Optimal Income Taxation in an Equilibrium Unemployment Model: Mirrlees meets Pissarides *

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Abstract

This paper characterizes optimal non-linear income taxation in an economy with a continuum of unobservable productivity levels and endogenous involuntary unemployment due to frictions in the labor markets. Redistributive taxation distorts labor demand and wages. Compared to the *laissez-faire*, gross wages, unemployment and participation are lower. Average tax rates are increasing. Marginal tax rates are positive, even at the top. Finally, numerical simulations suggest that redistribution is much more important in our setting than in a comparable Mirrlees (1971) setting.

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I Introduction

Following the seminal paper of Mirrlees (1971), the optimal income taxation literature has almost exclusively focused on the effects of taxation on labor supply responses. The emphasis has been put on the intensive margin (working hours and effort in employment) and more recently on the extensive margin (the participation decision). Examples of this more recent literature are Saez (2002), Boone and Bovenberg (2001, 2002b), Choné and Laroque (2002), Laroque (2002). However, as Mirrlees (1999) writes: "Another desire is to have a model in which unemployment can arise and persist for reasons other than a preference for leisure". The present paper follows this research avenue. We characterize the optimal income taxation in an equilibrium unemployment matching model with a continuum of labor productivities.

Following Mortensen and Pissarides (1999) and Pissarides (2000), frictions on the labor market generate rents for workers and firms who have matched. These rents are shared through wage bargaining. This setting allows to deal with the effects of income taxation on wages and labor demand. To highlight the intuition of our mechanisms, we first ignore the intensive margin. Higher levels of taxes reduce the surplus to be shared, push wages upwards and are therefore detrimental to employment. However, higher marginal tax rates reduce the incentives workers have to claim higher wages (see Malcomson and Sartor (1987) and Lockwood and Manning (1993)). The novelty is that this reduction in gross wages implies a rise in employment, whereas in the standard literature, a rise in marginal tax rates reduces labor supply. Through their impact on wages, higher marginal tax rates also induce firms to increase the resources spent on posting vacancies. This effect can outweigh the increase in gross output implied by the rise in employment. This will occur if, as we assume, a *laissez-faire* economy maximizes net output. Our optimum then solves a trade-off between the efficiency cost implied by the creation of too many vacancies and the usual equity gains coming from raising marginal tax rates.

For participants to the labor market, the optimal employment rate is higher than at the *laissez-faire*. This contrasts with the standard literature where total working hours are reduced compared to the *laissez-faire* (Mirrlees (1971), Stiglitz (1982,1987)). Our result is important because it gives a new rationale for fiscal policies aiming at stimulating labor demand, such as the ones that have been recommended by e.g. Drèze and Malinvaud (1994) or Phelps (1997). Unobservable productivities put a constraint on income redistribution. Lowering gross wages below their efficient level is then a key ingredient to mitigate the effects of this constraint.

It is also analytically shown that average tax rates are increasing through the whole distribution of skills.Progressive taxation (in the sense of Musgrave and Musgrave (1976)) is therefore a main feature of the optimal tax schedule. It is not as detrimental as it is generally believed. There is clearly no equivalent property in the Mirrlees framework.

In our framework, with a bounded distribution of productivities, marginal tax rates are positive at the top of the distribution. Distorting employment at the top generates no equity gain. The gross wage and the employment rate of the most productive workers are therefore at their *laissez-faire* levels. However, the positive tax level at the top generates wage pressure that has to be compensated by a positive marginal tax rate. In the standard literature, marginal tax rates should be nil at the top of the distribution, since there are no gains from distorting labor supply.

Our over-employment result is nevertheless tempered by a reduction in labor-force participation compared to the *laissez-faire* economy. Our model introduces a participation decision. With a single value of inactivity, every individual who is less productive than an endogenous threshold does not search for a job. Despite its efficiency cost, increasing this threshold (i.e. raising the number of welfare recipients) allows to reduce the informational rent that accrues to more productive workers. This explains why labor-force participation may be lower than in a *laissez-faire* economy.

In our model, the least skilled employed workers may receive a negative income tax which is lower than the welfare benefit. This result is in line with the standard optimal income taxation literature. Saez (2002) Boone and Bovenberg (2001 and 2002b) challenge this view. They recommend higher in-work benefits than welfare benefits. This result does not come out in our framework. The introduction of a continuous extensive margin in our model (e.g. though endogenous search intensity) might change this result.

To illustrate the above results, we also develop various numerical simulations. They should be taken more as suggestive insights than as policy recommendations. Marginal tax rates appear to be high. Depending on the chosen parameters, they lie between 55% and 75%. The optimal tax schedule turns out to be close to linear. For a given set of parameters, marginal tax rates fluctuate in a range that is smaller than 10 percentage points. Marginal tax rates are generally decreasing along the distribution. When the intensive labor supply margin is added, simulations suggest that the over-employment property remains valid for the bottom of the distribution but not for the top. This is because positive marginal tax rates reduce working hours compared to the *laissez-faire*. The introduction of the intensive labor supply margin also dramatically decreases the levels of marginal tax rates and of the welfare benefit.

Finally, we check to what extent our setting and the Mirrlees one generate different tax schedules. To do so, the Mirrlees model is extended to allow for exogenous involuntary unemployment. This model is calibrated so as to generate the same sensitivity of earnings with respect to marginal tax rates as our model without intensive margin. Our policy recommendations turn out to be dramatically different since we recommend a welfare benefit about three times higher and marginal tax rates always more than twice higher.

Our main contribution is methodological since we build a model where the efficiency distortions induced by income taxation are due to matching and wage bargaining instead of the standard consumption-leisure trade-off. To what extent our assumptions are empirically relevant remains an open question. There is first some evidence concerning the wage moderating effect of tax progressiveness in wage bargaining models. The time series regressions by Malcomson and Sartor (1987) for Italy, Lockwood and Manning (1993) for the UK, or Holmlund and Kolm (1995) for Sweden lend support to this mechanism (see Sorensen (1997) for a survey). A second literature surveyed by Blundell and MaCurdy (1999) is concerned with labor supply responses in micro data. Elasticities of labor supply along the intensive margin are rather low for men, with a typical estimate between 0.1 and 0.5, but are higher for married women. However, only hours are observable, not in-work effort. This suggests that labor supply responses should be better estimated by looking at gross earnings instead of working hours responses (see Feldstein (1995) and Gruber and Saez (2002)). However, these estimates can be either interpreted as labor supply or wage bargaining responses. Both mechanisms predict that individual gross earnings are increasing with the level of taxes (when leisure is assumed to be a normal good in labor supply models) and decreasing with marginal tax rates. Predictions differ only as far as the effects on employment, working hours and hourly wages are concerned. Few papers have exploited such data. The estimates by Hansen, Perdersen and Slok (2000) conclude that higher tax progression decreases hourly wages for blue collar workers, suggesting that the wage bargaining mechanism dominates the labor supply effect for these workers. However, the reverse turns out to be true for white collar workers.

The consideration of labor demand is not absent in the standard literature. The typical model assumes an aggregate production function with perfect substitution between the different types of labor. Hence, hourly gross wages equal marginal products of labor and are independent of taxation. Stiglitz (1982) considers a two-skill model with imperfect substitution between high and low-skilled labor. The marginal tax rate should then be negative at the top. This increases high-skill labor supply and employment and so reduces the hourly wage skill premium. Marceau and Boadway (1994) extend Stiglitz's (1982) model by introducing a minimum wage that generates unemployment for low skilled workers. However, when marginal productivity is decreasing, the derivation of a non-trivial labor demand requires a finite number of skills. The matching model of Mortensen and Pissarides (1999) and Pissarides (2000) provides an interesting alternative since it allows the derivation of tractable labor demand functions for each type of job. Engström (2002) extends the model of Stiglitz (1982) in a matching setting. His analytical results are developed for exogenous wages. They emphasize complementarities between the intensive margin and job creation. Our analytical framework is different since we consider a continuum of agents, fixed working hours but endogenous wages. His simulation results with endogenous wages are related to ours. In particular, he emphasizes that employment is a key feature of the redistributive scheme.

The presence of matching frictions raises new questions in welfare economics. The normative analyses developed in this framework have put strong emphasis on the conditions under which the allocation of resources is efficient. Hosios (1990) first established the condition under which the output generated by additional jobs is equal to the resources needed to create additional vacancies in a *laissez-faire* economy. When this condition is not fulfilled, labor-income taxation is one of the instruments that can be used to restore efficiency (see. Boone et Bovenberg (2002a)). We do not further explore this research avenue. Taking the Hosios condition for granted, the labor-income tax schedule is chosen so as to maximize a social welfare function. The desire to redistribute income between skill-groups will as usual create an equity-efficiency trade-off but the latter will come out in an enriched theoretical setting.

The paper is organized as follows. Section II presents the model and derives the analytical results. Section III is devoted to numerical simulations. In section IV, we do two things. First, we introduce endogenous working hours in our model. Second, we compare the optimal schedule in our setting with exogenous working hours to the optimum in a Mirrlees-type setting. Section V concludes.

II The model

We consider a static model [1] where jobs differ according to their exogenous productivity denoted by $a \in [a_0, a_1]$ with $0 \leq a_0 < a_1 < +\infty$. The intensive margin is not taken into account here but will be introduced in Section IV. Workers and firms are assumed to be risk neutral. Type-a active worker search for a type-a job. Firms open type-specific vacancies. Each vacancy has to be filled with a single searching worker. Matching workers and vacancies is a time-consuming and costly activity. Following Mortensen and Pissarides (1999) and Pissarides (2000), we consider a well-behaved matching function that gives the number of type-a jobs formed as a function of the number U_a of searching workers and the number V_a of vacancies. Employment in segment a is an increasing and constant-return-

[1] Our static model simplifies the dynamic version of the matching model but still captures its major mechanisms (see e.g. Boone and Bovenberg (2002a))

to-scale function $H(U_a, V_a)$. This matching function implicitly captures heterogeneities, frictions and information imperfections on the labor market.

The size of the population is normalized to 1. Workers' types are distributed according to a continuous density $f(.)$ and a c.d.f. $F(.)$. These functions are common knowledge. The productivity is only known by the workers and the employers. The income tax schedule consists in a non-linear tax function $T(.)$ and an untaxed welfare benefit b. Since the government observes gross wages but not productivity, $T(.)$ is only based on gross income [2]. The assistance benefit b is distributed to searching and non-searching jobless individuals. Thus we assume that job search cannot be monitored by the government. For each type a, w_a, L_a and x_a denote respectively the gross wage, the employment rate and the workers' surplus in case of employment, with $x_a \equiv w_a - T(w_a) - b$. Hence, type-$a$ employed workers receive $w_a - T(w_a) = x_a + b$. Let $d > 0$ be the value of inactivity. Irrespectively of their type, non-searching (respectively, searching unemployed) individuals receive $b + d$ (resp. b).

Posting a type-a vacancy costs κ_a. This parameter captures the cost of screening applicants and the investment cost of creating a workstation. A type-a filled (respectively unfilled) vacancy yields a surplus of $a - w_a - \kappa_a$ (resp. $-\kappa_a$) to the firm-owner. In the literature, the vacancy cost is either taken as fixed or as proportional to productivity (see e.g. Pissarides (2000)). We therefore assume that [3]:

$$0 \leq \frac{\dot{\kappa}_a}{\kappa_a} \leq \frac{1}{a} \tag{1}$$

The timing of the model is:

1. The government commits to a taxation scheme $T(.)$ and a level of benefit b.
2. Firms open vacancies and workers decide whether or not they search for a job.
3. Matching occurs. For each job, the firm and the worker negotiate their wage.
4. Transfers accrue to the agents.

The model is solved backward.

II.1 The matching process

Following empirical studies (see Blanchard and Diamond (1989) or the recent survey by Petrongolo and Pissarides (2001)), we assume a Cobb-Douglas matching function $H_a = (U_a)^\gamma \cdot (V_a)^{1-\gamma}$ with $\gamma \in (0,1)$. Let $V_a = \theta_a \cdot f(a)$ denote the measure of type-a vacancies.

[2] Tax evasion is here neglected. Therefore reported earning levels are perfectly and costlessly observed.
[3] A dot over a variable denotes the total derivative with respect to type a (e.g $\dot{\kappa}_a = d\kappa_a/da$).

All type-a individuals either search for a job or stay inactive. When they search, their measure is $U_a = f(a)$. Their probability of finding a job is then $L_a = H_a/U_a = (\theta_a)^{1-\gamma}$. The probability of filling a type-a vacancy is therefore $H_a/V_a = \theta_a^{-\gamma}$.

The expected return of posting a vacancy is $(\theta_a)^{-\gamma}(a - w_a) - \kappa_a$. The higher the gross wage w_a, the lower this return. Firms enter freely the market and post vacancies as long as this return is positive. Therefore, this return is nil at equilibrium. One can then derive the type-a labor demand:

$$L_a = \left(\frac{a - w_a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}} \tag{2}$$

Let $Y_a \equiv L_a \cdot a - \theta_a \cdot \kappa_a$. So, $Y_a \cdot f(a)$ denotes total output net of search costs in the type-a labor market. The free entry condition implies that net output equals the workers' expected gross income $L_a \cdot w_a$. So,

$$Y_a(w_a) = w_a \cdot \left(\frac{a - w_a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}} \tag{3}$$

Efficient gross wage and employment are given by the values that maximize this net output:

$$w_a^* = \gamma \cdot a \qquad L_a^* = (1-\gamma)^{\frac{1-\gamma}{\gamma}} \cdot \left(\frac{a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}} \tag{4}$$

The matching frictions imply that the efficient level of employment L_a^* is below 1 [4]. To increase employment above L_a^*, firms have to open more vacancies. The resources spent to create these vacancies are not offset by the increase in gross output. Matching frictions therefore imply that full employment is not optimal. Equations (1) and (4) imply that the efficient level of employment is non-decreasing in a.

II.2 The wage bargain

Once a firm and a worker have been matched, they bargain over the wage. In the absence of an agreement, nothing is produced and the worker gets the welfare benefit b. These outside options imply the existence of a positive rent. As it is standard in the literature, this rent is shared by maximizing a Nash product. The wage w_a maximizes the Nash product depending on the worker's and the firm's surplus if they match,

$$[w_a - T(w_a) - b] \cdot [a - w_a]^{\frac{1-\beta}{\beta}}$$

where $\beta \in (0, 1)$ denotes the worker's bargaining power.

¿From Equation (2), it can be seen that maximizing the Nash product is equivalent to maximizing workers' expected surplus $x_a L_a$ provided that workers' bargaining power β

[4]It is assumed that κ_a is "sufficiently" high to get $L_a^* < 1$.

coincides with the elasticity of the matching function γ. Our assumption that $\beta = \gamma$ is the so-called Hosios (1990) condition. It states that the relative weight of the firm's surplus in the Nash product $(1-\beta)/\beta$ is equal to the elasticity of labor demand with respect to the firm's surplus $(1-\gamma)/\gamma$. It implies that the *laissez-faire* economy with neither taxes nor benefits yields an efficient level of employment. The negotiated wage then solves:

$$\max_{w_a} \quad [w_a - T(w_a) - b] \cdot [a - w_a]^{\frac{1-\gamma}{\gamma}}$$

taking $T(.)$ and b as given. We define Σ_a by:

$$\Sigma_a \equiv \max_{w_a} \quad x_a \cdot L_a = [w_a - T(w_a) - b] \cdot \left[\frac{a - w_a}{\kappa_a}\right]^{\frac{1-\gamma}{\gamma}} \tag{5}$$

The first-order condition leads to:

$$w_a = \frac{\gamma(1 - T'_a)\, a + (1-\gamma)(T_a + b)}{1 - \gamma \cdot T'_a} \tag{6}$$

where $T'_a \equiv T'(w_a)$ denotes the marginal tax rate and $T_a \equiv T(w_a)$ denotes the level of taxes. Furthermore, Equation (6) can be rewritten as:

$$w_a - T_a - b = \frac{\gamma(1 - T'_a)}{1-\gamma}(a - w_a) \tag{7}$$

Since the firm's or the worker's surplus cannot be negative, we have:

$$T'_a \leq 1$$

Hence, the worker's surplus x_a is necessarily an increasing function of the gross wage w_a.

The tax schedule influences the labor market equilibrium in two ways. First, higher levels of taxes T_a or benefits b reduce the rent to be shared and therefore increase the gross wage to offset the reduction in workers' surplus x_a. Hence, this wage pressure effect reduces employment. This effect on gross earning is similar to what occurs in a labor supply framework when leisure is a normal good. There, a rise in the level of taxes at given marginal tax rates increases labor supply.

Second, keeping T_a constant, the wage is decreasing in the marginal tax T'_a rate because a unit rise in the gross wage increases net earnings at a rate of one minus the marginal tax rate. As the marginal tax rate rises, the worker earns less from each increase in gross wages while the effect on firms' profits remains unchanged. Therefore, workers have less incentives to claim higher wages. A rise in the marginal tax rate can be interpreted as a decrease in the worker's relative *effective* bargaining power. This wage moderating effect is well known in the literature (see Malcomson and Sartor (1987), Lockwood and Manning (1993), Holmlund and Kolm (1995), Pissarides (1998), Sorensen (1999) or Boone

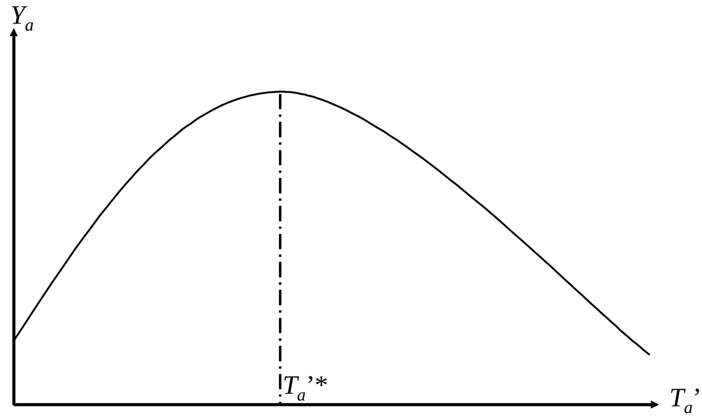


Figure 1: The impact of the marginal tax rate on efficiency at given taxes.

and Bovenberg (2002a) among others). As in labor supply frameworks, higher marginal tax rates decrease gross earnings. However, in the labor supply literature, the channel is different because there gross earnings are decreasing due to lower working hours.

Keeping the level of tax T_a unchanged, a rise in the marginal tax T_a' has a non monotonous effect on net output Y_a. Employment L_a and therefore gross output $a \cdot L_a$ and total vacancy costs $\kappa_a \cdot \theta_a$ increase. When employment is lower (respectively higher) than its efficient value L_a^*, the effect on gross output (resp. on vacancy costs) dominates, so net output increases (resp. decreases). Hence the relation is hump shaped as depicted in Figure 1. The "efficient marginal tax rate" $T_a'^*$ is the one that maximizes net output for a given level of tax. From equations (4) and (6):

$$T_a'^* = \frac{T_a + b}{\gamma \cdot a} \tag{8}$$

As a consequence, the efficient marginal rates is increasing with the level of taxes.

II.3 Incentive and participation constraints

The government faces an adverse selection problem since it observes only gross wages and not jobs' types a. In such a second-best environment, the government can only infer workers' productivities from the observation of negotiated wages. According to the revelation principle, it is equivalent to design a tax function $T(w_a)$ which depends on the observed gross wage w_a or to design a menu of contracts (w_a, T_a) through a truthful direct revelation mechanism.

The asymmetric information problem seems to be non standard here since two parties with conflicting interests are involved: the firm and the worker. An employer occupying a type-a worker wishes to mimic less productive jobs. Mimicking less productive jobs means paying a lower wage than the one designed for type-a jobs. However, the employer cannot unilaterally decide on such a deviation because the employee would disagree and the match would be interrupted. Symmetrically, the employee wishes to mimic more productive jobs, but he cannot unilaterally deviate. The firm and the employee have to agree on a single message (the gross wage w_a). Under Nash bargaining, this message/wage maximizes the Nash product. The incentive compatibility constraints therefore write:

$$\left(\frac{a-w_a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}}(w_a-T(w_a)-b)>\left(\frac{a-w_{a'}}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}}(w_{a'}-T(w_{a'})-b)\qquad\forall\ a'\neq a\quad(9)$$

Standard principal agent techniques then apply (see Salanié (1997) or Laffont and Martimort (2002)).

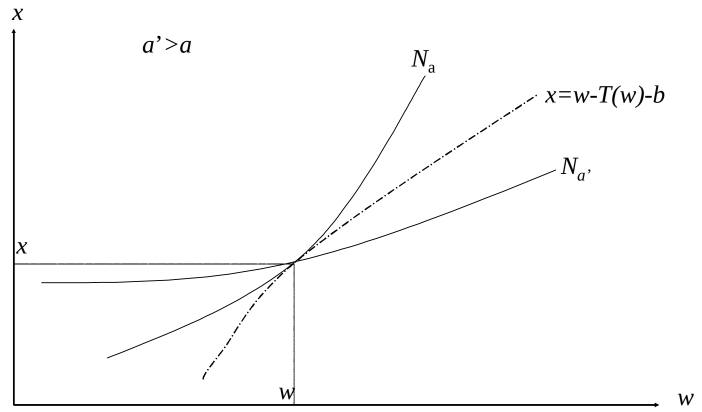


Figure 2: The single crossing property

The Nash product depends on type a, gross wage w and worker's surplus x in the following way $N(a,w,x)\equiv\left(\frac{a-w}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}}x$. The worker's surplus x has to increase when the gross wage w increases to keep the Nash product $N(a,.,.)$ unchanged. Lower surplus for the firm must be offset by higher surplus for the worker. However, for each pair (w,x) the marginal rate of substitution:

$$\left.\frac{\partial x}{\partial w}\right|_{N(a,.,.)}=\frac{1-\gamma}{\gamma}\frac{x}{a-w}$$

is a decreasing function of the type a. This single crossing property is illustrated in Figure 2. This figure displays the indifference curves in terms of the Nash product $N(a,.,.)$ for two jobs with different abilities $a'>a$. The higher the productivity of a match, the less elastic is the firm's surplus to the gross wage, so the less sensitive is the Nash product to changes in the gross wage.

For convenience, we consider that the government designs a menu of contracts in terms of (w_a,Σ_a), which by (5) is perfectly equivalent to a menu in terms of (w_a,T_a). Applying the envelope theorem to equation (5) gives the first order incentive compatibility constraint (see Appendix VI.1):

$$\dot{\Sigma}_a = \frac{1-\gamma}{\gamma}\left(\frac{1}{a-w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right)\Sigma_a \tag{10}$$

Equations (1) and (10) imply that the expected worker's surplus or equivalently the maximized Nash product Σ_a is increasing with respect to productivity [5].

A worker decides to search at stage 2 as long as its expected utility when searching $\Sigma_a + b$ is higher than its utility without searching $b + d$. The participation constraint can then be written as:

$$\Sigma_a \geq d \tag{11}$$

Since the expected worker's surplus Σ_a is increasing in a and since d is by assumption identical for all types, there exists a single threshold $a_d \geq a_0$ such that workers endowed with $a < a_d$ stay inactive whereas workers with $a \geq a_d$ search for a job.

II.4 The government's problem

How productivity levels are allocated in the population is out of the scope of this article. People are simply not held responsible for their productivity. So, the government is ready to compensate for differences in productivity levels. Each type of worker faces in addition the risk of being unemployed. This typically calls for another public intervention through an unemployment insurance scheme. Dealing with unemployment insurance requires that workers be risk averse. Introducing risk aversion in the framework presented above complicates the analysis. To keep things as simple as possible, the government is by assumption not concerned with the unemployment risk. The government is therefore concerned with the distribution of expected utilities, namely $L_a (w_a - T(w_a)) + (1 - L_a) b$ for those who are active and $b + d$ for inactive people. This approach could be further rationalized by considering a dynamic extension of our model with perfect capital markets. There, workers would move between employment and unemployment and become perfectly insured against the unemployment risk. Finally, as it will be shown later, our model leads to overemployment since the latter contributes to the fulfillment of the incentive compatibility constraint. To highlight this new effect, we have chosen to ignore the insurance effect that might be another reason for overemployment.

[5] It is shown in the appendix that the second order condition writes $\dot{w}_a > 0$. Given the single crossing property, first-order and second-order conditions are equivalent to the incentive compatibility constraints (9) (see Salanié (1997)). Throughout the paper, we consider only the first-order condition and verify ex-post in our simulations that $\dot{w}_a > 0$.

We consider therefore the following social objective:

$$\Omega = F(a_d)\,\Phi(b+d) + \int_{a_d}^{a_1} \Phi\left[L_a\left(w_a - T(w_a)\right) + (1-L_a)\,b\right] f(a)\,da$$

where $\Phi'(.) > 0$ and $\Phi''(.) < 0$. Given equation (5), this objective can be rewritten as:

$$\Omega = F(a_d)\,\Phi(b+d) + \int_{a_d}^{a_1} \Phi(\Sigma_a + b)\, f(a)\,da \tag{12}$$

The government faces a budget constraint

$$\int_{a_d}^{a_1} T(w_a)\cdot L_a \cdot f(a)\,da = \left[F(a_d) + \int_{a_d}^{a_1}(1-L_a)\,f(a)\,da\right] b + E$$

where $E \geq 0$ is an exogenous amount of public expenditures. Using Equations (3) and (5), it is convenient to rewrite this constraint as:

$$\int_{a_d}^{a_1} (Y_a - \Sigma_a)\cdot f(a)\,da = b + E \tag{13}$$

Optimal taxation therefore solves the following problem

$$\max_{a_d, w_a, \Sigma_a, b} \quad F(a_d)\,\Phi(b+d) + \int_{a_d}^{a_1} \Phi(\Sigma_a + b)\, f(a)\,da \tag{14}$$

$$s.t.: \int_{a_d}^{a_1} \left(Y_a(w_a) - \Sigma_a\right)\cdot f(a)\,da = b + E$$

$$\dot{\Sigma}_a = \frac{1-\gamma}{\gamma}\left(\frac{1}{a-w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right)\Sigma_a \qquad \begin{cases} \Sigma_{a_d} = d \\ \Sigma_{a_d} \geq d \end{cases} \text{if} \quad \begin{matrix} a_d > a_0 \\ a_d = a_0 \end{matrix}$$

For any value of a_d and b, this is a standard optimal control problem where workers' expected surplus Σ_a is the state variable and the gross wage w_a is the control variable.

II.5 Characterization of the optimum

The first-order conditions lead to the following formulation of the equity-efficiency trade-off (see Appendix VI.2):

$$\lambda \cdot \frac{\partial Y_a}{\partial w_a}\cdot f(a) = \frac{1-\gamma}{\gamma\,(a-w_a)^2}\int_a^{a_1} \left\{\lambda - \Phi'_t\right\}\Sigma_t \cdot f(t)\cdot dt \tag{15}$$

where λ is the Lagrange multiplier associated with the budget constraint, $\Phi'_a = \Phi'(\Sigma_a + b)$ and for $a < a_d$, $\Phi'_a = \Phi'(b+d)$.

Consider an increase of one unit in the wage of type-a workers that lets type-a's expected surplus constant. This decreases the employment level L_a and thereby gross output $L_a \cdot a$ but also the resources spent on posting vacancies. Therefore, the effect on net output Y_a is ambiguous. If $w_a < w_a^*$ (resp. $>$), the total effect is positive (resp. negative). Multiplying this by the number of type-a agents $f(a)$ and the shadow cost of public funds λ, the left-hand side of (15) measures the social value of the net marginal change in output. This captures the efficiency side of the trade-off.

The right-hand side of (15) represents the equity cost of a higher gross wage for type-a workers. When agents endowed with productivity a earn higher gross wages, more productive agents find it more attractive to mimic them. To prevent this, the expected surplus accruing to more productive workers has to grow. The term in front of the integral measures the rate at which the growth rate of the worker's expected surplus has to increase to prevent slightly more productive workers from mimicking type a individuals (see Equation (10)). Neglecting second-order effects, the incentive compatibility constraints will remain satisfied above a if all workers with a productivity higher than a benefit from an equivalent relative increase in their expected surplus. As it is explained in Appendix VI.2, the integral corresponds to the shadow cost of a relative increase in the expected surplus of more productive workers. For any type-t above a, this marginal relative increase leads to an absolute rise in expected surplus equal to Σ_t times the relative increase. This additional expected surplus generates an increase in the social welfare measured by Φ'_t, but implies a budgetary cost equal to λ.

The proof of the following properties is left to the appendix.

Proposition 1 *Employment is efficient at the top of the distribution.*

The right-hand side of (15) indicates that increasing the gross wage at the top has no distributional effect. The government can therefore set this gross wage at the level $w^*_{a_1}$ which maximizes net output.

Proposition 2 *For all types with productivity $a_d \leq a < a_1$, employment is above its efficient level.*

Consider that the tax schedule has been optimized for all workers up to type-a. The expected utility of type-a workers Σ_a being predetermined by the incentive compatibility constraints (9), let w_a decrease below its efficient value w^*_a. Firms' surplus and employment for type-a workers are then raised. In order to keep Σ_a constant, x_a has to decrease [6]. So doing, the workers' surplus in employment decreases but their expected utility remains unaffected. By the single-crossing property, the incentive compatibility constraint (9) can now be fulfilled by attributing a lower level of expected surplus to each more productive worker. Since the affected individuals have lower marginal social welfare, the social gain from relaxing the budget constraint outweighs the loss in welfare for these workers. However, decreasing w_a below its efficient value w^*_a has also an efficiency cost since firms are

[6]This combined movement in w_a and x_a is implemented by rising the marginal tax rate.

induced to post too many vacancies. The optimum solves this equity-efficiency trade-off. These mechanisms are illustrated in Figure 3.

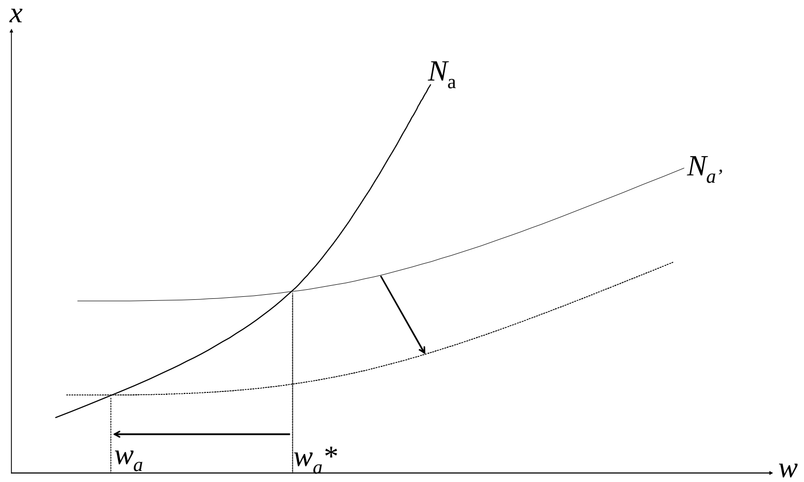


Figure 3: The overemployment effect

Another intuition for proposition 2 is given by Figure 1. Keeping the level of taxes unchanged up to T_a, a rise in the marginal tax T'_a creates an equity gain since it allows to tax richer workers more heavily. At the optimum, this equity gain is compensated by a loss in efficiency. According to Figure 1, the marginal tax rate is then necessarily higher than the efficient one. Consequently, there is overemployment.

Proposition 2 implies that policymakers should consider the stimulation of labor demand as a crucial redistributive tool. The next proposition deals with the adverse effect of this stimulation on participation.

Proposition 3 *The participation rate is lower than or equal to its value in the laissez-faire economy.*

Net output and therefore efficiency is increased if individuals of type $a_d \geq a_0$ participate. But their participation also gives to agents with a productivity above a_d the possibility to mimic them. To avoid this mimicking, the government has to give an additional informational rent to these more productive individuals. If this equity cost is higher than the efficiency gain, the government prefers that individuals of type a_d do not participate.

Proposition 4 *In-work benefits (if any) are lower than assistance benefits.*

This is a consequence of the previous proposition. In-work benefits that are higher than assistance benefits increase participation. However, the previous proposition shows that the governement chooses not to increase participation. This proposition implies that an EITC system would not be optimal at the second best.

Proposition 5 *Average tax rates are increasing in wages. Marginal tax rates are positive everywhere.*

The first part of this proposition states that the tax schedule $T(.)$ has to be progressive in the sense of Musgrave and Musgrave (1976). This means that the coefficient of residual income progression is below 1 everywhere. The importance of this conclusion should be stressed since standard optimal income taxation models do not yield precise analytical results about the shape of average tax rates. With a bounded distribution of productivity, this literature has shown that the marginal tax rate should be equal to zero at the top. Therefore, one only knows that the average tax rate should necessarily be decreasing close to the top of the distribution.

Since the value of inactivity is unique, the second part of Proposition 5 is in accordance with common wisdom [7], except at the top of the distribution. The reason why the marginal tax rate should be positive at the top is easily understood. As the government wants to redistribute income in favor of less productive agents, the level of taxes is positive at the top. This distorts the gross wage upwards. To restore an efficient level of employment (Proposition 1), a positive marginal tax rate is therefore needed at the top.

III Simulations

This section provides order of magnitudes and gives quantitative insights when analytical results are lacking.

III.1 Calibration

The parameters are chosen to roughly represent the key figures for France. The individuals' abilities are taken to be in the range of $[1000, 10000]$. This might be a rather realistic approximation of workers' productivities per month, measured in Euros. We use a truncated log-normal distribution function of the form:

$$f(a) = \frac{K}{a} \exp\left(\frac{\log a - \log\left(\mu \cdot a_1 + (1-\mu)\, a_0\right)}{2 \cdot \xi^2}\right)$$

where K is the appropriate scale parameter. This form is typical in the literature (Mirrlees (1971), Tuomola (1990) and Boadway et al. (2000)). The parameters of the distribution function are chosen equal to $\mu = 0.5$ and $\xi = 0.7$. Figure 4 shows in solid lines the distribution of a in our benchmark case.

[7] See Saez (2002) or Boone and Bovenberg (2002b) for a critical appraisal of this wisdom.

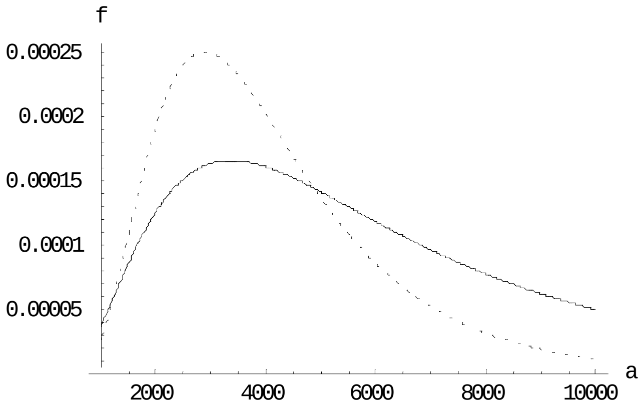


Figure 4: Density Function $f(a)$ in the Benchmark Case in solid line. Dotted line for $(\mu, \xi) = (0.3; 0.5)$.

σ	γ	δ	μ	ξ	E	L_a^*
1	0.5	1	0.5	0.7	0	0.7

Table 1: Parameters values in the benchmark case

The elasticity of the matching function γ is set at 0.5. This corresponds to the average estimates in empirical models (see Petrongolo and Pissarides 2001). Since the Hosios condition is assumed, the worker and the firm have equal bargaining power. Furthermore, we assume that vacancy costs are proportional to productivity. This assumption is usual in equilibrium search models (see Pissarides, 2000), even though empirical support is missing. The vacancy cost κ_a is adjusted to get an employment level of 0.7 in the *laissez-faire* economy. In the benchmark case, we assume that the value of leisure represents a fraction $\delta = 1$ of the surplus that the least able individual gets in the *laissez-faire* economy. Therefore, in the *laissez-faire* economy, everyone is searching for a job, whereas this has not to be the case in the second-best optimum. The government's expenditures E are set equal to 0. Finally, the government's utility function is assumed to be a CES function of the expected surplus. We have, $\Phi(\Omega) = \Omega^{1-\sigma}/(1-\sigma)$. In the benchmark case we take the elasticity σ equal to 1. This corresponds to the basic parameterization in Saez (2002). Table 1 summarizes the parameters values.

III.2 The benchmark

Figure 5 illustrates the propositions in section II. The upper-left panel displays the after-tax income level C as a function of the gross wage w. The upper-right panel displays the employment rate L as a function of the productivity level a. The panels at the bottom show the levels of taxes T and of marginal tax rates T_m as functions of gross wages. Dotted lines corresponds to the *laissez-faire* economy. Table 2 displays the main features of the optimum. LF denotes the *laissez-faire*, SB the second best optimum and Δ the relative differences of second-best values compared to *laissez-faire* ones.

Employment is significantly above its efficient level except at the top. Over-employment is more pronounced for low productivity levels. Total informational rents needed to prevent mimicking a given type of worker increases as the type decreases because more workers are then concerned by mimicking. The participation rate equals 95%. The government therefore excludes some low-ability individuals from the labor market. Total employment is nevertheless higher than in the *laissez-faire* economy.

Due to these employment distortions, total output net of vacancy cost is 1.9% lower than in the *laissez-faire*. However, redistribution increases social welfare (measured in certainty-equivalent $\Phi^{-1}(\Omega)$) by around 10%. While the top-bottom productivity ratio equals 10, the net income ratio is below 3. In addition the level of welfare benefit is quite high. The tax function turns out to be close to linear. Marginal tax rates are somewhat decreasing, from 66% to 57%.

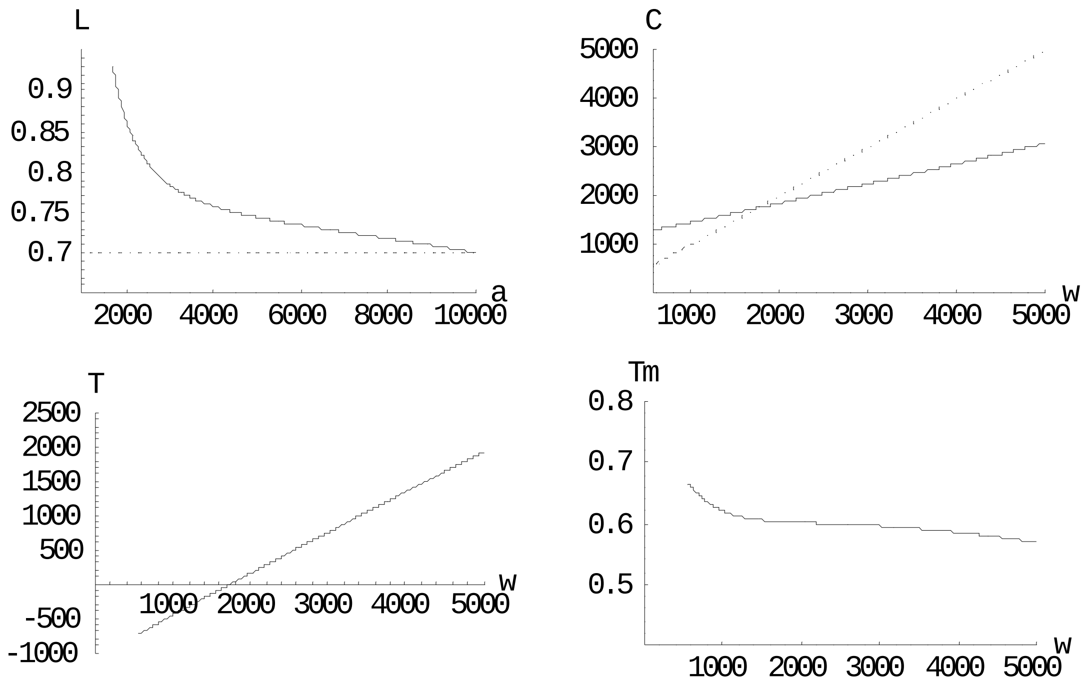


Figure 5: Benchmark Case. Laissez-faire in dotted lines.

III.3 Sensitivity analysis

The following section indicates to what extend the optimum is sensitive to changes in the main parameters.

	Employment L		Net output Y			Welfare $\Phi^{-1}(\Omega)$			b	Participation
	SB	Δ	LF	SB	Δ	LF	SB	Δ		
Benchmark	0.721	+2.9%	1731	1698	−1.9%	1536	1687	+9.8%	924	95.2%
$\sigma = 0.5$	0.715	+2.1%	1731	1709	−1.3%	1636	1703	+4.1%	842	96.3%
$\sigma = 2.0$	0.725	+3.7%	1731	1682	−2.8%	1334	1664	+24.8%	992	93.7%
$\delta = 0.5$	0.731	+4.5%	1731	1723	−0.5%	1536	1711	+11.4%	1200	99.1%
$\delta = 2$	0.631	−1.5%	1684	1594	−5.4%	1568	1687	+6.9%	575	82.2%
$\gamma = 0.4$	0.723	+3.3%	1385	1358	−2.0%	1229	1348	+9.7%	732	94.9%
$\gamma = 0.6$	0.719	+2.8%	2078	2040	−1.8%	1843	2027	+10.0%	1119	95.6%
$\mu = 0.3$ $\xi = 0.5$	0.728	+3.9%	1405	1377	−2.0%	1267	1365	+7.7%	671	95.9%

Table 2: Numerical results LF for laissez faire and SB for second best

III.3.1 The aversion to inequality

When the aversion to inequality increases, the government is ready to distort more the allocation of resources. When the inequality aversion is doubled, marginal tax rates shift upwards by about 5 percentage points (see Figure 6), and the assistance benefit varies slightly. These rather small changes might be explained by the fact that the benchmark case already leads to a very high degree of redistribution.

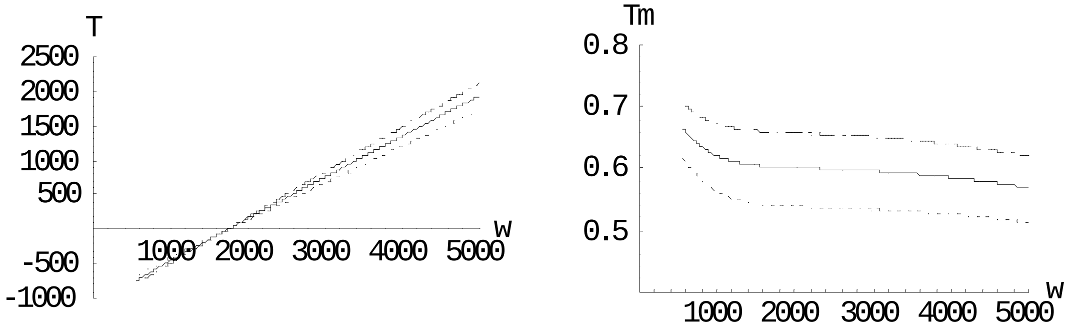


Figure 6: Dotted, solid and dashed lines respectively for σ equal to 0.5, 1 and 2.

III.3.2 The value of inactivity

When the value of inactivity increases, the participation constraint becomes more stringent. Therefore, the expected surplus of marginal participants increases. Keeping the type of marginal participants unchanged, the incentive constraints imply a higher expected surplus for all participants. The government is therefore forced to redistribute less. To mitigate this effect, the government lets the participation rate decline. As shown in Figure 7 and in Table 2, marginal tax rates shift downwards and welfare benefits decrease substantially. Hence, from a quantitative point of view, the optimal taxation is rather sensitive to this parameter.

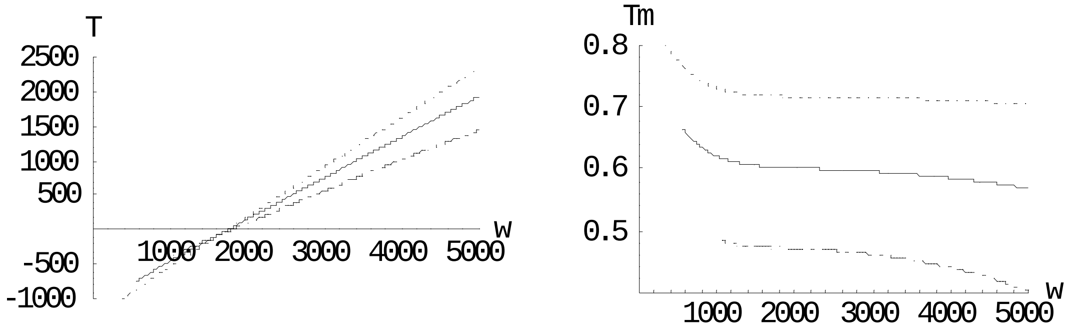


Figure 7: Doted, solid and dashed lines respectively for δ equal to 0.5, 1 and 2.

III.3.3 The labor demand elasticity and the bargaining power

We only consider variations that simultaneously change the elasticity of the matching function and the bargaining power so that the Hosios' condition remains satisfied. The vacancy costs are adjusted so as to keep the efficient employment rate unchanged when γ varies. The higher the level of γ, the higher is the effectivness of searching workers to generate matchings. Hence, the parameter γ is an efficiency parameter. As this parameter increases, the labor demand becomes less elastic. Put differently, taxation becomes less distortive. Consequently, welfare benefits increase and marginal tax rates shift upwards (see Figure 8 and Table 2).

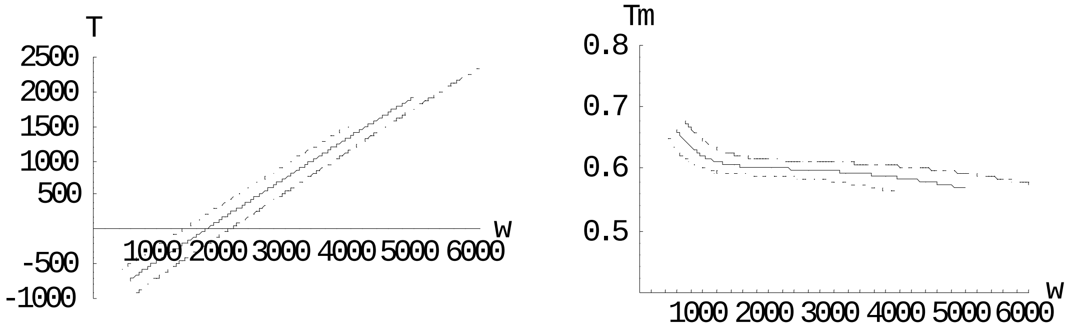


Figure 8: Dotted, solid and dashed lines respectively for γ equal to 0.4, 0.5 and 0.6.

III.3.4 The form of the distribution

As usual, in the optimal income taxation literature, the shape of the marginal tax rates is sensitive to the productivity distribution $F(.)$. To illustrate this, we consider an experiment displayed in Figure 4 where we keep roughly the value of the mode but decrease the variance of the distribution. Hence, we take $\mu = 0.3$ and $\xi = 0.5$. As Figure 9 and Table 2 show, the qualitative properties are not sensitive to such changes. This is confirmed by other unreported simulation results.

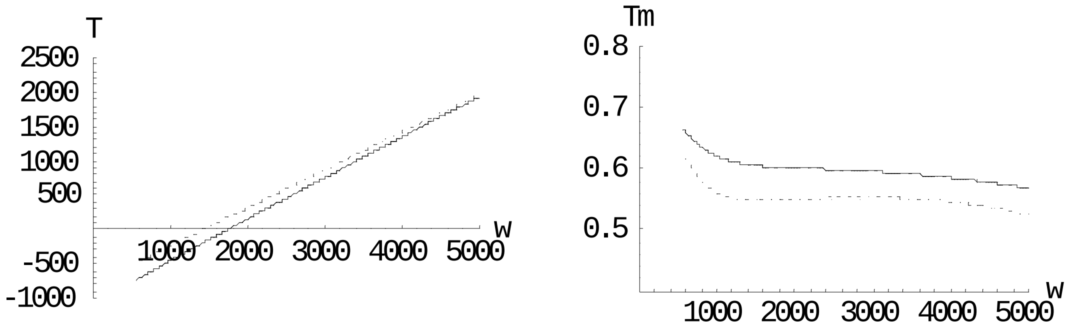


Figure 9: Solid lines for the Benchmark Case. Dotted lines for $(\mu; \xi) = (0.3; 0.5)$

IV In-work effort

This section has two different objectives. First, we conduct a sensitivity analysis by introducing in-work effort in the previous setting. Second, we conduct a numerical comparison between our methodological approach and the standard one.

IV.1 A mixed model with intensive margin

Following the standard literature *à la* Mirrlees (1971), neither working hours (or in-work effort) nor abilities can be observed by the government. The tax therefore only depends on the total gross wage w_a and not on working hours h_a. Contrary to the standard optimum income taxation literature, working hours are here not unilaterally chosen by the workers but are negotiated. Assuming that the utility function is quasi-linear in consumption, the worker's surplus in employment is $x_a \equiv w_a - T(w_a) - v(h_a) - b$ with $v(h_a)$, the disutility of work, $v(h_a)' > 0$ and $v''(h_a) > 0$. Let η denote the elasticity of labor supply, so $1/\eta = h \cdot v''(h) / v'(h)$. A type-$a$ filled vacancy yields a surplus of $a \cdot h_a - w_a - \kappa_a$ to the firm owner. One can then derive type-a labor demand:

$$L_a = \left(\frac{a \cdot h_a - w_a}{\kappa_a} \right)^{\frac{1-\gamma}{\gamma}}$$

Still assuming the Hosios condition, the level of output net of vacancy costs and of the disutility of work is:

$$Y_a(w_a, h_a) \equiv (a \cdot h_a - v(h_a)) L_a - \kappa_a \cdot \theta_a = (w_a - v(h_a)) \cdot \left(\frac{a \cdot h_a - w_a}{\kappa_a}\right)^{\frac{1-\gamma}{\gamma}} \tag{16}$$

The Nash product is now a function of the total wage and the working hours. Therefore the bargaining over wages and hours can be represented by the following problem:

$$\Sigma_a \equiv \max_{w_a, h_a} \quad [w_a - T(w_a) - v(h_a) - b] \cdot \left[\frac{a \cdot h_a - w_a}{\kappa_a}\right]^{\frac{1-\gamma}{\gamma}} \tag{17}$$

Using the first order condition with respect to working hours, one obtains:

$$\frac{\gamma \cdot v'(h_a)}{x_a} = \frac{a(1-\gamma)}{a \cdot h_a - w_a}$$

This implicitly defines the number of working hours $h_a \equiv h_a(w_a, \Sigma_a)$ (see appendix VI.8). The first-order condition with respect to wages leads to:

$$w_a = \frac{\gamma(1 - T'(w_a)) a \cdot h_a + (1-\gamma)(T(w_a) + b + v(h_a))}{1 - \gamma \cdot T'(w_a)}$$

Combining both conditions, one obtains:

$$v'(h_a) = a \cdot \left(1 - T'(w_a)\right) \tag{18}$$

Furthermore, we have:

$$\dot{\Sigma}_a = \frac{1-\gamma}{\gamma}\left(\frac{h_a(w_a, \Sigma_a)}{a \cdot h_a(w_a, \Sigma_a) - w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right)\Sigma_a \geq 0 \tag{19}$$

Optimal taxation therefore solves the following problem:

$$\max_{a_d, w_a, \Sigma_a, b} \quad F(a_d)\Phi(b+d) + \int_{a_d}^{a_1} \Phi(\Sigma_a + b) f(a)\, da \tag{20}$$

$$s.t.: \int_{a_d}^{a_1} \{Y_a[w_a, h_a(w_a, \Sigma_a)] - \Sigma_a\} \cdot f(a)\, da = b + E$$

$$\dot{\Sigma}_a = \frac{1-\gamma}{\gamma}\left(\frac{h_a(w_a, \Sigma_a)}{a \cdot h_a(w_a, \Sigma_a) - w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right)\Sigma_a \qquad \begin{cases} \Sigma_{a_d} = d \\ \Sigma_{a_d} \geq d \end{cases} \text{if} \quad \begin{matrix} a_d > a_0 \\ a_d = a_0 \end{matrix}$$

The solution of this "mixed" model is rather hard to interpret, so only numerical results are presented. For this, we take the parameters of our benchmark economy and compare the optimal taxation when $\eta = 0$ (benchmark economy) and $\eta = 0.2$. We normalize the maximum working hours $h_{\max}$ at 1. The results are displayed in Table 3.

Eight main results are emphasized by the simulations. First, from the consumption-leisure effect, at the *laissez-faire,* working hours increase with the productivity of a match. Second, working hours are lower when redistribution matters. With positive marginal tax

η	Total employment L SB	Δ	Total hours $h \cdot L$ SB	Δ	Net output Y SB	Δ	Welfare $\Phi^{-1}(\Omega)$ SB	Δ	b	Participation
0	0.721	+2.9%	0.721	+2.9%	1698	−1.9%	1687	+9.8%	924	95.2%
0.2	0.489	−1.7%	0.384	−10.2%	926	−3.7%	871	1.9%	345	100%

Table 3: Numerical results LF for laissez faire and SB for second best

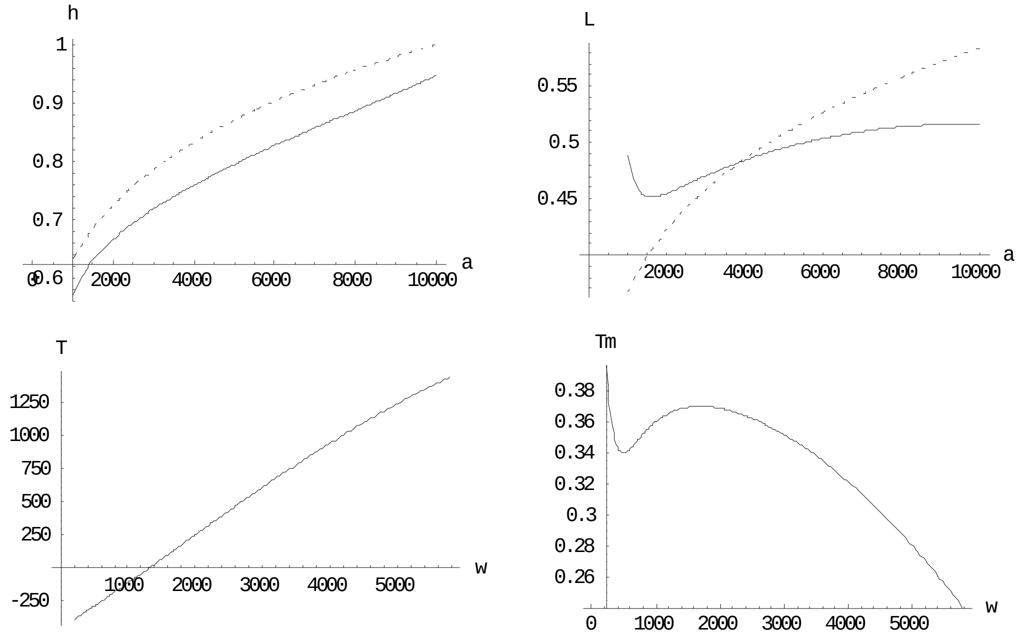


Figure 10: Intensive margin with $\eta = 0.2$. Dotted lines for the *laissez faire* and solid lines for the second best.

rates, the benefit from working an additional hour is lower while the cost is the same. Therefore, the firm and the worker bargain over less working hours.

Third, at the top of the distribution, the employment level is lower than at the *laissez-faire*. For hours to be efficient, the marginal tax rate should be equal to zero. To obtain the efficient level of employment, marginal tax rates should be positive. One instrument is therefore lacking to meet this requirement. The marginal tax rate at the upper end of the distribution is thus between zero and the marginal tax rate obtained when the intensive margin is ignored. Therefore, marginal tax rates are not sufficient to retore the efficient outcome and employment is lower. Fourth, by continuity, underemployment remains over a large range of the ability distribution. However, it turns out that the overemployment previously emphasized reappears and dominates at the bottom of the distribution. Fifth, total employment is nevertheless lower than at the *laissez-faire*.

Sixth, marginal tax rates are much lower than without in-work effort and are no more monotonously decreasing. Seventh, taxation is less negative at the low end and assistance benefits are lower when hours are endogenous. Since an additional margin is introduced, taxation is more distorsive and redistribution is more costly. Finally, participation is higher than our second best benchmark case. To reduce informational rents for participating workers, reducing labor supply along the intensive margin is a substitute for the reduction along the extensive margin.

IV.2 Comparing the two methodologies

Our main contribution in this article is methodological since we build a model where the efficiency distortions induced by income taxation are due to matching frictions and wage bargaining instead of the standard consumption-leisure trade-off. The quantitative importance of such a change is an issue that we now address.

To compare our model to a model that incorporates only the labor supply choice of the individuals, we build a model à la Mirrlees (1971). To do so, we eliminate the labor market frictions in our mixed model presented above by setting $\gamma = 1$. Employment rates are exogenous and assumed equal to $\tilde{L}_a$. Net output is thus given by $Y_a = \tilde{L}_a \left[a \cdot h_a - v\left(h_a\right)\right]$. Gross total wages w_a are equal to labor productivities $a \cdot h_a$ and (18) becomes a standard optimal labor supply condition

$$v'\left(h_a\right) = a \cdot \left(1 - T'\left(a \cdot h_a\right)\right) \tag{21}$$

Σ_a now corresponds to the workers' surplus $\Sigma_a = x_a \cdot \tilde{L}_a = \left[a \cdot h_a - v\left(h_a\right) - T\left(a \cdot h_a\right) - b\right] \tilde{L}_a$ and evolves according to:

$$\dot{\Sigma}_a = h_a \left(1 - T'\left(w_a\right)\right) \tilde{L}_a = h_a \frac{v'\left(h_a\right)}{a} \tilde{L}_a \tag{22}$$

Program (20) now writes:

$$\max_{a_d, h_a, \Sigma_a, b} \quad F\left(a_d\right) \Phi\left(b + d\right) + \int_{a_d}^{a_1} \Phi\left(\Sigma_a + b\right) f\left(a\right) da$$

$$s.t.: \int_{a_d}^{a_1} \left\{a \cdot h_a \cdot \tilde{L}_a - v\left(h_a\right) \cdot \tilde{L}_a - \Sigma_a\right\} \cdot f\left(a\right) da = b + E$$

$$\dot{\Sigma}_a = h_a \frac{v'\left(h_a\right)}{a} \tilde{L}_a \qquad \begin{cases} \Sigma_{a_d} = d \\ \Sigma_{a_d} \geq d \end{cases} \text{if} \quad \begin{matrix} a_d > a_0 \\ a_d = a_0 \end{matrix}$$

Optimal taxation verifies the following condition:

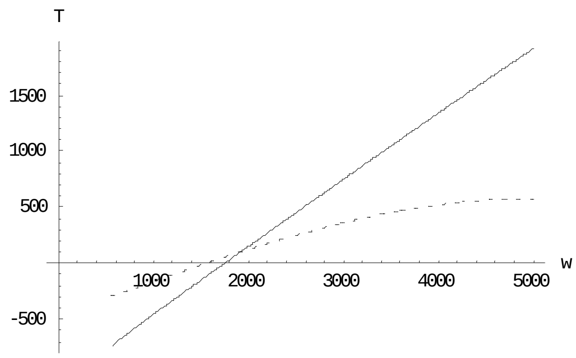
$$\frac{T'\left(w_a\right)}{1 - T'\left(w_a\right)} = \frac{\int_a^{a_1} \left(1 - \frac{\Phi'_t}{\lambda}\right) f\left(t\right) dt}{a \cdot f\left(a\right)} \left(1 + \frac{1}{\eta}\right)$$

which coincides with the classic formula provided by the standard literature when utility is quasi linear in consumption (see e.g. Diamond (1998)).

	Total employment L		Net output Y		Welfare $\Phi^{-1}(\Omega)$		b	Participation
	SB	Δ	SB	Δ	SB	Δ		
Benchmark	0.721	+2.9%	1698	−1.9%	1687	9.8%	924	95.2%
Mirrlees	0.697	−0.4%	1539	−11.1%	1077	5.2%	323	99.6%

Table 4: Numerical results LF for laissez faire and SB for second best

¿From now on, we refer to this model as "the Mirrlees setting". To compare it with the benchmark model of Sections II and III, we calibrate the Mirrlees model in such a way that at the *laissez-faire* both models have the same distributions of wages [8], and of employment rates and the same elasticity of gross wages with respect to one minus the marginal tax rate. From equation (6), this elasticity equals to $1-\gamma$ at the *laissez-faire* in our model [9], whereas it equals η in the Mirrlees model. Hence, we take $\eta = 0.5$.



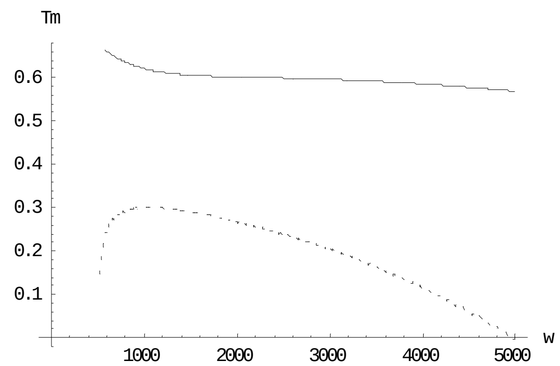


Figure 11: Solid lines for the Benchmark Case. Dotted for the Mirrlees model.

As Figure 11 and Table 4 show, the differences between the Mirrlees setting and ours are quantitatively very important. The optimum is much more redistributive when wages are bargained over and the intensive margin is neglected. Marginal tax rates are more than twice higher. Assistance benefits are almost three times greater. The gain in welfare is dramatically higher and the loss in output is much lower. Futhermore, the profile of marginal tax rates is substantially different.

Two major mechanisms are at work. First, the profiles of efficient marginal tax rates differ. In the Mirrlees model, efficient marginal tax rates are nil (since lump-sum transfers are the only way to redistribute income without distorting labor supply). Conversely, in our model, efficient marginal tax rates are positive according to proposition 5. Furthermore, according to unreported simulations, efficient marginal tax rates are increasing with

[8]This implies that the distribution of abilities in the "Mirrlees setting" has been appropriately reparametrized.

[9]We derive $\frac{\partial w_a}{\partial(1-T_a')}\frac{1-T_a'}{w_a}$ and evaluate this expression at $T_a' = T_a = b = 0$.

type a.

Second, in both models, marginal tax rates are above their efficient values, except at the top of the distribution. This prevents higher skilled workers from mimicking. As we move to the left of the distribution, the fraction of workers potentially involved in mimicking others increases. This generates a greater and greater upward pressure on marginal tax rates. However, in our model, the incentive compatibility constraint is expressed in terms of growth rates of workers' expected surplus (see Equation 10). In the Mirrlees version, the incentive constraint is formulated in terms of absolute changes (see Equation 22). Hence, the upward pressure on marginal tax rates is stronger at the low end of the distribution in our model.

V Conclusion

The optimal income taxation literature has essentially focused on distortions created through the consumption-leisure trade-off. This trade off is however not the unique way of explaining that earnings are affected by the profile of taxes. We have adopted an alternative setting where frictions on the labor market generate involuntary unemployment and rents to be shared by employers and employees. In this framework with exogenous working hours, the optimal income taxation has properties that strongly differ from those found in the Mirrlees competitive setting. Employment is higher than at the *laissez-faire* and average tax rates are increasing in wages. Compared to the prescriptions of a comparable Mirrlees setting, our numerical simulations show that assistance benefits are three times higher and marginal taxes are always more than twice higher.

In sum, estimating the elasticity of gross earnings with respect to taxes is not sufficient to derive clear policy recommendations about the optimal tax schedule. One needs in addition to clarify which theoretical setting is empirically the most relevant. We left this for further research.

This paper also allows for many interesting theoretical extensions. First the assumption that employment is efficient in the *laissez-faire* should be relaxed. Second the modeling of the extensive margin could be enriched. Finally, our contribution has been essentially methodological. Numerical simulations have therefore not tried to exploit rich datasets. All these extensions are also left for further research.

VI Appendix

VI.1 The incentive compatibility constraints

This section follows Salanié (1997) very closely. Let $\mathcal{N}(a,t)$ be the logarithm of the Nash product for a type-a job when the negotiated wage is the one designed for type t job. So

$$\mathcal{N}(a,t) \equiv \log N(a, w_t, x_t) = \frac{1-\gamma}{\gamma} \log\left(\frac{a-w_t}{\kappa_a}\right) + \log\left(w_t - T(w_t) - b\right)$$

and [10]:

$$\frac{\partial \mathcal{N}}{\partial a}(a,t) = \frac{1-\gamma}{\gamma}\left(\frac{1}{a-w_t} - \frac{\dot{\kappa}_a}{\kappa_a}\right) \qquad \frac{\partial^2 \mathcal{N}}{\partial a \partial t}(a,t) = \frac{1-\gamma}{\gamma}\frac{\dot{w}_t}{(a-w_t)^2} \tag{23}$$

Equation (9) means that the function $t \to \mathcal{N}(a,t)$ reaches a maximum for $t = a$. So $\log \Sigma_a = \mathcal{N}(a,a)$. The first-order condition can be written as $\frac{\partial \mathcal{N}}{\partial t}(a,a) = 0$. So, for any a

$$\frac{\dot{\Sigma}_a}{\Sigma_a} = \frac{\partial \mathcal{N}}{\partial a}(a,a) + \frac{\partial \mathcal{N}}{\partial t}(a,a) = \frac{\partial \mathcal{N}}{\partial a}(a,a)$$

which gives (10). Furthermore since $\frac{\partial \mathcal{N}}{\partial t}(a,a) = 0$ for all a, one has $\frac{\partial^2 \mathcal{N}}{\partial t \partial a}(a,a) + \frac{\partial^2 \mathcal{N}}{\partial t \partial t}(a,a) = 0$. So, the second-order condition is equivalent to $0 < \frac{\partial^2 \mathcal{N}}{\partial t \partial a}(a,a)$ for all a. From (23) the second-order condition requires then that for all a:

$$\dot{w}_a > 0$$

Finally, one has to verify that these local conditions are sufficient for (9). For any a and any $t \neq a$ there exists $\theta \in (0,1)$ such that for $\hat{t} = \theta a + (1-\theta)t$

$$\mathcal{N}(a,a) - \mathcal{N}(a,t) = \frac{\partial \mathcal{N}}{\partial t}\left(a,\hat{t}\right) \cdot (a-t)$$

Provided that for all t, $\dot{w}_t > 0$, one has $\frac{\partial^2 \mathcal{N}}{\partial a \partial t}(a,t) > 0$ and therefore $\frac{\partial \mathcal{N}}{\partial t}(a,t)$ is increasing in a. Since $\frac{\partial \mathcal{N}}{\partial t}\left(\hat{t},\hat{t}\right) = 0$, this implies that $\frac{\partial \mathcal{N}}{\partial t}\left(a,\hat{t}\right) \gtreqless 0$ if $a \gtreqless \hat{t}$, that is if $a \gtreqless t$. Hence $\frac{\partial \mathcal{N}}{\partial t}\left(a,\hat{t}\right) \cdot (a-t) > 0$ and $t = a$ is a global maximum for $t \to \mathcal{N}(a,t)$.

VI.2 The first-order conditions of the optimisation problem

We solve problem (14) in two steps. First we solve it given b and a_d. Second we choose the optimal values of b and a_d. Given b and a_d, we define the hamiltonian as:

$$\mathbb{H}_a = \left\{\Phi(\Sigma_a + b) + \lambda \cdot Y_a(w_a) - \lambda \cdot \Sigma_a\right\} f(a) + q_a \cdot \frac{1-\gamma}{\gamma}\left(\frac{1}{a-w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right)\Sigma_a \tag{24}$$

where λ is the Lagrange multiplier of the budget constraint and q is the co-state variable. The necessary conditions are:

$$\lambda \cdot \frac{\partial Y_a}{\partial w_a} \cdot f(a) + q_a \cdot \Sigma_a \cdot \frac{1-\gamma}{\gamma (a-w_a)^2} = 0 \tag{w_a}$$

[10] It is here assumed that the mechanism $a \to (w_a, x_a)$ is differentiable so $\dot{w}_a$ exists.

The co-state variable evolves according to

$$-\dot{q}_a = \left\{\Phi'_a - \lambda\right\} f(a) + q_a \frac{\dot{\Sigma}_a}{\Sigma_a} \tag{Σ_a}$$

and the transversality equations are:

$$q_{a_d} \cdot [\Sigma_{a_d} - d] = 0 \qquad q_{a_1} = 0$$

As usual, q_a is the shadow cost in terms of the social welfare of a marginal increase of Σ_a. Let $Z_a = q_a \Sigma_a$. The condition over Σ_a implies:

$$-\dot{Z}_a = \left\{\Phi'_a - \lambda\right\} \Sigma_a f(a) \tag{25}$$

So, together with the transversality condition:

$$Z_a = \int_a^{a_1} \left\{\Phi'_t - \lambda\right\} \Sigma_t \cdot f(t) \cdot dt \tag{26}$$

Z_a corresponds to the opposite of the integral in the right hand side of equation (15). Since $Z_a \cdot \frac{d\Sigma_a}{\Sigma_a} = q_a \cdot d\Sigma_a$, Z_a stands for the shadow cost of a relative marginal increase of Σ_a.

The first order condition w.r.t. w_a can be written as

$$\lambda \cdot \frac{\partial Y_a}{\partial w_a} \cdot f(a) = -Z_a \frac{1-\gamma}{\gamma (a - w_a)^2} \tag{27}$$

which, together with the expression for Z_a gives (15). Furthermore, from (3), we get

$$\frac{\partial Y_a}{\partial w_a} = \frac{\gamma \cdot a - w_a}{\gamma} (a - w_a)^{\frac{1}{\gamma} - 2} \cdot \kappa_a^{\frac{\gamma - 1}{y}} \tag{28}$$

Furthermore, the conditions with respect to b and a_d writes (see Leonard and Van Long (1992)).

$$\int_{a_0}^{a_1} \left(\Phi'_a - \lambda\right) f(a)\, da = 0 \tag{29}$$

$$\Phi(b + d) f(a_d) - \mathbb{H}_{a_d} \leq 0 \qquad \text{with} = \text{if } a_d > a_0 \tag{30}$$

VI.3 Proof of Proposition 1

The transversality condition $q_{a_1} = 0$ implies that the integral in the right hand side of equation (15) is nil for $a = a_1$, so $w_{a_1} = w^*_{a_1}$ and $L_{a_1} = L^*_{a_1}$.

VI.4 Proof of Proposition 2

Since Σ_a is increasing in a, Φ'_a is decreasing in a. Equation (29) implies that there exists a unique $\hat{a}$ such that $\Phi'_{\hat{a}} = \lambda$. For $t < \hat{a}$, we get $\Phi'_t - \lambda > 0$ and $\Sigma_t < \Sigma_{\hat{a}}$ and for $t > \hat{a}$, we get $\Phi'_t - \lambda < 0$ and $\Sigma_t > \Sigma_{\hat{a}}$. Therefore, for any $t \neq \hat{a}$, we have $\left(\Phi'_t - \lambda\right) \Sigma_t < \left(\Phi'_t - \lambda\right) \Sigma_{\hat{a}}$. Using this inequality and equations (29) and (26), we obtain

$$\begin{aligned}
Z_a &= \int_a^{a_1} \left(\Phi'_t - \lambda\right) \Sigma_t \cdot f(t) \cdot dt < \int_a^{a_1} \left(\Phi'_t - \lambda\right) \Sigma_{\hat{a}} \cdot f(t) \cdot dt \\
&< \Sigma_{\hat{a}} \left[\int_a^{a_1} \Phi'_t \cdot f(t) \cdot dt - \lambda(1 - F(a))\right] = \Sigma_{\hat{a}} \cdot (1 - F(a)) \cdot \left\{\mathbb{E}_f \left[\Phi'_t \,|t \geq a\right] - \lambda\right\} \\
&< \Sigma_{\hat{a}} \cdot (1 - F(a)) \cdot \left\{\mathbb{E}_f \left[\Phi'_t \,|t \geq a\right] - \mathbb{E}_f \left[\Phi'_t \,|t \geq a_0\right]\right\}
\end{aligned}$$

by equation (29). Therefore, Z_a is negative for all $a < a_1$ because Φ'_t is decreasing with respect to the ability. From (27), we obtain $\partial Y_a/\partial w_a > 0$, so from (28), we have over-employment for all types $a < a_1$.

VI.5 Proof of Proposition 3

We will prove that every individual that participates in our second best optimum also participates at the *laissez-faire* equilibrium. From the first order condition on a_d we have:

$$0 \geq f(a_d)\,\Phi(b+d) - \{\Phi(\Sigma_{a_d}+b) + \lambda \cdot (Y_{a_d} - \Sigma_{a_d})\}\, f(a_d) - Z_{a_d}\frac{1-\gamma}{\gamma}\left[\frac{1}{(a_d - w_{a_d})} - \frac{\dot{\kappa}_{a_d}}{\kappa_{a_d}}\right]$$

Since Z_a is always negative for $a < a_1$, the transversality condition on a_d implies that $\Sigma_{a_d} = d$. Rearrangering the terms, we get:

$$Y_{a_d} - \Sigma_{a_d} \geq -Z_{a_d} \cdot \frac{\dot{\Sigma}_{a_d}}{\Sigma_{a_d}} \cdot \frac{1}{\lambda f(a_d)} > 0 \quad \text{so} \quad \Sigma_{a_d} < Y_{a_d} \tag{31}$$

The Hosios condition implies that net output is maximized at the *laissez-faire* equilibrium. We therefore have $Y^*_{a_d} \geq Y_{a_d}$. Since there are no taxes at the *laissez-faire*, we have $Y^*_{a_d} = \Sigma^*_{a_d}$. Thus $\Sigma^*_{a_d} > \Sigma_{a_d}$. This implies that if the participation constraint is satisfied at the second best equilibrium, it is also satisfied at the *laissez-faire* equilibrium.

VI.6 Proof of Proposition 4

¿From (31), one has $w_{a_d} > x_{a_d}$ so $T_{a_d} + b > 0$ and $b > -T_{a_d}$. $-T_{a_d}$ should be understood as the in-work benefits for the least skilled workers who participate to the labour market.

VI.7 Proof of Proposition 5

After expressing a as a function of w_a, $T_a + b$ and T'_a from (6), one has:

$$a - \frac{w_a}{\gamma} = \frac{1-\gamma}{\gamma} \cdot w_a \cdot \frac{T'_a - \dfrac{T_a + b}{w_a}}{1 - T'_a} \tag{32}$$

Proposition 2, implies that the left hand side of (32) is positive. Since the marginal tax rate T'_a is lower than 1, we get:

$$T'_a > \frac{T_a + b}{w_a} > \frac{T_a}{w_a} \tag{33}$$

Furthermore, the derivative of average tax rate w.r.t. the gross wage is:

$$\frac{\partial\left(T(w_a)/w_a\right)}{\partial w_a} = \frac{T'(w_a) - \frac{T(w_a)}{w_a}}{w_a}$$

So the average tax rate is increasing in wages.

VI.8 Intensive margin

The derivative of the net output with respect to working hours and the wage are respectively:

$$\left.\frac{\partial Y_a}{\partial w_a}\right|_{h_a} = \frac{\gamma \cdot a \cdot h_a + (1-\gamma)\, v\,(h_a) - w_a}{\gamma} \left(ah_a - w_a\right)^{\frac{1}{\gamma}-2} \cdot \kappa_a^{\frac{\gamma-1}{y}}$$

$$\left.\frac{\partial Y_a}{\partial h_a}\right|_{w_a} = \frac{(w_a((1-\gamma)\cdot a + \gamma v'(h_a)) - \gamma v'(h_a) a h_a - a(1-\gamma) v(h_a))}{\gamma} \left(ah_a - w_a\right)^{\frac{1}{\gamma}-2} \cdot \kappa_a^{\frac{\gamma-1}{y}}$$

We now determine the derivative of the function $h_a(w_a, \Sigma_a)$. Define

$$\Xi\,(h_a, w_a, \Sigma_a, a) = \frac{1-\gamma}{\gamma} a \cdot \Sigma_a - \kappa_a^{\frac{\gamma-1}{y}} v'\,(h_a)\,(ah_a - w_a)^2$$

The first order condition of the bargaining problem (17) with respect to hours implies $\Xi(h_a, w_a, \Sigma_a, a) = 0$, which implictly defines $h_a(w_a, \Sigma_a)$. Hence:

$$\left.\frac{\partial h_a}{\partial w_a}\right|_{\Sigma,a} = \frac{v'\,(h_a)}{v''\,(h_a)\,(a \cdot h_a - w_a)\,\gamma + v'\,(h_a)\,a}$$

$$\left.\frac{\partial h_a}{\partial \Sigma_a}\right|_{w,a} = \frac{-\,(1-\gamma)\,a}{\kappa_a^{\frac{\gamma-1}{y}}\,(a \cdot h_a - w_a)^{\frac{1}{\gamma}-1}\left[v''\,(h_a)\,(a \cdot h_a - w_a)\,\gamma + v'\,(h_a) \cdot a\right]}$$

We define the hamiltonian of (20) as:

$$\mathbb{H} = \{\Phi\,(\Sigma_a + b) + \lambda \cdot Y_a\,(w_a, h_a\,(w_a, \Sigma_a)) - \lambda \cdot \Sigma_a\}\, f\,(a) + q_a \cdot \frac{1-\gamma}{\gamma}\left(\frac{h_a}{ah_a - w_a} - \frac{\dot{\kappa}_a}{\kappa_a}\right)\Sigma_a \tag{34}$$

where λ is the Lagrange multiplier of the budget constraint and q is the co-state variable. The necessary conditions are (see Leonard and Van Long (1992)):

$$\int_{a_0}^{a_1} \left(\Phi_a' - \lambda\right) f\,(a)\,da = 0 \qquad (b)$$

$$\lambda \cdot \left(\frac{\partial Y_a}{\partial w_a} + \frac{\partial Y_a}{\partial h_a} \cdot \frac{\partial h_a(w_a, \Sigma_a)}{\partial w_a}\right) \cdot f\,(a) + Z_a \cdot \frac{1-\gamma}{\gamma} \frac{h_a + \frac{\partial h_a(w_a,\Sigma_a)}{\partial w_a} w_a}{(ah_a - w_a)^2} = 0 \qquad (w_a)$$

$$\Phi\,(b+d)\, f\,(a_d) - \mathbb{H}(a_d) \leq 0 \qquad (a_d)$$

The co-state variable evolves according to

$$-\dot{q}_a = \left\{\Phi_a' - \lambda\right\} f\,(a) + q_a \frac{\dot{\Sigma}_a}{\Sigma_a} + \frac{\partial h_a(w_a, \Sigma_a)}{\partial \Sigma_a} \cdot \left[\lambda \frac{\partial Y_a}{\partial h_a} f(a) - Z_a \frac{1-\gamma}{\gamma} \frac{w_a}{(ah_a - w_a)^2}\right] \qquad (S_a)$$

and the transversality equations are:

$$q_{a_d} \cdot [\Sigma_{a_d} - d] = 0 \qquad q_{a_1} = 0$$

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