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Voting under ignorance of job skills of unemployed: the overtaxation bias

Jean-François Laslier[a], Alain Trannoy[b], Karine Van Der Straeten[c,*]

[a]CNRS and THEMA, Laboratoire d'Econométrie de l'Ecole Polytechnique, 1 Rue Descartes, 75005 Paris, France

[b]THEMA, Univ. de Cergy-Pontoise, 33, Boulevard de Port, 95 011 Cergy-Pontaise Cedex, France

[c]THEMA and DELTA, 48 Boulevard Jourdan, 75014 Paris, France

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Abstract

Usual models on voting over basic income–flat tax schedules rest on the assumption that voters know the whole distribution of skills even if at equilibrium some individuals do not work. If individuals' productivity remains unknown until they work, it may be more convincing to assume that voters have only beliefs about the distribution of skills and that a learning process takes place. In this paper, at each period, individuals vote according to their beliefs which are updated when getting new information from the job market. The voting process converges towards some steady-state equilibrium that depends on both the true distribution of skills and the initial beliefs. The equilibrium tax rate is higher than (or equal to) the tax rate achieved in the perfect information framework. An illustration is provided on French data: if voters are over-pessimistic as to the potential productivity of unemployed people, majority voting may lock the economy in an ''informational trap'' with a high tax rate and a high level of inactivity.

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1. Introduction

The debate on the basic income–flat tax proposal remains topical in public finance as Atkinson's (1995) book testifies. Understanding the equilibrium of

*Corresponding author.

E-mail address: karine.vanderstraeten@eco.u-cergy.fr (K. Van Der Straeten).

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political forces which can prevent or support this kind of proposition remains a key issue. Romer's seminal article (1975) tackled this problem, in the case of a political equilibrium given by the choice of the median voter. Roberts (1977) provides a generalization of the model, while Meltzer and Richard (1981) show how the model can provide a fruitful explanation of the growth of government.

This model, like many others in public economics since Mirrlees's pathbreaking article (1971), is based on a distinction between public and private information. A common assumption shared by this voting model and by optimal taxation ones is that the true distribution of skills is common knowledge, while the individual's labor productivity is not observable[1]. Where does this public knowledge of the skills distribution come from? In optimal taxation models, one can defend the idea that the government has its own investigation instruments such as surveys, polls and sophisticated econometric methods. This explanation is much less satisfactory in voting models, since it is assumed that all voters know the true distribution of skills. Indeed, one can hardly maintain the view that voters are endowed with such elaborated methods or are willing to spend so much time gathering information about the distribution of skills. At best, it seems sensible to assume that voters are ready to use the information directly available from the job market, and in particular the distribution of labor incomes. Yet, deducing the distribution of skills from the distribution of labor incomes may prove to be difficult, and even sometimes impossible, specifically where bunching is present. Bunching means that people of different types choose the same level of labor market income given the tax system. The simplest case of bunching is of course when some people do not work. As a matter of fact, the fiscal history may have been such that some individuals have always chosen not to work due to a poverty trap phenomenon or other reasons, so that no information about their true productivity is available. At best, and under some additional hypothesis, one can assume that the productivity of the unemployed is lower than the lowest productivity observed among employed workers.

The basic motivation for this paper is to introduce the unobservability of the skills distribution of unemployed individuals as a basic ingredient of voting models over taxation schemes. One can even defend the view that one of the roots of the argument about the level of the basic income comes from the unobservability of the productivity of the unemployed. Different beliefs about the bottom tail of the skill distribution may indeed open the door to marked differences, when evaluating the consequences of a given tax schedule. For instance, the incentive argument for lower tax rates rests on the evaluation of the productivity of the unemployed.

Given this fundamental and inherent unobservability of the unemployed's

[1]The assumption is made explicit in optimal taxation models. In voting models, the assumption is much less conspicuous. Yet, it is somehow implicit since the tax basis is the labor income (which induces distorting effects) and not the wage.

productivity, it seems more convincing to assume that individuals only have beliefs about the distribution of skills at the starting point, while they are ready to revise their beliefs as long as new pieces of information coming from the job market are available. As early emphasized by Downs (1957), beliefs and uncertainty are at the heart of the political institution and debate. The paper explores this idea further in the classical median voter model when the question at issue is the choice of a simple redistribution scheme.

Voting occurs repeatedly as in a regular democracy and the sequence of votes can figure the long period needed for the fiscal parameters to converge toward their steady state values (if any). At each period individuals vote on the tax rate to be adopted, according to their current beliefs. The policy selected by the majority rule is implemented and individuals take their labor decisions. They observe what happens on the labor market and update their beliefs. Assuming a quasi-linear utility function, voters are able to learn the type of all working individuals[2].

We show that this dynamic process converges towards some steady state equilibrium (in a sense that will be made precise below). The steady state depends of course on the true distribution, but also on the initial beliefs. The model may provide a theoretical explanation why, according to Perotti (1996), ''there appears to be [little] empirical support for explanations based on the effects of income distribution on fiscal policy''[3]. In particular, some countries with very similar distributions of wages choose quite different tax schedules. If these countries differ with respect to their initial beliefs about the distribution of skills among the unemployed, our model predicts that the eventually chosen tax schedules may also be quite different. This ignorance of the persistent role of initial beliefs may be one among the presumably numerous explanations why the predictions made by standard voting models are not as good as it could be expected.

Secondly, it is shown that, under this kind of imperfect information, the tax rate at equilibrium is always higher than (or equal to) the tax rate which would be chosen if all the parameters of the economy were known. The economic intuition supporting this result is quite simple. If we assume that individuals' productivity is discovered by observing the labor market, highest productivities are more easily discovered. So, at equilibrium, there is no way to overvalue the number of highly productive individuals whereas there is some chance of overestimating the thickness of the lower tail of the productivity distribution. If voters are over-pessimistic as to the potential productivity of unemployed people, majority voting may lock the economy in an ''informational trap'' with a high tax rate and a high level of unemployment.

Section 2 of the paper describes our model. Section 3 proves the existence of a

[2]In other words our model precludes the possibility of bunching for individuals choosing to work and only portrays the difficulty to observe the productivity of the unemployed.

[3](page 149). For a survey about empirical tests of the median voter model see Persson and Tabellini (1999).

political equilibrium at each period of the process. Section 4 establishes the overtaxation bias introduced by the imperfect information setting. Is this bias more than a theoretical curiosity? Section 5 provides an illustration of the possibility of the bias on French data. Section 6 offers some concluding comments. Some proofs are relegated to the appendix.

2. The model

Our model is similar to Romer (1975) and Roberts (1977) except for the information structure and the dynamics. Dates are numbered $\tau = 0,1,2\ldots$ and the economy is populated by n individuals living forever. The number of individuals is supposed to be large enough for individuals to neglect the influence of their private decisions on the macroeconomic variables. They divide up into K productivity types[4]. An individual of type k is characterized by his (exogenous) productivity, θ_k, with $0 \leq \theta_1 < \theta_2, \ldots, \theta_{K-1} < \theta_K \leq 1$. The θ_k's give the relative effectiveness of the labor supplied per unit of time, so a high θ_k household is more effective in production. Since we assume perfect competition on the labor market, the individual productivity is also his gross wage rate (see Eq. 1). The ''true'' number of individuals of type k is denoted n_k^* with $\Sigma_{k=1}^{k=K} n_k^* = n$ and the ''true'' distribution of types is $\omega^* = (n_1^*, n_2^*, \ldots, n_K^*)$. All the parameters of the economy are common knowledge except ω^*. At the beginning of the story, the individuals know the value of n and the division of society into K productivity types but they ignore how individuals divide up into the different types. More precisely, all individuals share the same prior beliefs at date 0 about the distribution of types[5]. In order to describe these beliefs, we introduce Ω the set of all possible ''states of the world'', here the possible distributions of types:

$$\Omega = \left\{ \omega = (n_1, n_2, \ldots, n_K) \in N^K : \sum_{k=1}^{k=K} n_k = n \right\}.$$

To each possible state of the world $\omega \in \Omega$ is associated its prior probability at date 0, $\pi_\omega^0 \geq 0$, with $\Sigma_{\omega \in \Omega} \pi_\omega^0 = 1$. We make the assumption that $\pi_{\omega^*}^0 > 0$, which states that a strictly positive probability is assigned to the true distribution of types[6].

At time τ, individuals derive positive utility from private consumption at time τ

[4] The choice of a continuum of types raises the technical problem of the computation of the revision of beliefs according to Bayes's rule.

[5] This is an obvious limitation of the model. Allowing diverse beliefs adds a second source of heterogeneity among individuals which raises a difficulty for the existence of political equilibrium at each period.

[6] This assumption assures that Bayesian updating of beliefs is always possible.

(C^{τ}), and negative utility from hours spent working at time τ (L^{τ}). Preferences are summarized by a snapshot utility function which is supposed to be quasi-linear in consumption:

$$U^{\tau}(C^{\tau},L^{\tau}) = C^{\tau} - v(L^{\tau}).$$

Since this snapshot utility function is constant through time, there is no ambiguity in omitting the superscripts. The available time is normalized to 1 so that $0 \leq L \leq 1$. When type k individual works for L hours, he earns a gross income:

$$y = \theta_k L. \tag{1}$$

The function v is continuously differentiable, increasing and convex; we assume that $v(0) = 0$. Quasi-linear preferences imply that all income effects are absorbed by consumption. Obviously, this is a noticeable restriction although the econometric estimations at our disposal reveal rather weak values of the income elasticity of the labor supply (see for instance Blundell et al. (1998) or Blundell and McCurdy (1998)). Another reason to favor the quasi-linearity assumption will emerge further on (see claim 1 and the subsequent comment).

The description of the economy makes clear that there is only one source of heterogeneity among the population. This heterogeneity can be interpreted in another way. Rather than assuming that individuals differ in their productivity, they might instead differ in their marginal disutility of labor. If we set $\theta_k = \theta$ and $v_k(L) = h(\lambda_k,L)$ for all k, where λ_k is a parameter describing differences across types in the marginal labor disutility and provided that $\partial^2 h/(\partial\lambda\ \partial L) < 0$, the analysis can then be pursued in the same way, leading to exactly the same results, with λ_k playing a similar role to θ_k. Nevertheless in the following we will stick to the productivity interpretation.

The job market is the only source of new information. The distribution of labor incomes is common knowledge. It will be shown that two different types do not choose the same positive labor income. Hence as soon as individuals of some type choose to work, their number is known to everybody. When receiving new information from the job market, individuals update their beliefs according to Bayes' rule, getting posterior beliefs.

Basic income–flat tax schemes, with b the (positive) basic income and t (between 0 and 1) the constant marginal tax rate on labor incomes, are the only available tax schemes. In order for the government budget to be currently balanced, the amount of social benefits distributed at date τ must equal tax revenues collected at that date. More precisely the government respects the following schedule: first the tax rate to be levied is decided, then the tax revenues are collected and finally the payment of a basic income to everyone in the population is organized. Doing so makes sure that the government budget constraint is satisfied ex post. The tax rate at each period is determined by majority voting. The vote selects, if any, the ‘‘Condorcet winner tax’’, namely the tax rate that obtains at least 50% of the votes in any pairwise comparison.

It is important to keep in mind the precise timing which occurs during a given period:

1. According to their prior beliefs, everyone votes on the tax rate on the basis of the resulting level of expected current utility. Uncertainty affects individuals’ utility through the level of the basic income; indeed, for a given tax rate, the value of the basic income depends on the aggregated labor supply, which in turn depends on the distribution of types among individuals.
2. The ‘‘Condorcet winner tax’’ is announced.
3. Individuals choose their labor supply. Some types choose to work, and some others choose not to work.
4. The taxes on labor incomes are collected, and the budget is uniformly distributed among individuals.
5. The number of individuals in working types is learned, and voters update their beliefs according to Bayes’ rule, leading to posterior beliefs which become the prior beliefs the next period.

This myopic scenario is repeated at each date. Beliefs make the only link between two subsequent periods. The information provided by the job market at each date governs the dynamics of the current equilibrium.

3. The current equilibrium

3.1. The set of feasible basic incomes

The voters are assumed to perceive correctly the relation between tax rates and government revenue, anticipating the reactions of taxpayers to fiscal schemes. This anticipation is here a little bit more complicated than when voters know the true distribution of types. They have to follow a backward induction argument inspired by steps 3 and 4 above. Individuals know that the government budget constraint has to be satisfied ex post, i.e., for each state of the world, the basic income budget must be equal to tax revenues. In the state of the world $\omega = (n_1, n_2, \ldots, n_K) \in \Omega$, for a given tax rate t, the government budget constraint is:

$$nb_\omega = t \sum_{k=1}^{k=K} n_k \theta_k L_k(t, \pi), \tag{2}$$

where b_ω is the basic income amount in state ω, π the prior beliefs of the current period and $L_k(t,\pi)$ the labor supply for type k. In this writing we take into account the fact that the labor supplies have to be chosen as soon as the tax rate t is announced and so are independent from the state of the world, but can depend on beliefs.

Consider now the determination of the labor supplies.

Claim 1. *Thanks to the quasi-linear utility assumption, the labor supplies do not depend on individuals' beliefs concerning the various states of the world. Uncertainty affects individuals' votes but not their economic decision of labor supply.*

This claim makes the quasi-linearity assumption more sensible in this particular context. This assumption is reasonable if one thinks that beliefs about a macroeconomic parameter have a more important impact on political decisions than on economic ones, at least on those regarding the labor market.

Writing the optimization program of the individual will make transparent the argument supporting this claim. Like the government, individuals have to balance their budget in each state of the world. The number of individuals is supposed to be large enough so that individuals neglect the influence of their private decisions on the macroeconomic variables. Thus they consider the value of the basic income in each state of the world as given. Type k individuals' budget constraint in state $\omega \in \Omega$ is given by:

$$C_{k,\omega} = b_\omega + (1-t)\theta_k L_k,$$

where $C_{k,\omega}$ denotes type k individual's consumption in the state ω. The current utility to be considered is the expected current utility function under π. Therefore the optimal labor supply solves the maximization problem:

$$\max_{0 \le L_k \le 1} \sum_{\omega \in \Omega} \pi_\omega (C_{k,\omega} - v(L_k))$$

subject to

$$C_{k,\omega} = b_\omega + (1-t)\theta_k L_k, \quad \forall \omega \in \Omega$$

which can be written:

$$\max_{0 \le L_k \le 1} \sum_{\omega \in \Omega} \pi_\omega (b_\omega + (1-t)\theta_k L_k - v(L_k)),$$

or:

$$\max_{0 \le L_k \le 1} \left[\left(\sum_{\omega \in \Omega} \pi_\omega b_\omega \right) + (1-t)\theta_k L_k - v(L_k) \right].$$

Uncertainty only results in an additive constant $(\Sigma_{\omega \in \Omega} \pi_\omega b_\omega)$ on individuals' objective function and so it does not affect their labor supply decisions. The first order condition gives[7]: $(1-t)\theta_k - v'(L_k) = 0$, which leads to:

$$L_k(t) = \max(0,(v')^{-1}((1-t)\theta_k),1). \quad (3)$$

Exploiting (3) leads to the second claim.

Claim 2. *The labor supply is a weakly increasing function of the productivity and a weakly decreasing function of the tax rate. Besides, there is no bunching for working types:*[8] *when the gross labor income is strictly positive, it is a strictly increasing function of the productivity.*

In other words gross incomes, which will be denoted by $y_k = \theta_k L_k$, are ordered by productivity. Hence the knowledge of the distribution of gross incomes allows everyone in the economy to infer the true type of employed individuals.

Moreover for each type, there is a critical tax rate, t_k, such that, if the chosen tax rate is less than t_k, individuals of type k choose to work[9] and if it is higher than or equal to t_k, they choose not to work. One may easily check that:

$$t_k = \max\left(0,1 - \frac{v'(0)}{\theta_k}\right). \quad (4)$$

If the critical tax rate is null, type k individuals will never work, whatever the tax rate may be. If the critical tax rate is positive, it is strictly increasing with productivity. It means that for a given tax rate, if individuals of some type choose to work (resp. choose not to work), then so do all individuals with higher types (resp. with lower types). Since there is no income effect, individuals who choose not to work do so because of the level of the tax rate and not because of the level of the basic income.

Substituting (3) into (2) gives the relation between the tax rate and the basic income in each state of the world:

$$b_\omega(t) = t\sum_{k=1}^{k=K}\frac{n_k}{n}\theta_k \max(0,(v')^{-1}((1-t)\theta_k),1) \qquad \forall \omega \in \Omega.$$

3.2. Voters' preferred tax rate

All individuals know that the government will balance its budget ex post; for each possible state of the world $\omega \in \Omega$, type k individual has an indirect utility function depending only on t:

[7] As the function v is convex, the second order condition is also satisfied.

[8] Remember that bunching means that two different types choose the same level of gross labor income.

[9] We say that an individual chooses to work if and only if he has a strictly positive labor supply.

$$V_k(\omega,t) = U(b_\omega(t) + (1-t)\theta_k L_k(t), L_k(t)) = b_\omega(t) + (1-t)\theta_k L_k(t) - v(L_k(t)).$$

The expected indirect utility function of a type k individual is then:

$$E_\pi V_k(t) = E_\pi b(t) + (1-t)\theta_k L_k(t) - v(L_k(t)), \tag{5}$$

where $E_\pi b(t) = \Sigma_{\omega\in\Omega}\pi_\omega b_\omega(t)$ is the expected value of the basic income when the tax rate is t and the beliefs are π.

Following Meltzer and Richard (1981) which give a simple and illuminating formula for the type k individual's preferred tax rate in the perfect information case, we derive the analog in our case. Let us denote by $t(k,\pi)$ type k's preferred tax rate when beliefs are π [10]:

$$t(k,\pi) = \mathrm{argmax}_{t\in[0,1]} E_\pi V_k(t). \tag{6}$$

Exploiting the fact that the preferred tax rate $t(k,\pi)$ is implicitly defined by the first order condition[11], and using the envelope theorem, one gets:

$$0 = \frac{\mathrm{d}E_\pi V_k(t)}{\mathrm{d}t} = \frac{\mathrm{d}E_\pi b(t)}{\mathrm{d}t} - \theta_k L_k(t). \tag{7}$$

Besides, by definition of the expected basic income,

$$\frac{\mathrm{d}E_\pi b(t)}{\mathrm{d}t} = \sum_{j=1}^{j=K}\frac{E_\pi n_j}{n}\theta_j L_j(t) + t\sum_{j=1}^{j=K}\frac{E_\pi n_j}{n}\theta_j\frac{\mathrm{d}L_j(t)}{\mathrm{d}t}. \tag{8}$$

Let us denote by $E_\pi\bar{y}(t)$ the expected average gross income: $E_\pi\bar{y}(t) = \Sigma_{j=1}^{j=K}(E_\pi n_j)/n\ \theta_j L_j(t)$.

Combining (7) and (8) one gets:

$$[E_\pi\bar{y}(t(k,\pi)) - y_k(t(k,\pi))] + t(k,\pi)\frac{\mathrm{d}E_\pi\bar{y}(t(k,\pi))}{\mathrm{d}t} = 0. \tag{9}$$

Eq. (9) defines implicitly the voter k's preferred tax rate when beliefs are π. It is similar to the formula popularized by Meltzer and Richard but the average gross income and its variation to the tax rate are taken in expectation. Consider the left hand side of Eq. (9). $E_\pi\bar{y}(t(k,\pi)$ is the expected marginal benefit of a higher tax rate keeping constant labor supplies. The difference $E_\pi\bar{y}(t(k,\pi)) - y_k(t(k,\pi))$ is positive for a voter poorer than the expected average and negative otherwise. The term $t\ \mathrm{d}E_\pi\bar{y}/\mathrm{d}t$ is the expected marginal cost of higher taxes, inducing a decrease in the labor supply. This term is always negative. In particular, voters whose gross

[10] If an individual happens to have several preferred tax rates, which will generically not occur, $t(k,\pi)$ denotes the lowest one.

[11] at least when it is strictly positive and different from a critical tax rate.

income is greater than (or equal to) the expected average one will favor a zero tax rate.

3.3. Majority voting equilibrium

The tax rate t is determined by majority voting. Given beliefs π and two tax rates $t, t' \in [0,1]$, with $t' > t$, type k individuals vote in favor of t rather than t' if and only if $E_\pi V_k(t) \geq E_\pi V_k(t')$[12]. The outcome selected by the voting process is the Condorcet winner (if any), that is the tax rate which gets at least 50% of the vote in any pairwise comparison. It is well known that a Condorcet winner often fails to exist, unless voters' preferences satisfy certain conditions.

The most familiar of these conditions is the ''Single Peaked Preferences Condition'', which dates back to Black (1958). Yet, the family of indirect utility functions $t \mapsto E_\pi V_k(t)$ for $k = 1,2, \ldots ,K$ does not, in general, satisfy this condition, although preferences over consumption and leisure are well-behaved (see Romer, 1977 and Roberts, 1977)[13].

When this Single Peaked Preferences Condition fails to be satisfied, a weaker condition, namely the ''Single Crossing Condition''[14] is also sufficient to guarantee the existence of a Condorcet winner. In the present case, this condition states that when an individual faces a choice between two tax rates, if he chooses the lower one, then more productive individuals also choose the lower tax rate when facing the same choice.

We now show that the family of expected indirect utility functions

$$\left(E_\pi V_k : \begin{array}{l} [0,1] \to R \\ t \mapsto E_\pi V_k(t) \end{array} \right)_{k=1,2, \ldots K}$$

defined in (5) satisfies this Single Crossing Condition.

[12]If an individual happens to be indifferent between two tax rates (which generically does not occur), we assume w.l.o.g. that he votes in favor of the lowest one. For the convergence result to hold, and in order to perform the comparison between perfect and imperfect information, it is important that the median voter, should he have the choice, always picks up the same alternative. But it does not matter whether he chooses the lowest rather than the highest alternative.

[13]Consider the following example: starting from a low level of tax, an individual may dislike a small increase in the marginal tax rate because the expected increase in the lump-sum subsidy may not be sufficient to compensate the loss in his after tax labor income. However, if this individual has a rather low productivity, further increase in the tax rate may cause him to stop working, while still having some positive effect on the lump-sum subsidy. Once he stops working, the individual agrees with further increase in the tax rate, as long as this increase results in a higher lump-sum. When a tax increase will eventually have a negative impact on the basic income (which will occur since with a tax rate of 100%, nobody chooses to work) the individual will dislike further increase in the tax rate. The individual depicted in this example will have two-peak preferences.

[14]This condition is also sometimes referred to as the ''Spence–Mirrlees Condition''.

A geometrical interpretation of the Single Crossing Condition is useful to get the intuition of the result. Consider the family of two variable functions:

$$\left(W_k\colon \begin{array}{l} [0,1] \times \mathbb{R} \to \mathbb{R} \\ (t,b) \mapsto W_k(t,b) = b + (1-t)\theta_k L_k(t) L_k(t) - v(L_k(t)) \end{array} \right)_{k=1,2,\ldots,K}. \tag{10}$$

At any given tax schedule (t,b), the slopes of the indifference curves of various individuals for the functions W_k are ordered according to individuals' productivities. Indeed, consider two individuals with different productivities $\theta_{k'} > \theta_k$ (as in Fig. 1). The utility loss associated with a marginal increase of the tax rate is higher for the more productive individual k', since his gross income is higher. Thus, the marginal increase of the basic income required to compensate the increase of the tax rate is higher, meaning that the productive individual indifference curve is steeper. Fig. 1 gives an illustration of a pair of expected indirect utility functions which ''single cross''.

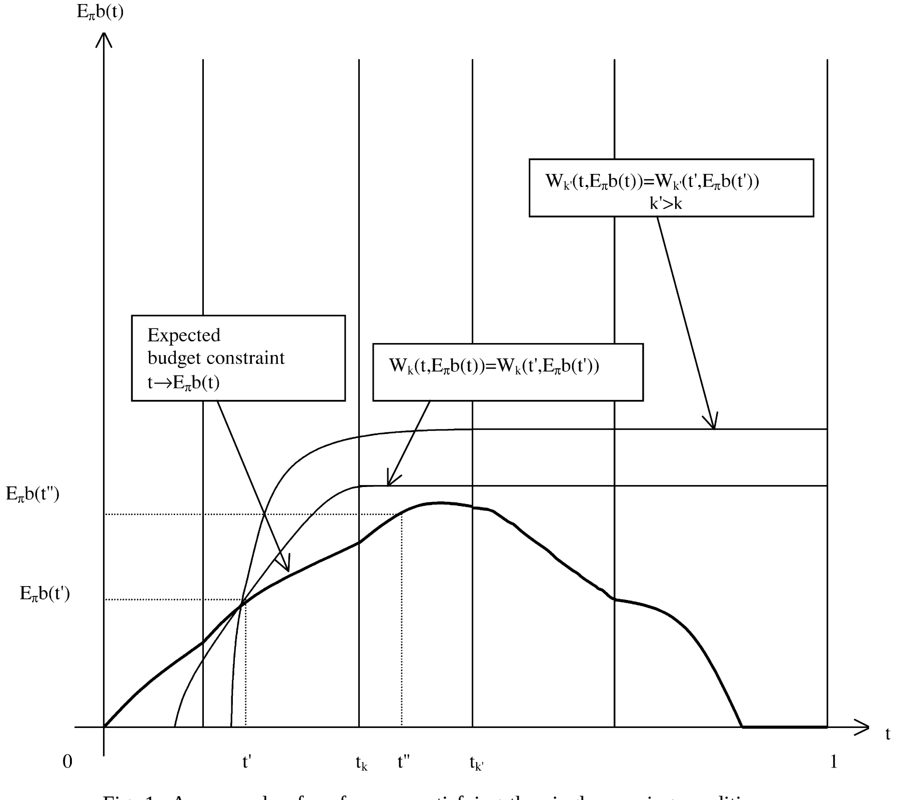

Fig. 1. An example of preferences satisfying the single crossing condition.

A direct consequence from this geometrical property is that if the θ_k-productivity individual prefers tax rate t' to tax rate $t'' > t'$ (as in Fig. 1), the $\theta_{k'}$-productivity individual also prefers tax rate t' to tax rate t''. Indeed, his indifference curve passing through point $(t', E_\pi b(t'))$ is located "above" that of type k individual passing through this same point, for all tax rates higher than t'. This shows that the family of utility functions $(E_\pi V_k)_{k=1,2,...K}$ satisfies this Single Crossing Condition.

We can now establish the existence of a Condorcet winner tax. We first recall the definition of the median type.

Definition 1. We call the *median type* and we denote by m the integer:

$$m = \max\left(k: \sum_{j=k}^{j=K} n_j^* \geq \frac{n}{2}\right).$$

Proposition 1. *The median type's preferred tax rate is a Condorcet winner tax.*

This proposition follows from the Single Crossing Condition. Consider the median voter's most preferred tax rate $t(m,\pi)$ (see notation in (6)). By definition, type m individuals prefer this tax rate to any higher tax rate. Since individuals' preferences satisfy the Single Crossing Condition, individuals with type higher than m also prefer $t(m,\pi)$ to any higher tax rate. By definition of the median type, it represents at least 50% of the population, thus $t(m,\pi)$ gets at least 50% of the vote in any pairwise comparison with a higher tax rate. A similar argument can be given concerning pairwise comparisons with lower tax rates. All formal definitions and proofs are to be found in Appendix A.

The use of the Single Crossing Condition to prove the existence of a Condorcet winner in a model of vote over two parameters (here the tax rate and the basic income) which can be linked by a constraint (here the government budget constraint) dates back to Roberts (1977). Roberts' intuition was then formalized and generalized by Milgrom (1994), Milgrom and Shannon (1994) and Gans and Smart (1996). The same argument is used to solve voting equilibria problems, as in Epple and Romer (1991), Epple and Romano (1996a,b) or Roemer (1997).

A simple application of formula 9 shows that the median voter's preferred policy when beliefs are π is implicitly defined by:

$$t(m,\pi) = \frac{y_m(t(m,\pi)) - E_\pi \bar{y}(t(m,\pi))}{\frac{\mathrm{d}E_\pi \bar{y}(t(m,\pi))}{\mathrm{d}t}}. \tag{11}$$

If the median voter is such that his gross income is smaller or equal to the expected average one, he will favor a positive tax rate. Note that the identity of the median type does not depend on voters' beliefs about the state of the economy but solely on the true distribution of types. This guarantees that the median type remains the same all through the dynamic path.

3.4. Special case: the economy under perfect information

The economy under perfect information, which has been studied by Romer (1975, 1977) and Roberts (1977), appears as a special case of the model described above. More precisely, the perfect information framework is equivalent to the case where the prior probability is a unit mass on the true state of the world. Formally, the perfect information case is the case where beliefs are π^*, with $\pi^*_{\omega^*} = 1$ and $\pi^*_\omega = 0, \forall \omega \in \Omega, \omega \neq \omega^*$. We denote by $t^* = t(m,\pi^*)$ the median voter's preferred tax rate under perfect information, which is the Condorcet winner tax. Note that the median type is the same as in the context of imperfect information.

4. Dynamics and steady state

The dynamics is described by the sequence $(\pi^\tau, t^\tau)_{\tau \geq 0}$ with π^τ the prior probability distribution at date τ and t^τ the outcome of the vote on the tax rate organized at date τ.

The sequence is obtained through an induction argument. Suppose that π^τ is given. As shown in the sub-section 3.3, the tax rate decided at date τ is the median type's preferred tax rate $t^\tau = t(m, \pi^\tau)$.

All the individuals of type k with a critical tax rate t_k such that $t_k > t^\tau$ choose to work, all the individuals of type k such that $t_k \leq t^\tau$ choose not to work (see (4) for the definition of the critical tax rates). Since these critical tax rates decrease with productivity, there exists a type $k^{\text{work}}(\tau)$ such that working individuals at period τ are precisely individuals of type $k \geq k^{\text{work}}(\tau)$.

Claim 2 makes transparent that, as soon as individuals of some type choose to work, their true proportion in the economy is immediately known to everybody. Therefore the number of individuals with type k is common knowledge at period τ if there exists a period $\tau' \leq \tau$ such that $k \geq k^{\text{work}}(\tau')$. Let $k^{\min}(\tau) = min_{\tau' \leq \tau}\left(k^{\text{work}}(\tau')\right)$, this is the lowest productivity type which has ever been observed on the job market in the past, up to period τ. The information set at date τ can then be defined as:

$$I(\tau) = \{\omega = (n_1, n_2, \ldots, n_K) \in \Omega : \forall k \geq k^{\min}(\tau), n_k = n^*_k\}.$$

Thus at the beginning of period $\tau + 1$, the beliefs are the conditional probabilities:

$$\pi^{\tau+1} = \pi^\tau | I(\tau).$$

If $I(\tau) = I(\tau - 1)$, there is no need to update the beliefs. On the opposite, new information is available if individuals who have never worked before enter the job market at period τ, namely $k^{\min}(\tau) < k^{\min}(\tau - 1)$. Then the updating of beliefs goes as follows. If a state of the world is such that proportions are wrong for types for which the true information has been obtained, then the probability of this state becomes zero. The probability is increased by a common multiplicative factor for the other states of the world. Formally if the state of the world $\omega = (n_1,n_2, \dots ,n_K) \in \Omega$ is such that there exists some type $k \in \{k^{\min}(\tau),k^{\min}(\tau) + 1, \dots ,K\}$ such that $n_k \neq n_k^*$, then $\pi_\omega^{\tau+1} = 0$, while if the state of the world $\omega = (n_1,n_2, \dots ,n_K) \in \Omega$ is such that $\forall k \in \{k^{\min}(\tau),k^{\min}(\tau) + 1, \dots ,K\},n_k = n_k^*$, then $\pi_\omega^{\tau+1} = A\pi_\omega^\tau$, where A is the positive number such that $\Sigma_{\omega \in \Omega}\pi_\omega^{\tau+1} = 1$. More precisely, Bayesian updating yields:

$$\frac{1}{A} = \sum \left\{ \pi^\tau_{\omega=(n_1,n_2, \dots ,n_K)} : n_k = n_k^* \right\},$$

which induces a value $A > 1$.

4.1. The steady state

We now show that the dynamics of current equilibria of the successive beliefs and tax rates converges to a unique steady state equilibrium. Since the only source of change occurring between two periods concerns the beliefs, a steady state equilibrium will be reached when voters do not change their minds about the distribution of productivities, once they have observed the distribution of labor incomes generated by the current tax rate. In other words, when beliefs become stationary, the current equilibrium does so.

Definition 2. We call a *steady state equilibrium* a pair (π^∞,t^∞) which satisfies:

1. when the beliefs are π^∞, the tax rate t^∞ is supported by a majority of voters,
2. when the tax rate t^∞ is chosen, the Bayesian revision of beliefs π^∞ leads to the same beliefs π^∞.

This steady state equilibrium could be termed an *economic, political, Bayesian equilibrium*. Indeed the job market and the government constraint are in equilibrium, the tax rate is a Condorcet winner and beliefs remain unchanged once the tax rate is implemented. Moreover it can be noticed that in the steady state, the expected amount of basic income is just equal to the amount of transfer carried out ex post: $E_{\pi^\infty}b(t^\infty) = t^\infty\Sigma_{k=1}^{k=K}\frac{n_k^*}{n}\theta_k L_k(t^\infty)$.

The proof of the convergence is simple and goes as follows. Let us consider the sequence $(\pi^\tau,t^\tau)_{\tau \geq 0}$, resulting from some initial beliefs π^0. Suppose that at some date τ_c the current tax rate increases relatively to the tax rate of the previous period. Only individuals who were already working during the previous period are likely to decide to work, and therefore this period does not provide any new information about the distribution of productivities. The beliefs remain unchanged and so does the tax rate at the next period, which means that $(\pi^{\tau_c},t^{\tau_c})$ is the steady state equilibrium. If the supposition is not verified, it means that the sequence $(t^{\tau})_{\tau \geq 0}$ is a strictly decreasing sequence of positive real numbers and thus it converges to some equilibrium which is unique, since the median voter choice is. Since there are at most $K-1$ unknown parameters in the economy, the stationary beliefs are obtained in no more than $K-1$ periods, while an additional period may be required for the tax rate to reach its steady state value. Beyond, the sequence of the tax rates becomes stationary and no new information is revealed at further periods. The above discussion is summarized by the following proposition.

Proposition 2. *Whatever initial belief π^0 may be, the dynamics of current equilibria $(\pi^{\tau}, t^{\tau})_{\tau \geq 0}$ of the successive beliefs and tax rates converges to a unique steady state equilibrium. Moreover, the steady state is reached within K periods.*

4.2. The imperfect information bias

In general, the steady state tax rate depends on both the true distribution of types and initial beliefs. Our purpose is to compare the economic decisions taken when the unemployed's productivity is unknown to those which would be taken under perfect information. In particular, a comparison is performed between the equilibrium tax rate under imperfect information and the tax rate under perfect information. Interestingly, we show that when they differ, they always differ in the same direction. We start by the following remark.

Remark 1. At equilibrium, there is no uncertainty on the level of basic income: for all tax rates $t \in [t^{\infty}, 1]$, $E_{\pi^{\infty}} b(t) = E_{\pi *} b(t)$, where $E_{\pi *} b(t) = t \Sigma_{k=1}^{k=K} n_k^{*} / n \, \theta_k L_k(t)$ is the true basic income for a given t.

This comes from the fact that, for any given tax rate, the uncertainty about the level of the basic income only stems from an uncertainty about the proportions of the types choosing to work at this tax rate (remember that once the tax rate is given, there is no uncertainty about the gross labor income that each individual will get). But, for all tax rates greater than (or equal to) the steady state tax rate, only individuals working at equilibrium may choose to work. Thus the true proportions of types that are likely to work for a tax rate $t \in [t^{\infty}, 1]$ are known at equilibrium.

Our main result is a direct consequence of the above remark.

Proposition 3. *The tax rate at the steady state equilibrium under imperfect information is always higher than (or equal to) the tax rate under perfect information.*

Proof. Consider an equilibrium $(\pi^{\infty}, t^{\infty})$. By definition, t^{∞} is the median voter's most preferred tax rate when beliefs are π^{∞}: $t^{\infty} = t(m, \pi^{\infty})$. It implies that:

$$\forall t \in [0,t^{\infty}[, E_{\pi^{\infty}}V_m(t^{\infty}) > E_{\pi^{\infty}}V_m(t),$$
$$\forall t \in [t^{\infty},1], E_{\pi^{\infty}}V_m(t^{\infty}) \geq E_{\pi^{\infty}}V_m(t). \quad (12)$$

Consider now the Condorcet winner with perfect information t^*. Since $t^* = t(m,\pi^*)$:

$$\forall t \in [0,t^*[, E_{\pi^*}V_m(t^*) > E_{\pi^*}V_m(t),$$
$$\forall t \in [t^*,1], E_{\pi^*}V_m(t^*) \geq E_{\pi^*}V_m(t). \quad (13)$$

Suppose now by contradiction that $t^* > t^{\infty}$. By simple application of (12) and (14), it implies that:

$$E_{\pi^{\infty}}V_m(t^{\infty}) \geq E_{\pi^{\infty}}V_m(t^*) \text{ and}$$
$$E_{\pi^*}V_m(t^*) > E_{\pi^*}V_m(t^{\infty}). \quad (14)$$

By Remark 1, $E_{\pi^{\infty}}b(t^*) = E_{\pi^*}b(t^*)$ and $E_{\pi^{\infty}}b(t^{\infty}) = E_{\pi^*}b(t^{\infty})$. This in turn implies that $E_{\pi^{\infty}}V_m(t^*) = E_{\pi^*}V_m(t^*)$ and $E_{\pi^{\infty}}V_m(t^{\infty}) = E_{\pi^*}V_m(t^{\infty})$, which contradicts inequalities 14. □

Fig. 2 illustrates a case of a strict overtaxation bias. The dashed curves characterize the equilibrium in perfect information while the plain curves characterize the steady state one.

What does this result mean for the comparison of the level of the basic income in the steady state with respect to the level in perfect information or, in other words, do higher tax rates equally imply higher basic income level? Both steady state tax rate and perfect information tax rate are located on a increasing part of the true Laffer curve. If the Laffer curve is single-peaked then the answer is positive. Yet, if it exhibits disjoint increasing sections, nothing can be deduced as to the level of the basic income. The absence of single-peakedness of the Laffer curve rules out any general statement.

Fortunately, thanks to the decreasingness of the labor supply with respect to the tax rate, we can state a simple corollary to our main proposition concerning both the participation rate on the job market and the gross national income.

Corollary 1. *The participation rate and the gross national income at the steady state equilibrium under imperfect information are always smaller than* (*or equal to*) *what they would be under perfect information.*

Many economists pay attention to the risk that a too generous welfare benefits program can amplify a poverty trap. Our result indicates that majority voting can lock a society in a poverty trap for informational reasons. The bottom line of the argument is the following. If the population, and hence the median voter, are overpessimistic about the unemployed's job skills, the marginal benefit of lower

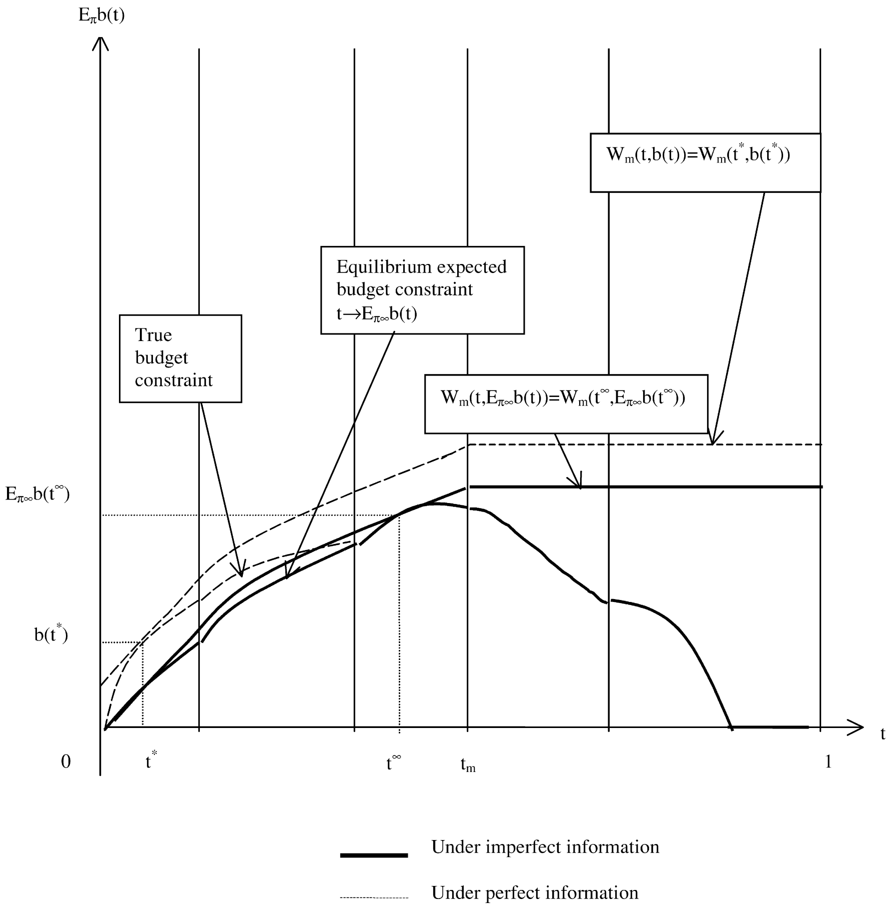

Fig. 2. An example of over-taxation.

distorting taxes can be underestimated, leading the society to the choice of too high a tax rate and by way of consequence too low a participation rate.

Proposition 3 states the possibility of a systematic bias. But, under certain circumstances, the bias vanishes. We now give a sufficient condition on initial beliefs guaranteeing that the equilibrium tax rate under imperfect information coincides with the tax rate chosen under perfect information.

Definition 3. Beliefs $\pi = (\pi_\omega, \omega \in \Omega)$ are said to be *optimistic* if a positive probability is only associated to states of the world for which the distribution of productivities stochastically dominates at the first order the true distribution of productivities:

$$\forall\omega = (n_1, n_2, \ldots, n_K) \in \Omega,$$
$$\pi_\omega > 0 \Rightarrow \forall k \in (1,2,\ldots,K), \sum_{j=k}^{j=K} n_k \geq \sum_{j=k}^{j=K} n_k^*.$$

Proposition 4. *If initial beliefs are optimistic, the equilibrium tax rate under imperfect information coincides with the tax rate under perfect information.*

Proof. See Appendix B □

Proposition 4 gives a condition on initial beliefs which is sufficient to cancel a strict overtaxation bias. One may also think of conditions about individuals' utility over consumption and leisure. For example, if the disutility of labor v is such that $v'(0) = 0$, all individuals choose to work whatever the tax rate may be. Thus, right from the first period, whatever the tax rate t^0 (depending on prior beliefs π^0), all the information concerning the skill distribution is revealed. So the steady state (t^∞, π^∞) is reached within two periods, and it coincides with perfect information equilibrium: $(t^\infty, \pi^\infty) = (t^*, \pi^*)$.

Systematic results about comparative statics when a small change affects initial beliefs π^0 are difficult to obtain since the Bayesian revision of beliefs can lead to serious jumps in the expectations. If an initial belief is a little bit more optimistic than another one (in the sense of first order stochastic dominance), it does not imply that it will be true along the equilibrium path. The search for a positive result is therefore more or less hopeless.

Results about comparative statics when a small change affects steady state beliefs π^∞ are of course less exciting, since the change does not affect a primitive of the model. Nevertheless a difficulty is still along the road of a positive result and the intuition behind this can be grasped, if we remember the interpretation of the tax rate given by formula 11 for equilibrium beliefs $\pi = \pi^\infty$. If equilibrium beliefs become more optimistic, on the one hand we can expect that the expected marginal cost of distorting taxes increases for a large family of utility functions. On the other hand, the expected average gross income increases as well, which means that, from a qualitative point of view, we cannot sign the global effect of an optimistic marginal change in steady state beliefs.

One can also define pessimistic beliefs in a way similar to optimistic beliefs[15] and wonder whether this property is sufficient to guarantee the existence of a strict overtaxation bias. It turns out that this result is false in general[16]. A stochastic

[15]Pessimistic beliefs are such that a positive probability is only associated to states of the world for which the true distribution of productivities stochastically dominates at the first order the distribution of productivities.

[16]The analog of lemma 26 in the Appendix B is not true.

dominance argument is not sufficient to obtain a strict overtaxation. Nevertheless, the possibility of a strict bias cannot be dismissed on the view of the simulation realized on French data.

5. Simulation on French data

It is always difficult to establish a link between the real world and an abstract model. Nevertheless we find it instructive to try to estimate the potential size of the overtaxation bias revealed by the theoretical analysis through a simulation exercise. For convenience we take the example of the French economy which offers at least the advantage, from our point of view, to display one of the lowest participation rate to the job market among the OECD countries.

The hypothetical question we are interested in is the following: suppose that individuals face the (complex) fiscal system existing in France at a given time (we focus our attention on year 1995) and that the government decides to switch from the existing system to a basic income–flat tax schedule in the vein of the reform proposed by Bourguignon and Chiappori (1998). What would happen if a vote took place in order to decide the parameters of this new tax scheme?

5.1. The parameters of the economy

Our French economy is described by four parameters, the number of individuals, the productivity types, the utility function and the prior probability.

5.1.1. The set of voters

Since our model only allows one parameter of heterogeneity, the productivity, it will be sensible to choose the most homogeneous subpopulation with respect to other parameters which can influence the choice between leisure and consumption, such as, in particular, the family size. The population of singles offers some additional advantages since by definition the set of voters is equal to the set of workers. Hence we confine our interest to a society of singles having no children, at least 25 year old and below retirement age. We only keep individuals who were either employed during the twelve months of year 1995 or unemployed during the whole year (we delete individuals who switched at least once from employment to unemployment or from unemployment to employment during the period). We use the data from the survey ‘‘Budget des Familles’’, year 1995; under the restrictions mentioned above, we get a population of $n = 2\ 410\ 223$ individuals.

5.1.2. The productivity types

We choose to divide the population into thirty productivity types. The description of the distribution is not too coarse and the computations remain manageable. To each type is associated a fixed productivity level. The productivity values are rather ad hoc and are chosen for convenience. The productivity levels are normalized in such a way that the median type productivity is equal to 1(see below for more details). See columns 1 and 2 of the Table in Appendix C.

5.1.3. The utility function

The specification of the utility function is:

$$U(C,L) = C + 2\alpha\sqrt{1-L} \tag{15}$$

where α denotes the marginal disutility of labor when all the available time is devoted to leisure. Simple algebra shows that, in the absence of taxes, nobody will work for a wage lower than α. Then the value of α can be also interpreted as the smallest wage on the job market. In all that follows, α is set equal to one third of full time minimum wage ($\alpha = 0.245$[17]).

5.1.4. Prior beliefs

The last parameter is the prior probability, which must be compatible with voters' supposed knowledge of the distribution of types. We assume that voters observe the labor market incomes in 1995, namely the values of labor incomes contained in the survey ''Budget des Familles'' for the single (see Fig. 3).

The remarkable feature exhibited by the data is the surprising low participation rate to the job market. Less than 75% of the population of singles are working at that date. There is no means to find the true type of unemployed individuals. On the opposite, we assume that voters are able to deduce the distribution of implicit productivities for employed people. For instance, voters can use the Bourguignon and Spadaro (1999)[18] inversion procedure: the labor income is supposed to derive from the maximization of the utility function (18) under the budget constraint depending on the fiscal and benefit regime in force for year 1995. This procedure is rather simple since it only uses first order conditions.

More precisely, let $T(y)$ be the net tax function which gives the net transfer from the individual to the government for all gross labor incomes. This function incorporates the dispositions of the tax code regarding the income tax and the basic income. Housing benefits, which depend on other households' choice variables, are excluded from the computation.

As there is no income effect, the optimal labor supplies only depend on the net tax function $T(y)$. A θ productivity individual chooses a labor supply satisfying the first order condition of the utility maximization subject to individual budget constraint: $\theta - \theta T'(\theta L) - \alpha/\sqrt{1-L} = 0$, which leads to:

[17] A value expressed relatively to the median type productivity.

[18] See also for an application Spadaro (1999) and D'Autume (1999).

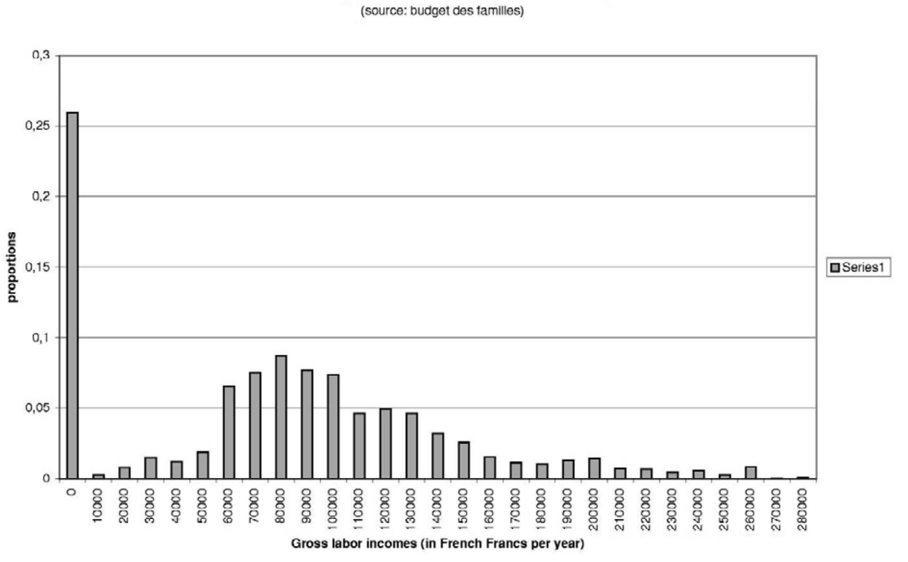

Fig. 3. Gross labor incomes (in French Francs per year).

$$L = \mathrm{Max}\left(0, 1 - \frac{\alpha^2}{\theta^2(1 - T'(\theta L))^2}\right),$$

or equivalently:

$$y = \mathrm{Max}\left(0, \theta - \frac{\alpha^2}{\theta(1 - T'(y))^2}\right).$$

Consequently, if $y = 0$, the productivity θ cannot be inferred and if $y > 0$,

$$\theta = \frac{1}{2}\left(y + \sqrt{y^2 + \frac{4\alpha^2}{(1 - T'(y))^2}}\right). \tag{16}$$

Note that only individuals with productivity higher than $\alpha/1 - T'(0)$ choose to work. This value determines the critical productivity level for the job market and fiscal conditions in 1995. By microsimulation it turns out that the effective marginal tax rate for a null labor income is about 50%[19]. Under the specified

[19] Such a tax rate may seem very high at first glance but recall that T is the net tax function including benefits. Some benefits are provided conditionally on not working. It implies a very high effective marginal tax rate in some circumstances and a 50% rate is an empirical average. Such a tax rate may generate an inactivity trap (for a development see Bourguignon and Chiappori, 1998).

utility function individuals with a productivity lower than two thirds of the minimum wage do not work in 1995. (It corresponds to a 0.49 value in terms of normalized productivity).

Using the labor incomes in the sample as inputs, formula (16), applied with the marginal tax rates at all relevant values under the incumbent system, allows to assign a productivity level to each active individual in the sample[20]. We normalize each productivity level to the median one and we approximate the resulting ''almost continuous'' distribution of productivities in a discrete distribution for ''working types'' (the types with a normalized productivity value higher than 0.49). The distribution of workers along this classification is displayed in Fig. 4, while the exact figures are contained in column 3 of the table in Appendix C.

Individuals of types 1, 2, 3, 4, 5 choose not to work (see the critical tax rates in column 4 of the table). Their respective percentages in the population are thus unknown; voters only know with certainty that these five types represent a little more than 25% of the population.

The information set at date 1995 can then be defined as:

$$I(1995) = \{\omega = (n_1, n_2, \ldots, n_K) \in \Omega : \forall k \geq 6, n_k = n_k^*\},$$

with n_k^*/n given by the values in column 3 in the table in Appendix C.

The prior beliefs at period 1996 must be compatible with this information set, that is: $\pi^{1996} = \pi^{1995}|I\ (1995)$.

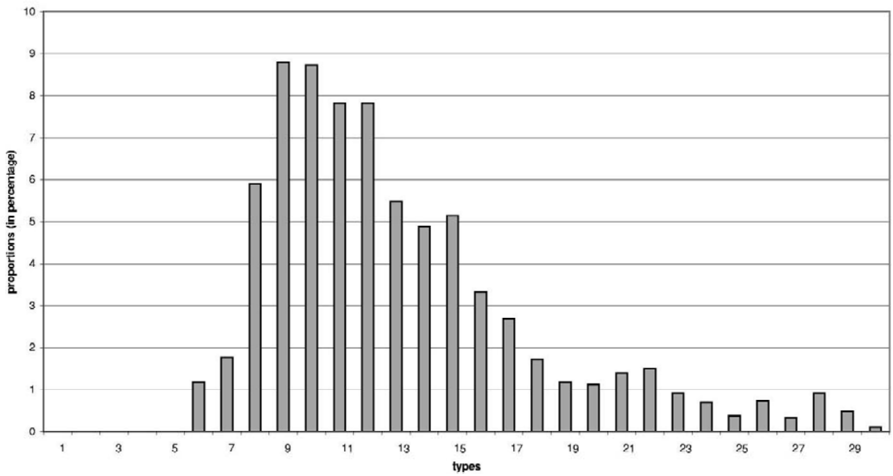

Fig. 4. The true distribution of job skills among employed.

[20] This kind of procedure ignores the possibility of bunching for working types. Considering the shape of $T(y)$ (see for example graphique 4 in Fleurbaey et al. (1999)), it is an assumption.

5.2. The importance of initial beliefs

We first have to compute the equilibrium with perfect information. The ''true distribution'' for types 1 to 5 is supposed to be uniform. The numerical computation indicates that the stable political outcome is $t^* = 31.6\%$, leading to a non participation rate $u^* = 20\%$, individuals from type 1 to type 4 choosing not to work.

For the investigation of the properties of the steady state in imperfect information, we limit our attention to only three states of the world: the true state, a ''bad'' state where all the 25% unemployed are of type 1[21] (individuals who were unemployed at date 1995 will never work[22], whatever the tax rate may be) and a ''good'' state where all the 25% unemployed are of type 5[23] (individuals who were unemployed at date 1995 have a productivity just below the first productivity observed on the job market at that date).

A vector of prior beliefs can be simply described by a three-dimension vector, the first element of which is the probability given to the bad state, the second the probability given to the true state, and finally the third the probability given to the good state. We consider a one parameter family of beliefs, the $\varepsilon -$ family denoted $(1-2\varepsilon,\varepsilon,\varepsilon), 0 \leq \varepsilon \leq 50\%$ where ε is the probability associated to both the true state and the good state. The computation of the steady state equilibrium has been made for any value of $\varepsilon \in [0,1]$ and claim 3 gives the critical value of ε under which a strict overtaxation bias occurs.

Claim 3. *If* $\varepsilon \geq 8.9\%$, *the steady state coincides with the perfect information equilibrium. If* $\varepsilon < 8.9\%$, *the tax rate at the steady state is strictly greater than the tax rate under perfect information.*

For instance, consider the following two examples of beliefs:

(1) Gloomy beliefs $(1-2\varepsilon,\varepsilon,\varepsilon)$, with $\varepsilon \simeq 0$.

At date 1996, tax rate $t^{1996} = 34.7\%$ is the stable political outcome. As $t^{1996} = 34.7\% > t_5 = 33.78$, individuals from types 1 to 5 remain unemployed and voters do not get any new information by observing the job market. Consequently, their beliefs remain unchanged and the steady state is reached within one period:

[21]To be more rigorous, unemployed individuals at date 1995 divide up into types 1, 2, 3, 4, 5 in the following way: all individuals but four are of type 1, one individual is of type 2, one individual is of type 3, one individual is of type 4, and one individual is of type5.

[22]but three of them, namely the individual of type 3, the individual of type 4 and the individual of type 5. The total number of individual n is supposed to be large enough so that the influence of these individuals on macroeconomic variables can be neglected.

[23]Just as in footnote 21, unemployed individuals at date 1995 divide up into types 1, 2, 3, 4, 5 in the following way: all individuals but four are of type 5, one individual is of type 4, one individual is of type 3, one individual is of type 2, and one individual is of type 1.

$t^{\infty} = 34.7\% > t^{*} = 31.6\%$ and $u^{\infty} = 25\% > u^{*} = 20\%$.

(2) Laplacian beliefs (1/3,1/3,1/3).

At date 1996, tax rate $t^{1996} = 28.9\%$ is the stable political outcome. As $t_3 = 18.33\% < t^{1996} = 28.9\% < t_4 = 30\%$, individuals from types 4 and 5 choose to work. Consequently, the voters discover the true state of the world, and within two steps, the economy converges to the perfect information equilibrium.

The conclusion is that overpessimism when evaluating the unemployed's productivity may lead to a significant bias towards over-taxation and under-activity. The economy can fall down in an informational poverty trap. But fortunately it is not necessarily the case for less pessimistic beliefs.

6. Discussion and concluding remarks

Three points deserve a full discussion: the rationality assumptions, the sensitivity of our main result to the way voters get information, and a possible extension of the model to an indirect democracy framework.

Our model can surely be classified among voting models with rational voters. Nevertheless one could argue that our voters are not fully rational on two points.

Firstly, individuals seem satisfied with their relative ignorance of the true distribution of skills and do not try to get more information along the process of convergence, which may deserve some explanation. A possible answer would be that in a large society the virtual gain to participate to the voting process is moderate (see for example Mueller, 1989). Hence it is rational for individuals not to invest a lot of time or money to learn about unknown parameters.

Secondly, one could defend the idea that fully rational voters have to use an intertemporal utility function to decide the stream of their votes instead of choosing it through a sequence of snapshot utility functions. Introducing intertemporal optimization behavior would open the possibility for forward-looking individuals to vote for a low tax rate in order to discover the true number of low-skilled agents. The free rider problem is likely to prevent a single individual from adopting such a strategy since information shares the properties of a pure public good. Besides, from a more technical point of view, such a behavior implies a formidable computation capacity since the information value of lower tax rates depends on individuals' beliefs about the median voter's beliefs at the following period. The single crossing property may be lost which may preclude an equilibrium to exist. Furthermore, it is unlikely that in true life the whole electorate endorses such a sophisticated voting behavior.

Independently of the institutional setting, one may further wonder how sensitive our main result is to the details of how voters get information.

First, our model provides no mechanism for determining the prior beliefs. Note that our main result states that for any initial belief the steady state tax rate is higher than or equal to the perfect information tax rate. To this respect the result holds independently from the way beliefs are determined.

Suppose that individuals exogenously get more information than described in the model: in addition to the information provided by the labor market, they learn the type of some individuals along the process[24]. The same kind of reasoning that sustains proposition 4.2 shows that additional information may affect the dynamics and the steady state, but the main result remains: information processing by the economic system only rules out strict undertaxation outcomes, leaving open the possibility for an overtaxation bias.

Suppose now that individuals have less information than described in the model. It is far from obvious that voters do not make mistakes when observing the distribution of labor incomes. For example, we can imagine, in a minimalist view, that individuals only know the value of the basic income. The definition of the steady state remains the same, namely a pair of stationary beliefs and resulting tax rate, but no prediction can be made concerning the comparison between the steady state tax rate and the tax rate that would be chosen with perfect information. Hence one can conclude that our overtaxation bias result depends on the learning process about the job skills.

In this model, the voting process is a black box which elicits the median opinion at each step. It is well known that, in perfect information models, this black box can be interpreted as a ''Downsian'' competition, namely an electoral competition between two office-motivated parties (see Downs, 1957, or Ordeshook, 1986, for a survey). The choice of the median policy can be seen as the equilibrium of a two-player zero-sum game, in which parties simultaneously make proposals and majority voting takes place, the party objective being either to win the election (to maximize its probability of winning) or to gather as many votes as possible (to maximize its expected plurality).

In our imperfect information model, such an interpretation remains possible. Indeed assume that the two parties share voters' beliefs about the distribution of skills and that the median type is already known to everybody from the very beginning[25]. Then it is easily checked that the median policy happens to be the equilibrium at each date, as in perfect information models. Thus under these assumptions the dynamics and therefore the steady state are exactly identical to those described above.

This standard interpretation in the imperfect information case is not fully satisfying since it supposes that parties and voters share the same information. It

[24] For instance, if the results of all ballots about pairwise comparisons were publicly announced, then purely rational individuals would be able to deduce much more information about the full distribution of skills at the first step of the voting process, which might even bring them back to the perfect information case.

[25] This latter condition is satisfied if, for example, the types of the 50% most skilled individuals are common knowledge.

may be more sensible to think that candidates for public office are better informed than voters. In that case, the question of the degree of rationality of the electorate must be raised, leading to two different classes of models.

According to a first option, voters do not take into account the fact that parties' proposals may convey information, and they vote on the sole basis of their current beliefs. Then the 2-player electoral game under imperfect information can be solved, and leads to the same comparative static result: even if the parties are well informed, a strict overtaxation bias can appear. For more details, see Van der Straeten (2000).

According to a second option, fully strategic voters, knowing that parties information is different from their own, are engaged in a strategic game of incomplete information. In a Bayes–Nash equilibrium of such a game, they infer information from parties' policy announcements. This kind of electoral games played between $n+2$ players (n voters and 2 parties) is considered, in a very stylized framework, by Laslier and Van der Straeten (2000). Although it is shown that these games typically have a lot of Nash equilibria, refinement analysis helps to prove that well informed parties will reveal their information. Extrapolating these results to the present model, one may conjecture that if parties know the true distribution and if voters know that parties know it, then: in any ''truly perfect'' equilibria[26], parties' proposals are based on the true distribution. More specifically, if parties are perfectly informed, then there should be no strict overtaxation.

We conclude by mentioning two other issues which could deserve some attention in future research. A first obvious limitation of the model lies in the assumption that all individuals share the same beliefs. Allowing beliefs to be diverse will add a second source of heterogeneity among individuals which raises a difficulty for a median voter result at each period. A second limitation concerns the specification of the utility function. Even if some arguments can be put forward to defend the quasi-linearity assumption in this model, considering a more general utility function is obviously on the research agenda. Paying the price of giving up the ex post budget constraint for the government seems at least necessary to hope for an extension in this direction.

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[26]True perfection is a refinement of Selten's trembling hand perfection.

are indebted to Cyrille Hagneré and Amedeo Spadaro for their precious help in the microsimulations on French data.

Appendix A. Proof of proposition 1 (Existence and characterization of the Condorcet winner)

Lemma 1. *The slope of the indifference curve of the function* $(W_k)_{k=1,2,\dots,K}$ *defined in* (10) *is non decreasing with k at each point of the space* $[0,1]\times\mathbb{R}$.

Proof of Lemma 1. Note that: $W_k(t,b)=\max_{0\le L\le 1} b+(1-t)\theta_k L - v(L)$. Thus, by the envelope theorem, the slope denoted by $s(t,b)$ of the indifference curve of the function W_k at the point (t,b) is:

$$s(t,b) = -\frac{\partial W_k(t,b)}{\partial t} / \frac{\partial W_k(t,b)}{\partial b} = \theta_k L_k(t)$$

As the gross labor income is a non decreasing function of the productivity (see claim 2), the slope of the indifference curve of the function W_k at the point (t,b) is a non decreasing function of k, which proves the result. □

Definition 4. A family of functions

$$\left(f_k\colon \begin{array}{l}[0,1]\to\mathbb{R}\\ t\mapsto V_k(t)\end{array}\right)_{k=1,2,\dots K}$$

satisfies the ''Single Crossing Condition'' if

$$\begin{array}{l}\forall k\in\{1,2,\dots,K\},\forall t,t'\in[0,1]\text{ with }t'\ge t,\\ f_k(t)\ge f_k(t')\Rightarrow \forall k'\in\{k,k+1,\dots,K\}, f_{k'}(t)\ge f_{k'}(t').\end{array}$$

Lemma 2. *The family of functions*

$$\left(E_\pi V_k\colon \begin{array}{l}[0,1]\to\mathbb{R}\\ t\mapsto E_\pi V_k(t)\end{array}\right)_{k=1,2,\dots K}$$

defined in (5) *satisfies the ''Single Crossing Condition''.*

Proof of Lemma 2. Consider two types $k,k'\in\{1,2,\dots,K\}, k'\ge k$ and let $t',t''\in[0,1]$ be two tax rates such that $t'\le t''$ and $E_\pi V_k(t')\ge E_\pi V_k(t'')$. We want to show that $E_\pi V_{k'}(t')\ge E_\pi V_{k'}(t'')$.

By assumption, $E_\pi V_k(t')\ge E_\pi V_k(t'')$. As for any $t\in[0,1], E_\pi V_k(t)=W_k(t,E_\pi b(t))$, this is equivalent to $W_k(t',E_\pi b(t'))\ge W_k(t'',E_\pi b(t''))$.

In the space (t,b) (see Fig. 1), this means that type k individuals' indifference curve passing through point $(t',E_\pi b(t'))$ is ''above'' point $(t'',E_\pi b(t''))$.

By Lemma 1, the slope of the indifference curve of the function W_k at point $(t',E_\pi b(t'))$ is a non decreasing function of k. As $k' \geq k$, this implies that type k' individuals' indifference curve passing through point $(t',E_\pi b(t'))$ is also ''above'' the point $(t'',E_\pi b(t''))$ (see Fig. 1). Thus $E_\pi V_{k'}(t') \geq E_\pi V_{k'}(t'')$. □

Proof of Proposition 1. Let us now prove that the median voter's preferred tax rate $t(m,\pi)$ defeats any tax rate in pairwise comparison.

Let $t \in [0,1]$, $t \neq t(m,\pi)$ be any tax rate different from the median type individuals' preferred tax rate. We distinguish two cases: either $t > t(m,\pi)$ or $t < t(m,\pi)$.

· If $t > t(m,\pi)$, by definition of the median type individuals' preferred tax rate, $E_\pi V_m(t(m,\pi)) \geq E_\pi V_m(t)$. Besides, by Lemma 2, the family of functions $(E_\pi V_k)_{k=1,2,\ldots K}$ satisfies the Single Crossing Condition, so:

$$\forall k \geq m, E_\pi V_k(t(m,\pi)) \geq E_\pi V_k(t).$$

By definition of the median type, $\Sigma_{k=m}^{k=K} n_k^* \geq n/2$, so at least 50% of the population prefers $t(m,\pi)$ to t.

· Consider now the case $t < t(m,\pi)$. By definition of the median type individuals' preferred tax rate, $E_\pi V_m(t(m,\pi)) \geq E_\pi V_m(t)$. Besides, note that the fact that the family of functions $(E_\pi V_k)_{k=1,2,\ldots K}$ satisfies the Single Crossing Condition also implies that $\forall k' \in \{1,2,\ldots,K\}, \forall t,t' \in [0,1]$ with $t' \geq t, V_{k'}(t') \geq V_k(t) \Rightarrow \forall k \leq k', V_k(t') \geq V_k(t)$ (it is easy to check that assuming the contrary would lead to a contradiction). So:

$$\forall k \leq m, E_\pi V_k(t(m,\pi)) \geq E_\pi V_k(t).$$

By definition of the median type, $\Sigma_{k=1}^{k=m} n_k^* \geq n/2$, so at least 50% of the population prefers $t(m,\pi)$ to t. □

Appendix B. Proof of Proposition 4 (Optimistic beliefs)

We proceed through three lemma.

Lemma 3. *If beliefs* π *are optimistic*, $\forall t \in [0,1], E_\pi b(t) \geq E_{\pi^*} b(t)$.

Proof of Lemma 3. For any

$$t \in [0,1], E_\pi b(t) = \sum_{\omega \in \Omega} \pi_\omega b_\omega(t) = \sum_{\omega \in \Omega} \pi_\omega \left(\sum_{k=1}^{k=K} \frac{n_k}{n} t\theta_k L_k(t) \right) = \sum_{k=1}^{k=K} \frac{E_\pi n_k}{n} t\theta_k L_k(t)$$

and $E_{\pi^*} b(t) = \sum_{k=1}^{k=K} \frac{n_k^*}{n} t\theta_k L_k(t)$. So:

$$\begin{aligned}\forall t \in \quad & [0,1], E_\pi b(t) - E_{\pi *} b(t) \\ = \quad & \frac{t}{n}\sum_{k=1}^{k=K}(E_\pi n_k - n_k^*)\theta_k L_k(t) \\ = \quad & \frac{t}{n}[E_\pi n_K - n_K^*][\theta_K L_K(t) - \theta_{K-1}L_{K-1}(t)] \\ & + \frac{t}{n}[(E_\pi n_K - n_K^*) + (E_\pi n_{K-1} - n_{K-1}^*)] \\ & \quad *[\theta_{K-1}L_{K-1}(t) - \theta_{K-2}L_{K-2}(t)] \\ & + \frac{t}{n}[(E_\pi n_K - n_K^*) + (E_\pi n_{K-1} - n_{K-1}^*) + (E_\pi n_{K-2} - n_{K-2}^*)] \\ & \quad *[\theta_{K-2}L_{K-2}(t) - \theta_{K-3}L_{K-3}(t)] \\ & + \frac{t}{n}[(E_\pi n_K - n_K^*) + \ \dots \ + (E_\pi n_2 - n_2^*)][\theta_2 L_2(t) - \theta_1 L_1(t)] \\ & + \frac{t}{n}[(E_\pi n_K - n_K^*) + \ \dots \ + (E_\pi n_1 - n_1^*)]\theta_1 L_1(t).\end{aligned}$$

As the beliefs π are optimistic, $\forall k \in (1,2,\dots,K), \Sigma_{j=k}^{j=K} E_\pi n_k \geq \Sigma_{j=k}^{j=K} n_k^*$, so $\forall k \in \{1,2,\dots,K\}, (\frac{E_\pi n_K - n_K^*}{n} + \ \dots \ + \frac{E_\pi n_k - n_k^*}{n}) \geq 0$.

It has previously been shown (see claim 2) that the labor supply, for a given tax rate, is a non decreasing function of the productivity, so: $\forall k \in (2,\dots,K), (t\theta_k L_k(t) - t\theta_{k-1} \ L_{k-1}(t)) \geq 0$.

This shows that $\forall t \in [0,1], E_\pi b(t) - E_{\pi *} b(t) \geq 0$. □

Lemma 4. *If equilibrium beliefs are optimistic,* $t^\infty = t^*$.

Proof of Lemma 4. Suppose on the contrary that $t^\infty > t^*$(Proposition 1 states that $t^\infty \geq t^*$).

As the median voter's preferred tax rate is chosen at equilibrium, $W_m(t^\infty, E_{\pi^\infty} b(t^\infty)) > W_m(t^*, E_{\pi^\infty} b(t^*))$.

But $E_{\pi^\infty} b(t^\infty) = E_{\pi *} b(t^\infty)$, so: $W_m(t^\infty, E_{\pi *} b(t^\infty)) > W_m(t^*, E_{\pi^\infty} b(t^*))$.

Besides, Lemma 3 states that, as the beliefs π^∞ are optimistic, $\forall t \in [0,1], E_{\pi^\infty} b(t) \geq E_{\pi *} b(t)$. Now

$$\forall (t,b) \in [0,1] \times \mathbb{R}, \frac{\partial W_m(t,b)}{\partial b} \geq 0, \textit{ so } W_m(t^*, E_{\pi^\infty} b(t^*)) \geq W_m(t^*, E_{\pi *} b(t^*)).$$

Thus $W_m(t^\infty, E_{\pi *} b(t^\infty)) > W_m(t^*, E_{\pi^\infty} b(t^*))$ implies that $W_m(t^\infty, E_{\pi *} b(t^\infty)) > W_m(t^*, E_{\pi *} b(t^*))$, which contradicts the fact that t^* is the Condorcet winner under perfect information. □

Lemma 5. *If initial beliefs* π^0 *are optimistic, then, for all* $\tau \geq 0$, *beliefs* π^τ *at period* τ *are optimistic. In particular, equilibrium beliefs* π^∞ *are optimistic too.*

Proof of Lemma 5. Suppose that initial beliefs π^0 are optimistic and let us prove by iteration that $\forall \tau \geq 0$, beliefs at period τ are optimistic.

Suppose that at the beginning of period τ, the beliefs π^τ are optimistic. By definition, this means that $\forall \omega = (n_1, n_2, \ldots, n_K) \in \Omega, \pi^\tau_\omega > 0 \Rightarrow \forall k \in (1,2,\ldots,K), \Sigma_{j=k}^{j=K} n_k \geq \Sigma_{j=k}^{j=K} n_k^*$.

Posterior beliefs $\pi^{\tau+1}$ are obtained by Bayesian updating of prior beliefs π^τ, so $\pi^{\tau+1} > 0 \Rightarrow \pi^\tau > 0$.

Given the definition of optimistic beliefs, the result follows. □

Proof of Proposition 4. With these lemmas, one can prove Proposition 4. Suppose that initial beliefs π^0 are optimistic. Then by Lemma 5, equilibrium beliefs π^∞ are optimistic too. By Lemma 3, $\forall t \in [0,1], E_{\pi^\infty} b(t) \geq E_{\pi^*} b(t)$. This in turn implies, by Lemma 4, that $t^\infty = t^*$. □

Appendix C

Table A.1
Description of the economy according to types

Type (k)	Productivity (θ_k)	Percentage of the population (n_k^*/n)	Critical tax rate (t_k)
1	0.100	?	0
2	0.200	?	0
3	0.300	?	18.33
4	0.350	?	30.00
5	0.370	?	33.78
6	0.500	1.18	51.00
7	0.625	1.77	60.80
8	0.750	5.90	67.33
9	0.875	8.79	72.00
10	1.000	8.72	75.50
11	1.125	7.82	78.22
12	1.250	7.82	80.40
13	1.375	5.49	82.18
14	1.550	4.88	83.67
15	1.625	5.15	84.92
16	1.750	3.32	86.00
17	1.875	2.68	86.93
18	2.000	1.72	87.75
19	2.125	1.18	88.47
20	2.250	1.13	89.11

Table A.1. *Continued*

Type (k)	Productivity (θ_k)	Percentage of the population (n_k^*/n)	Critical tax rate (t_k)
21	2.375	1.39	89.68
22	2.500	1.50	90.20
23	2.625	0.91	90.67
24	2.750	0.70	91.09
25	2.875	0.38	91.49
26	3.000	0.73	91.83
27	3.125	0.34	92.16
28	3.250	0.91	92.46
29	3.375	0.48	92.74
30	3.500	0.11	93

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