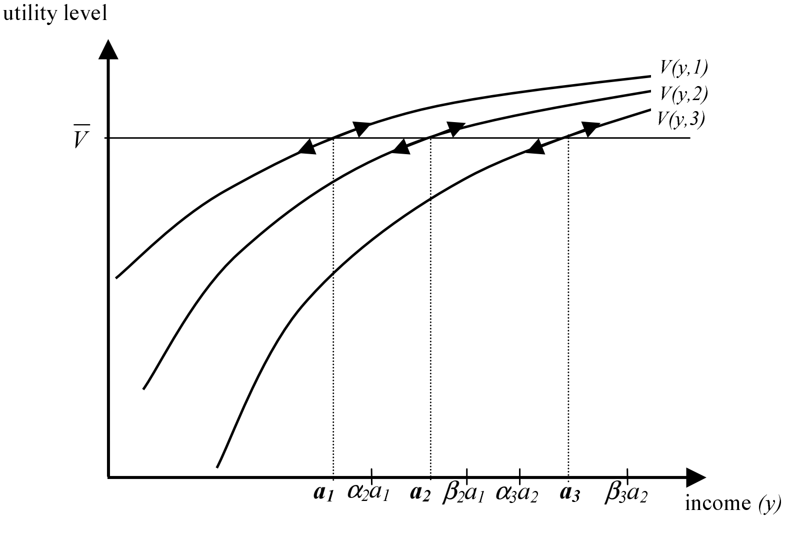

FIG. 4 Implication of assumption U5 in presence of U1 to U4.

Let us compare U5(i) to U_{JL} and U_M. To make the comparison easier, we take the case $\alpha_k = 1$ and $\beta_k = +\infty$ for all $k > 1$. The chain condition is reduced to $a_{k-1} \leq a_k$ for all $k > 1$. In the general case, if U5(i) is satisfied, then U_{JL} cannot be verified for a value of $\overline{a}$ smaller than a_K. However, no relation of inclusion holds between the class of functions satisfying U5(i) and the one satisfying U_{JL}. On the other hand, for unbounded functions, U_M implies U5(i). Indeed, assume that $V(y, k) \geq V(y, k+1)$ for all $y \geq 0$, and V is increasing in y. Then, for all $\bar{V}$, if $(a_k)_{k=1,...,K}$ is such that $V(a_k, k) = \bar{V}$, necessarily $a_1 \leq ... \leq a_K$. In this restricted comparison the class of utility functions considered here is smaller than that considered by Moyes and a more discriminating criterion can be expected as shown in the following theorem.

Denoting $\mathcal{V}(\alpha, \beta, a_1)$ the family of functions $V(y, k)$ satisfying assumptions U1 to U5, we obtain:

THEOREM 5. *Let us assume that* $a_1 \geq \max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K})$, [10]

$$f \ D_{\mathcal{V}(\alpha,\beta,a_1)} \ f^* \tag{A'}$$

$$\Updownarrow$$

$$\sum_{k=1}^{K} \triangle[p_k H_k(x_k)] \leq 0 \quad \forall (x_k)_{k=1,...,K} \text{ such that} \tag{B'}$$

$$0 \leq x_1 \leq a_1,$$

$$\text{and } \alpha_k x_{k-1} \leq x_k \leq \beta_k x_{k-1} \ \ \forall k = 2, ..., K.$$

Proof. This proof is similar to the Theorem 1's one.

Sufficiency : (B') implies (A'). We have to modify the functions $V^n(y,k)$ in comparison to the proof of Theorem 1. Let us take

$$V^n(y,k) = V(y,k) + \frac{a_k}{n}\log(\frac{y}{a_k}+1) - \frac{a_k}{n}\log(2). \tag{25}$$

Since $\triangle[p_k f_k(y)] = 0 \ \forall y \geq \overline{s}_k$ and $a_k \geq \overline{s}_k$, $\forall k$, expression (21) can be written:

$$\triangle W_V = \sum_{k=1}^{K} \int_0^{a_k} \triangle[p_k f_k(y)] V(y,k) dy.$$

When V^n is considered, integrating by parts and using assumption U5(i) give:

$$\triangle W_V^n = \overline{V} \sum_{k=1}^{K} (p_k - p_k^*) - \sum_{k=1}^{K} \int_0^{a_k} V_y^n(y,k) \triangle[p_k F_k(y)] dy.$$

Because $\sum_{k=1}^{K} p_k = \sum_{k=1}^{K} p_k^* = 1$, it follows:

$$\triangle W_V = - \sum_{k=1}^{K} \int_0^{a_k} V_y^n(y,k) \triangle[p_k F_k(y)] dy.$$

In the remaining of the proof, it is enough to replace $p_k \triangle F_k(y)$ by $\triangle[p_k F_k(y)]$, which is equal to $\int_0^y [p_k f_k(z) - p_k^* f_k^*(z)]\, dz$, and to take $b_k = a_k$ $\forall k$, with $a_1 \geq \max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K})$. Notice that the condition $a_k = \varphi_k(a_{k-1})$ is now implied by assumption U5(ii).

Hence, the following condition emerges (instead of condition (9)):

$$\sum_{k=1}^{K} \triangle[p_k H_k(\varphi_k \circ \varphi_{k-1} \circ ... \circ \varphi_2(y))] \leq 0, \text{ for all } y \in [0, a_1],$$

$$\text{and all functions } \varphi_k \text{ such that } \alpha_k y \leq \varphi_k(y) \leq \beta_k y \ \forall k = 2, ..., K.$$

[10] This condition implies that $a_k \geq \overline{s}_k$ $\forall k$ and guarantees that the differences in needs across groups are relevant over the whole support of income distributions. Giving up this assumption would raise some difficulties about the identity of groups of needs.

We end the proof in a similar way, noting that a difference with the Theorem 1 comes from the fact that we cannot avoid to check the condition $\sum_{k=1}^{K} \triangle[p_k H_k(x_k)] \leq 0$, for $x_1 \geq \max(\overline{s}_1, \frac{\overline{s}_2}{\beta_2}, \frac{\overline{s}_3}{\beta_2\beta_3}, ..., \frac{\overline{s}_K}{\beta_2\beta_3...\beta_K})$. Indeed, the functions $\triangle[p_k H_k]$ are not constant beyond $\overline{s}_k$.

Necessity: (A') implies (B'). Suppose that $f \; D_{\mathcal{V}(\alpha,\beta,a_1)} f^*$ and there exists a K-vector $(e_1, e_2, ..., e_K)$ such that

$$0 \leq e_k \leq a_k \text{ for all } k = 1, ..., K, \tag{26a}$$

$$\alpha_k e_{k-1} \leq e_k \leq \beta_k e_{k-1} \text{ for all } k = 2, ..., K, \tag{26b}$$

$$\text{and } \sum_{k=1}^{K} \triangle[p_k H_k(e_k)] > 0. \tag{26c}$$

The proof is exactly the same than the Theorem 1's one. To verify U1 to U5, we consider the function $U_0(x)$ which satisfies expression (12) and is equal to 0 for $x \geq \varepsilon$. Setting $b_k = a_k$, the property $e_k \leq b_k$ is now satisfied by assumption (26a). ∎

As a byproduct, this result provides an extension of Bourguignon's criterion to the case of different distributions of needs.

Contrary to the case where the population composition does not vary, the functions $\triangle[p_k H_k]$ are not constant beyond the upper bound of the support of Δf_k. Consequently, the dominance criterion defined in Theorem 5 is dependent on the value of a_1, which can be interpreted as the income limit chosen by the decision maker for the reference group. This may deserve a more detailed explanation.

As for the Jenkins and Lambert's criterion, the condition (B') can be rewritten as the sum of two terms when $x_1 \geq \max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K})$:

$$\sum_{k=1}^{K} \triangle[p_k H_k(x_k)] = \sum_{k=1}^{K} [p_k^* \mu_{f_k^*} - p_k \mu_{f_k}] + x_1 \sum_{k=1}^{K} (p_k - p_k^*) \prod_{l=1}^{k} \gamma_l.$$

with $\gamma_1 = 1$ and $\alpha_l \leq \gamma_l \leq \beta_l$, $l = 2, ..., K$. The γ_l's are nothing else that the income ratio of individuals belonging to two adjacent groups.

One can see that, for a given vector $(\gamma_k)_{k=1,...,K}$, the above function is monotone on the interval $[\max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K}), \infty)$. Therefore, checking our criterion is equivalent to checking the two following conditions.

$$\sum_{k=1}^{K} \triangle[p_k H_k(x_k)] \leq 0 \ \ \forall (x_k)_{k=1,...,K} \text{ such that} \tag{27a}$$
$$x_1 \leq \max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K}),$$
$$\text{and } \alpha_k x_{k-1} \leq x_k \leq \beta_k x_{k-1} \ \ \forall k = 2, ..., K,$$
$$\sum_{k=1}^{K} [p_k^* \mu_{f_k^*} - p_k \mu_{f_k}] + a_1 \sum_{k=1}^{K} (p_k - p_k^*) \prod_{l=1}^{k} \gamma_l \leq 0 \ \forall (\gamma_l)_{l=1,...,K} \tag{27b}$$
$$\text{such that } \gamma_1 = 1 \text{ and } \alpha_l \leq \gamma_l \leq \beta_l \ \forall l = 2, ..., K.$$

The first term only depends on the distributions of income, whereas the second term is purely demographic, and overrides the first one when the a_1 is large enough. This can be understood by the fact that when the a_1 grows large, assumption $\mathrm{U5}_{(i)}$ is less and less restrictive about the differences in utility levels between groups over values of incomes within the support. In application, the most discriminating criterion corresponds to minimal value of income income limit for the reference group, that is $\max(\overline{s}_1, \frac{\overline{s}_2}{\alpha_2}, \frac{\overline{s}_3}{\alpha_2\alpha_3}, ..., \frac{\overline{s}_K}{\alpha_2\alpha_3...\alpha_K})$.

In the limit case, when a_1 goes to infinity, the condition (27b) is reduced to the following one:

$$\sum_{k=1}^{K} (p_k - p_k^*) \prod_{l=1}^{k} \gamma_l \leq 0 \ \forall (\gamma_l)_{l=1,...,K} \text{ such that} \tag{28}$$
$$\gamma_1 = 1 \text{ and } \alpha_l \leq \gamma_l \leq \beta_l \ \forall l = 2, ..., K.$$

Comparing with the Moyes' condition (23b) is only meaningful in the case where $\mathrm{U_M}$ implies U5(i) ($\alpha_k = 1$ and $\beta_k = +\infty$ for all $k > 1$). Then, in this very particular case, it can be established that the above condition is equivalent to Moyes'one.

For applications, note that the algorithm presented in Theorem 2 is immediately adapted to the present setting, by replacing $p_k \triangle H_k$ by $\triangle[p_k H_k]$.

6. CONCLUSION

The paper has considered the problem of comparing income distributions for heterogeneous populations. Following Atkinson and Bourguignon [3], we have divided the population in different groups of needs and evaluated the social welfare with a utilitarian function. By introducing principles which bound the social marginal value of income for a group of need with respect to that value for a different group of need, we have found an implementable condition of dominance which allows to rank more distributions than the Bourguignon criteria [6]. Furthermore, we have shown that this condition amounts to applying the one-dimensional dominance criterion on equivalent income distributions by considering a social welfare function weighted by equivalence scales, these not being given but belonging to intervals. Finally, we have extended our results to the case where distributions of needs differ between the two populations being compared. In particular in tune with our framework, we make use of a condition which bounds the difference of utility levels across groups.

We have supposed in the paper that there is no ambiguity about the ranking of groups with respect to their needs. But in applications, if there is a doubt about the ranking, one only has to perform the dominance analysis for all potential rankings.

Our criterion degenerates to Bourguignon's one [6] when we consider unbounded equivalence scales. A further investigation would be to start again all the analysis of this paper in order to have the dominance criterion obtained degenerating to Atkinson and Bourguignon's one [3] at the limit. But it cannot be done simply by combining U1, U2, U3, U4 and U_{AB} because the family of utility functions satisfying these assumptions is degenerate and has equal marginal utilities across groups of needs.[11]

In a companion paper [17], we show that the discriminating power of our criterion is much greater than both Bourguignon's criterion and Atkinson's and Bourguignon's one, on actual data about the French income distribution.

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