1.1 Motivation : Taxation, redistribution and incentives

From some egalitarian perspective, the market income distribution, the income distribution which comes out from free markets, free market of labor and capital, which is often termed the primary income distribution is generally viewed as unsatisfactory. To correct such a discrepancy between the actual primary distribution and what is seen as an ideal income distribution, redistribution takes place through instruments like taxes, mainly income taxes and social benefits like basic income, negative income tax. The redistribution which occurs entails some behaviour reaction in terms of labor supply, typically people when taxed may reduce their labor supply either in terms of lower hours of work or in terms of effort or energy spent in the work place. Then it is likely that redistribution induces some incentive costs, namely, efficiency costs. Then, on the one hand, redistribution leads to gains from an egalitarian perspective while on the other hand, from an efficiency point, redistribution leads to a cost. On the one hand the cake is better distributed, but on the other hand, the size of the cake diminishes. This is classic trade-off between equity and efficiency. There is a balance between the costs and the benefits to find and this is one of the purpose of optimal taxation to help decision makers to find such a optimum.

Hence from a mere positive economic point of view, the basic question which emerges is the following : does the redistribution in developed countries be optimal in terms of size and the agents who benefit from it ? The practice in that matter among developed countries seems very diverse. For instance, even if the progressivity of the income tax is widespread, you can find some very important exceptions like Russia who recently adopts a flat tax of 15%. Such differences paves the way for a more general question ? May economic theory provide a rational explanation for such differences ? It is a cross section empirical evidence.

Since twenty years, a fall or a decline in redistribution may be observed in anglo-saxon countries, USA, GB and Australia, in Italy, in France since 2001. To understand
such an evolution, the question which arises is here: Which are the key parameters of optimal taxation?

Two aspects of redistribution are usually distinguished: vertical redistribution, from rich to poor, and horizontal one which, for a given family income, equalizes the income per capita. It is a redistribution for small size families like singles to families with numerous children. This course will focus on the first one.

Let me briefly describe what are the instruments of redistribution in France.

1.1.1 The instruments of redistribution in France

Quels sont les outils de cette redistribution dans le cas français?

Taxes

- A progressive income tax, namely, the average tax rate is increasing with income.
- The generalized social contribution (CSG) and the contribution to the repayment of the social debt (CRDS) is nothing that a proportional income tax. Labor income and capital incomes are the tax bases. One point of CSG represents roughly one point of national income.

The French income tax comprises two parts. It is not the first time that such a division takes place. Before the reform launched by Antoine Pinay the minister of budget in 1958, it was already the case. Pinay merged the old proportional and the old progressive one in a unique progressive one.

- The local dwelling tax, which comprises an exemption threshold. Income lower than this threshold don’t pay the dwelling tax.
- Social security contributions on wages.
- VAT and excise taxes on tobacco, alcohol, gas. The VAT rate depend on the commodities. The normal rate is 19.6% but there is reduced rate at 5.5% for mainly food, and you have also a superreduced rate at 2.5% for newspapers for example. A recent French study written by Forgeot and Starzec that has been published in the past issue of Journal Économie Publique shows the real negative redistributive impact of these taxes. Indeed poor people spend a larger proportion of their budget in vices, tobacco, alcohol, gambling and so on. And they also spend a larger part of their income in consumption. These two facts mainly explain the regressive power of the indirect taxation. The richer you are, the smaller the part of your income devoted to indirect taxes is. The last two chapters of the course are devoted to the usefulness of sales taxes in presence of income taxes. Atkinson et Stiglitz show the useless of such a second instrument of taxation under some assumptions.

Social benefits

Two kinds of benefits must be distinguished. These which are mainly redistributive in their aim (Beveridge), and those which have been mainly designed to insure people...
against social risks such as retirement, unemployment. (Bismarck). Here we only take into account the first ones.

Contrary to the US, expenditure programs for the poor in France are cash transfers rather than in-kind transfers.

- the redistributive benefits : they are generally means-tested. Only individuals whose financial resources fall below a certain level can receive benefits.
- family benefits such as : parentale benefit for young allocation parentale pour jeune enfant (APJE), parentale benefit for education allocation parentale d’éducation (APE), parentale benefit for single women allocation parent isolé (API); they are equivalent to the American AFDC aid to families with dependent children.
- transfers to compensate some disability, handicap : allocation adulte handicapé (AAH);
- Unemployment assistance : allocation solidarité spécifique (ASS) about 400 euros per month;
- Housing assistance : allocations logement which budget is five time more important than this of RMI.
- A negative income tax La prime pour l’emploi (PPE) it gives a supplement of income of 1200 euros per year for a single with the minimal wage
- Basic income (RMI), which became revenu minimum d’autonomie (RMA), 4.6 billions of euros are devoted to this benefit.

All in all, the total amount of redistributive benefits secure an income of about 560 euros for a single.

- Social security payments They depend on social security contributions : The pay as you go systemes for pensions, Unemployment benefits, illness benefits. We do not argue that those transfers do not have any redistributive impact, since they are. But they are mainly lagged wages. We only suggest that their main role is to insure people against some risk about the loss of a wage.
- Health care even if it financed by the social security is a transfer in kind like the provision of education. It is a matter of public good rather than transfers.

### 1.1.2 An assessment of the redistribution in France

On a *Quantitative basis*, 5% of income of the last decile are (is) redistributed to the people of the first decile. Between the second and the eighth decile, the balance between taxes and benefits is almost neutral. It is amazing that such a so complex system leads to a so simple result.

On a *Qualitative basis*, from the efficiency viewpoint, taxation induces a discrepancy between the MRT and the MRS and therefore it generate distortions. From a micro-econonomic analysis, marginal rates of taxation matter since they shape the slope of the budget set and they directly affected optimal choices of agents. In France, as in almost other countries of OECD, we observe a U-shape curve of marginal tax rates. Rich and
the poor are the people which are the most discouraged from working. How to explain this U-shape Curve?

First it is important to put forth that the marginal tax rates are implicit marginal tax rates (IMTR). They are given by the slope of the function expressing disposable income with respect to primary income.

- On the top part of the income distribution, the high level of marginal tax rates for the income tax (47-48%), plus the CSG (10%), plus social security contributions, plus the ISF (Tax on large fortunes)
- On the bottom part of the income distribution, the agents does not pay income tax (as 50% of the population they are below the exemption threshold), but they may pay the dwelling tax which may represent up to one month of income for people of the third and forth decile. But more importantly, the high level of marginal tax rate in the bottom part comes from the fact that people loose a lot of benefits when they begin to work as long as they benefit from basic income. It it the loss of the benefits which explain the huge level of implicit marginal tax rates

**Implicit marginal tax rates in the bottom part**

How is changing the amount of transfers when the income is increasing? Three types of transfers must be distinguished

- Unconditional transfer: with respect to income. Example: Family benefits in France up to some ceiling.
- Differential transfer: For each euro earned on the job market, you face a loss of one euro of your benefit. The marginal tax rate is exactly 100%. Example: RMI before 1997.
- Between the two: diminishing transfer. Example: RMI in France with an IMTR of 50%.

To perform an unconditional transfer of 220 euros per month per adult will cost about 10% of the GDP, namely more or less the revenue generated by the VAT.

Is such a rather small redistribution operated by the French taxation system optimal? The importance of the IMTR at the bottom part of income distribution raise the question of poverty traps. If the difference between the minimum wage and the basic income is too small, it may not pay the cost to go to work and low productivity people are discouraged from taking a job or discourage to search a job. Then in a dynamic perspective, according to this view, far from being a solution to poverty, expenditure programs for poor are the cause of the problem. As Charles Darwin put it more a century ago in the Voyage of the Beagle (Darwin’ is the British Naturalist inventor of the theory of evolutionary selection; *Charles Darwin’s chronicle of his amazing journey aboard the Beagle where he made observations that led to his revolutionary theory of natural selection*. Great is our fault, if the misery of our poor does not come from natural laws but from our institutions. There is a great debate around these questions of poverty trap. The empirical evidence provided by econometric studies is not so crystal clear. I have contributed to this debate and even it is true that poor people react to incentives, it is difficult to disentangle the
impact of three factors in the actual unemployment. The effect of the minimum wage, frictional unemployment linked to difficulties in matching supply and demand and the lack of incentives induced by the aid to poor.

Are these traps optimal after all? Marginal tax rates must be sufficiently high in order to finance redistribution but not high in order not to reduce too much the size of the cake.

The answer depends on the weight devoted to poor. This weight may also depend on numbers. How many are low productive people compared to the whole population? In optimal taxation, equity considerations are next to demographic aspects.

Quel cadre informationnel retenir? Quelles sont les informations qui sont connaissance commune?

1.2 Notations

1.2.1 Population

We denote by \( \omega \) the productivity of the individual (or the real wage) (we always suppose competitive labor markets) and it is supposed to be independent from his present choice of labor supply. It is completely exogeneous. The population may be discrete or continuous. In the discrete case, we consider \( n \) types of productivity \( \omega_1, \ldots, \omega_n \). (lowercase) In the continuous case, \( \omega \) is distributed according to a probability distribution with finite support \([\omega, \bar{\omega}]\) and \( F \), the CDF. We have to care of the extreme values of the support, \( \omega \) and \( \bar{\omega} \) which may have an impact on the results. In particular, \( \omega = 0 \) or \( \omega > 0 \)? Is \( \bar{\omega} \) finite or not?

1.2.2 Preferences

Let \( l \) denote the hours of work, or the effort; In an other interpretation of the model, it may be human capital. \( 1 - l \) leisure; \( c \) consumption; \( U(c, l) \) utility. \( l \) is normalized such that \( 0 \leq l \leq 1 \) and we assume \( U \) to be strictly quasi-concave. With differentiability, (C2)

\[
2U_{cl} - \frac{U_l}{U_c}U_{cc} - \frac{U_c}{U_l}U_{ll} < 0
\]

agents are identical in their preferences (choice consumption leisure). The only parameter of heterogeneity is productivity. People are identical, except that they have preference more or less dispendious in term of leisure.

\( ^1 \)La quasi-concavité stricte de la fonction d’utilité est équivalente à la convexité stricte des courbes d’indifférence. Soit une courbe d’indifférence \( U(c(\omega), 1 - L(\omega)) = \overline{U} \), où \( L(\omega) \) est le loisir. Cette inégalité définit implicitement une fonction dont la dérivée est \( \frac{dc(\omega)}{d\omega} = -\frac{U_l}{U_c} \). L’inégalité suivante doit donc être vérifiée pour que les courbes d’indifférence soient strictement convexes : \( \frac{d^2U(c, l)}{dcdl} = \frac{U_l}{U_c}[2U_{cl} - \frac{U_l}{U_c}U_{kl} - \frac{U_c}{U_l}U_{cc}] > 0 \), qui équivaut à \( 2U_{cl} - \frac{U_l}{U_c}U_{kl} - \frac{U_c}{U_l}U_{cc} > 0 \). Or, \( U_{cl} = -U_{cl} \) et \( U_L = -U_l \). Par conséquent, la stricte quasi-concavité se traduit par : \( 2U_{cl} - \frac{U_l}{U_c}U_{lt} - \frac{U_c}{U_l}U_{ll} < 0 \).
several spécifications of the utility function:
- **séparabilité** : \( U(c, l) = u(c) - v(l) \);
- **log-linéarity** : \( U(c, l) = \log c - \gamma \log l \);
- **quasi-linéarity in consumption units** : \( U(c, l) = c - \gamma \log l \) ou \( U(c, l) = c - v(l) \). We make sure that there is no income effect on labor supply: The income effect is completely absorbed by consumption. By the way, when net of taxes wage is changing, we only have to take care of the substitution effect. On the other hand, when we increase or decrease an unconditional basic income, we are sure that labor supply remains unchanged. This specification avoids assuming that leisure is a normal good. Even this assumption is not supported by empirical evidence, it offers a simpler way to solve the optimal tax problem.
- **quasi-linéarité in labor units** : \( U(c, l) = -l + \gamma u(c) \). It is the opposite. All income effects are absorbed by labor supply. Consumption is invariant to transfers. This assumption is not particularly realistic and it is made for instrumental purpose.
- **quasi-linéarité quadratique** : \( U(c, l) = c - l^2 \).

Let \( y \) be the primary income. Then, \( y = \omega l \). Without taxes, consumption or disposable income are equal to primary income : \( c = \omega l \).

In general we can write that consumption demand and labor supply are a function a productivities; \( c(\omega), l(\omega) \). Let \((c^*(\omega), l^*(\omega))\) be the optimal choice for an agent of productivity \( \omega \). The indirect utility function is defined as usual as \( u: [\omega, \omega] \rightarrow \mathbb{R}^+ \), \( u(\omega) = U(c^*(\omega), l^*(\omega)) \).

### 1.2.3 Taxation

What are the different types of taxation that we can imagine? The three parameters which be included in the tax base are \( y, \omega, l \). If there parameters are public knowledge then we can write : \( T(y, \omega, l) \). But if \( \omega, l \) are private information, the tax base is only made by the income. In France the PPE depends on \( l \). But it has been argued that it is not a good idea since people firms or/and individuals can cheat at figures about \( l \). There is no saving in the model which is a static one. Then consumption is just equal to disposable income : \( c = y - T(y) \).
- In case of lump sum taxes, \( T \) is independant from \( y \) or \( l \).
- In case of a linear income tax schedule, the tax rate is proportionnel : \( T(y) = m + ty \), where \( t \) is the constant MTR. When \( m < 0 \), \( m \) is the negative income tax we get the simplest progressive tax:

\[
\frac{T(y)}{y} = \frac{m}{y} + t
\]

If \( m < 0 \), \( \frac{T(y)}{y} \) is increasing in \( y \). With, the incentive cost of the taxation are the same whatever is the productivity of the agent. A linear taxation represents a order 1 approximation of tax systems. In France, it’s a cubic polynomial which best fits the data.
1.2.4 Resources constraints for the government

Pure redistribution case. No public good. For a discrete distribution:

\[ \sum_{i=1}^{n} T(y(\omega_i)) \geq K \]  

(1.3)

where \( K = 0 \) without debt repayment. In the continuous case:

\[ \int_{\omega} T(y(\omega)) \, dF(\omega) \geq K \]  

(1.4)

1.2.5 Social objective

We are a welfarist framework: The aim of the government is to maximise:

\[ \int_{\omega} \phi(U(c(\omega), l(\omega))) \, dF(\omega) \]  

(1.5)

where \( \phi : \mathbb{R} \rightarrow \mathbb{R} \) represents the taste for equality of the decider. When \( \phi \) is the identity, we get utilitarianism. If we consider the functional:

\[ \phi = \begin{cases} 
\frac{[U(c(\omega), l(\omega))]^{1-\rho}}{1-\rho} & \text{for } \rho \in [0, \infty] \setminus \{1\} \\
\ln U(x(\omega), l(\omega)) & \text{for } \rho = 1
\end{cases} \]  

(1.6)

then for \( \rho = 0 \) get utilitarianism et for \( \rho \to \infty \), the Rawlsian case. The preference for equality is growing with \( \rho \), qualified as "parameter of inequality aversion" (Atkinson).

1.3 Laissez-faire

Without taxes, an agent maximizes his utility \( U(c(\omega), l(\omega)) \) under the constraint \( c(\omega) = \omega l(\omega) \). \( U \) is quasi-concave (interior solution) and concave (decreasingness of the marginal utility of income). We get the FOC:

\[ \omega U_c(c(\omega), l(\omega)) + U_l(c(\omega), l(\omega)) = 0 \iff \omega = -\frac{U_l(c(\omega), l(\omega))}{U_c(c(\omega), l(\omega))} = \frac{dc(\omega)}{dl(\omega)} \]  

(1.7)

where \( U_c(c(\omega), l(\omega))) = \frac{\partial U}{\partial c} \). For a Cobb-Douglas, we get:

\[ -\frac{U_l}{U_c} = \frac{c}{1-l} \Rightarrow \omega = \frac{\omega l}{1-l} \Rightarrow l = \frac{1}{2} \]  

(1.8)

Hence, \( c = \omega l = \frac{\omega}{2} \). At equilibrium, the individuals have an income proportional to productivity: the productivity scale determines the welfare scale.

More generally, for a more general utility function, by deriving the indirect utility function \( u(\omega) = U(c^*(\omega), l^*(\omega)) \), and using the envelop theorem we obtain that
\[ u'(\omega) = U_c \omega^a (\omega) > 0 \text{ since } U_c > 0. \]